



Modified S-Control Chart for Specified value of Cp

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Abstract: In this paper, a modified S control chart for specified value of the process capability index Cp is proposed. This control chart is applicable for normally distributed data with bilateral specification and also can be used as a unified approach of control chart and capability index. For illustration purpose the performance of this control chart is assessed through a simulated normally distributed data. From the simulation result it is observed that the data persist after elimination of points above and below the upper and lower control limits of the proposed control chart respectively produce the expected capability index value.

Keywords: Capability indices, Variable, Control chart, Normal Distribution, Upper Specification limit(USL), Lower specification limit(LSL).

I. INTRODUCTION

Control chart is an on-line process monitoring technique widely used to detect the occurrence of assignable causes. W. A. Shewhart initialized the concept of statistical process control. Assignable causes of variation are due to non-random causes and are rectifiable. A typical control chart consists of three horizontal lines. They are, a central line (CL), Upper control limit (UCL), Lower control limit (LCL). The S chart is used to monitor the variability of a process. Chen et. al (2007), constructed the control chart of unilateral specification index Cpl and Cpu to monitor the stability of process and process capability. Subramani et. al (2012) proposed $\bar{X}$ and R control chart based on process capability indices Cp and Cpk. Liaquat Ahmad et. al (2013) proposed a $\bar{X}$ control chart based on process capability index (Cp) using repetitive sampling and the performance of the proposed chart was evaluated by ARL₁ and the performance is efficient for the quick detection of false alarms. Muhammad Aslam et. al (2017), presented a mixed control chart with attribute and variable data using the process capability index Cpk. O.A. Adeoti et. al (2017), proposed a process capability index based control chart using Downton estimator with a specified Cp value. O.A. Adeoti et. al (2017), presented a new process capability index control chart for monitoring the process mean using repetitive sampling. Souha Ben Amara et. al (2016), investigates the effects of measurement system variability evaluated by measurement system discrimination ratio (DR) on the process capability indices Cp and Cpm, on the expected non conforming units of product per million (ppm), on the expected mean value of the Taguchi loss function (E(Loss)) and on the Shewhart charts properties. In this paper, as a unified approach of control chart and capability index a modified S control chart for specified value of the process capability index Cp is proposed with operating procedure. This chart is applicable for normally distributed data with bilateral specification. A simulated normally distributed data with mean $\mu = 25$, standard deviation $\sigma = 1$ and parameters, USL = 28, LSL = 22 and T = 25 is used to assess the performance of this control chart.

II. BASIC PROCESS CAPABILITY INDICES

The four basic capability indices Cp, Cpk, Cpm and Cpmk were developed for measuring the capability of a process in meeting specification under the assumption of normal distribution. The index Cp only measures the variability of the process and is given by,

$$C_p = \frac{USL - LSL}{6\sigma}$$

By replacing σ with the unbiased estimator $\hat{\sigma} = \frac{\bar{s}}{c_4}$

$$\hat{C}_p = \frac{USL - LSL}{6 \frac{\bar{s}}{c_4}} \quad (1)$$

Kane (1986) discussed the index Cpk, which attempts to measure the variability and shift in process mean simultaneously. Thus Cpk with the unbiased estimator $\hat{\sigma} = \frac{\bar{s}}{c_4}$ is given by,

$$\hat{C}_{pk} = \min \left\{ \frac{USL - \bar{X}}{3 \frac{\bar{s}}{c_4}}, \frac{\bar{X} - LSL}{3 \frac{\bar{s}}{c_4}} \right\} \quad (2)$$

According to Taguchi (1988) there is a loss to society associated with missing the target and developed the concept of the quadratic loss function. Chan et al. (1988) introduced so called Taguchi capability index Cpm considering specification range, process variation and variation of mean from the target. Thus index Cpm with the unbiased estimator $\hat{\sigma} = \frac{\bar{s}}{c_4}$ is given by,

$$\hat{C}_{pm} = \frac{USL - LSL}{6 \sqrt{\left(\frac{\bar{s}}{c_4}\right)^2 + (\bar{X} - T)^2}} \quad (3)$$

Pear et al. (1992) proposed the index C_{pmk} by modifying the numerator and denominator of the index C_{pm}.

$$\hat{C}_{pmk} = \frac{\min(USL - \bar{X}, \bar{X} - LSL)}{3 \sqrt{\left(\frac{\bar{s}}{c_4}\right)^2 + (\bar{X} - T)^2}} \quad (4)$$

III. Modified S - Chart for Specified Cp value:

In this section, a Modified S - Chart for any Specified value of Cp is discussed. According to Montgomery (2009), the control limits for Shewhart S control chart by replacing σ with the unbiased estimator $\hat{\sigma} = \frac{\bar{s}}{c_4}$ is given by,

$$UpperControlLimit = \bar{s} + 3 \left(\frac{\bar{s}}{c_4} \right) \sqrt{1 - C_p^2} = B_4 \bar{s} \quad (5)$$

$$CentrlLine = \bar{s} \quad (6)$$

$$LowerControlLimit = \bar{s} - 3 \left(\frac{\bar{s}}{c_4} \right) \sqrt{1 - C_p^2} = B_3 \bar{s} \quad (7)$$

Where,

$$\bar{s} = \frac{\sum_{i=1}^m S_i}{m}$$

$$S_i = \sqrt{\frac{1}{n-1} \sum_{j=1}^n (X_{ij} - \bar{X}_i)^2}$$

$$c_4 = \left(\frac{2}{n-1} \right)^{1/2} \left(\frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)} \right)$$

Since Cp only measures the variability of the process, it is considered for calculating standard deviation of a process for any specified value of Cp. Thus for any specified value of Cp, USL and LSL, the standard deviation ($\bar{s}$) is calculated from (1) and is given by,

$$\bar{s} = \frac{c_4(U - L)}{6 C_p}, \quad C_p \neq 0 \text{ and } \frac{c_4(U - L)}{6 C_p} \neq 0 \quad (8)$$

The proposed control chart can be used to calculate the control limits for any specified values of the capability index Cp.

A.1 Control Limits for Modified S - Chart:

The Shewhart control chart is based on the concept of 99.73% of the data lies within *Expected value* $\pm$ *3Standard Error* of the data under consideration. But capturing of 99.73% of data alone does not identify a process that satisfies customer specification. Thus $\pm 3Standard Error$ is replaced by

$\pm \left(\frac{C_p(\text{observed})}{C_p(\text{expected})} \right) Standard Error$. Hence control limits of the proposed modified S chart for any specified value

of capability index Cp is derived by substituting (8) and replacing the three standard error limit in (5), (6) and (7)

with the ratio of $C_p(\text{observed})$ and $C_p(\text{expected})$ standard error limit .

$$UCL = \bar{s} + \left(\frac{C_p(\text{observed})}{C_p(\text{expected})} \right) \left[\left(\frac{\bar{s}}{c_4} \right) \sqrt{1 - c_4^2} \right] \quad (9)$$

$$CL = \bar{s} \quad (10)$$

$$LCL = \bar{s} - \left(\frac{C_p(\text{observed})}{C_p(\text{expected})} \right) \left[\left(\frac{\bar{s}}{c_4} \right) \sqrt{1 - c_4^2} \right] \quad (11)$$

A.2 Operating Procedure:

Step-1: Calculate the average of subgroup standard deviation and calculate the estimate of population standard deviation.

Step-2: Calculate $C_p(\text{observed})$ using (1).

Step-3: Calculate the control limits using (9), (10) and (11) and construct the control chart.

Step-4: Eliminate the point's lies above and below the UCL and LCL respectively.

Step-5: Identify the data suitable in producing specified capability value.

IV. SIMULATION STUDY

The proposed control chart is assessed with a simulated normally distributed data with mean $\mu = 25$ and standard deviation $\sigma = 1$. The following parameters, $USL = 28$, $LSL = 22$ and $T = 25$ are considered for comparing Shewhart control chart and the proposed control chart. Various statistical constants of the data under consideration is as follows:

$$\bar{\bar{X}} = 24.898 ; \quad \bar{s} = 0.96999 ; \quad C_p = 0.99, C_{pk} = 0.95 \text{ and } C_{pm} = 0.98.$$

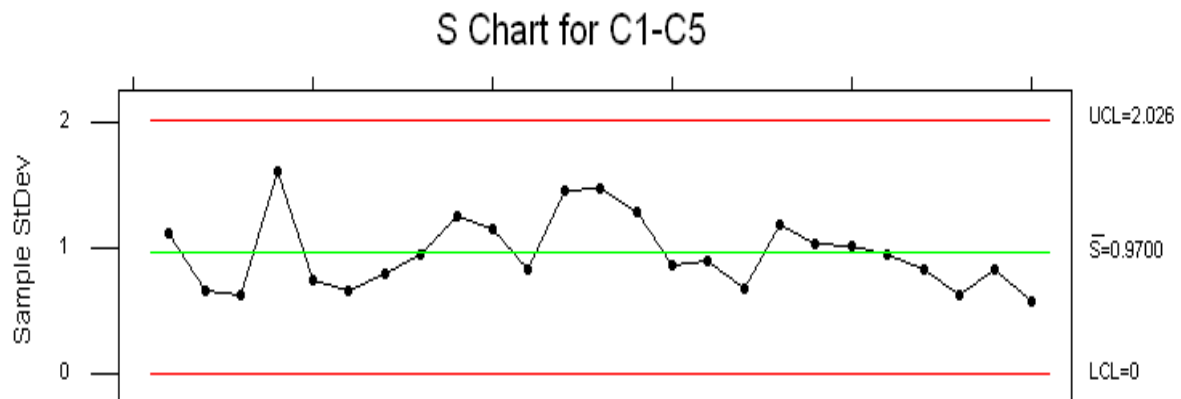


Figure 1 Shewhart X-bar and S Control Chart

Figure 1 represents the Shewhart S control chart constructed by plotting the subgroup standard deviation ($\bar{s}$) and the control limits calculated using (5), (6) and (7). The UCL is 2.026, LCL is 0 and CL is 0.9700. Since all the points are within the control limits it is inferred that the process is statistically control.

Process Capability Analysis for C1 - C5

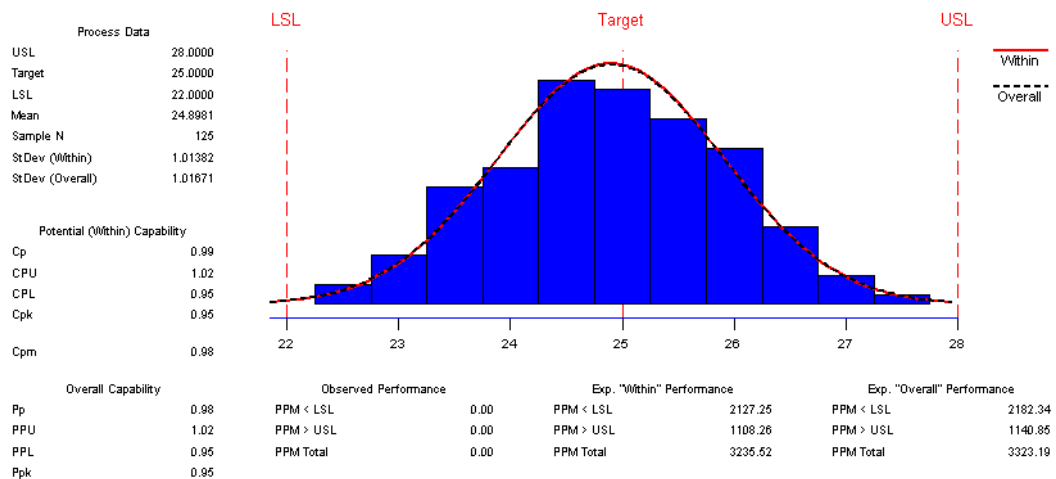


Figure 2 Capability analysis of the simulated process

Figure 2 represents the capability analysis of the sample of size 25 with subgroup size 5. Capability indices are calculated using the statistical software MINITAB 16. Since all the capability index C_p values is less than 1, it is inferred that the process is not capable to meet the specification. Thus it is clear that even a statistically control process (figure 1) is not performing with the capability of meeting customer specification (figure 2). Hence there is a scope for developing a control chart that eliminates the points that influence in reducing the C_p value. Thus the modified S chart is constructed using various C_p (expected) values 1.00, 1.33 and 2.00 and also by substituting $\bar{s}$ from (8) in (9), (10) and (11).

Control Chart	C_p	UCL	LCL	CL	Range of Control Limits
Modified S – Chart	1	1.292	0.588	0.940	0.703
	1.33	0.906	0.508	0.707	0.398
	2	0.558	0.382	0.470	0.176
Shewhart S Control Chart	-	2.026	0	0.97	2.026

Table 1

UCL, LCL and CL of Shewhart S Control Chart and modified S chart with various specified value of C_p are presented in Table 1. For Shewhart S Control Chart, $UCL = 2.026$, $LCL = 0$ and $CL = 0.9700$. For modified S chart, when the C_p (expected) = 1.00, the standard deviation ($\bar{s}$) calculated using (8) is 0.940 and $UCL = 1.292$, $LCL = 0.588$ and $CL = 0.940$. When the C_p (expected) = 1.33, the standard deviation ($\bar{s}$) calculated using (8) is 0.707 and $UCL = 0.906$, $LCL = 0.508$ and $CL = 0.707$. When the C_p (expected) = 2.00, the standard deviation ($\bar{s}$) calculated using (8) is 0.470 and $UCL = 0.558$, $LCL = 0.382$ and $CL = 0.470$. It is clear that the range of control limits of modified S chart is decreased when C_p value is increased and also the range of control limits of modified S chart is less than Shewhart S Control Chart.

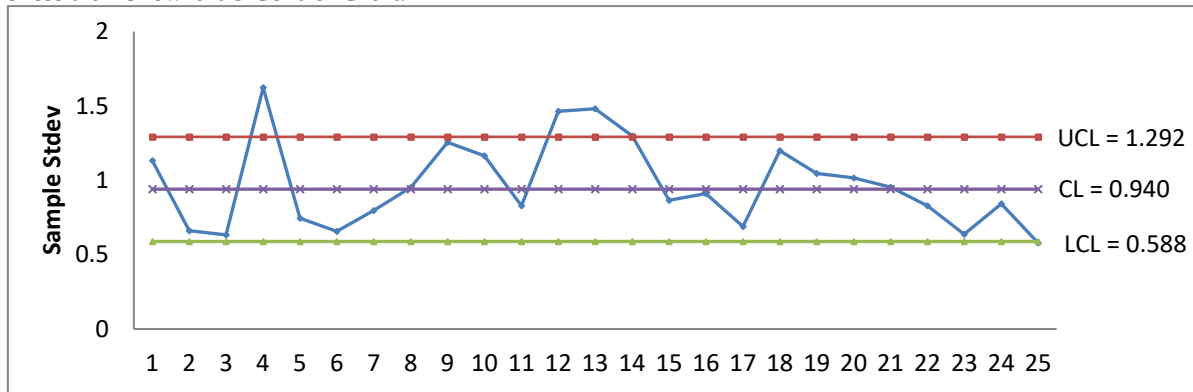


Figure 3 Modified S - Chart for Specified C_p value of 1.00

From figure 3, since the control limits are calculated for C_p (expected) = 1.00, it is observed that five points 4, 12, 13, 14 and 25 falls outside upper and lower control limits. After removal of out of control points, the control limits of Modified S chart is calculated and presented in table 2.

C_p	Control Limits for Modified S – Chart after removal of out of control points			
	UCL	LCL	CL	Range of Control Limits
1	1.323	0.557	0.940	0.767

Table 2

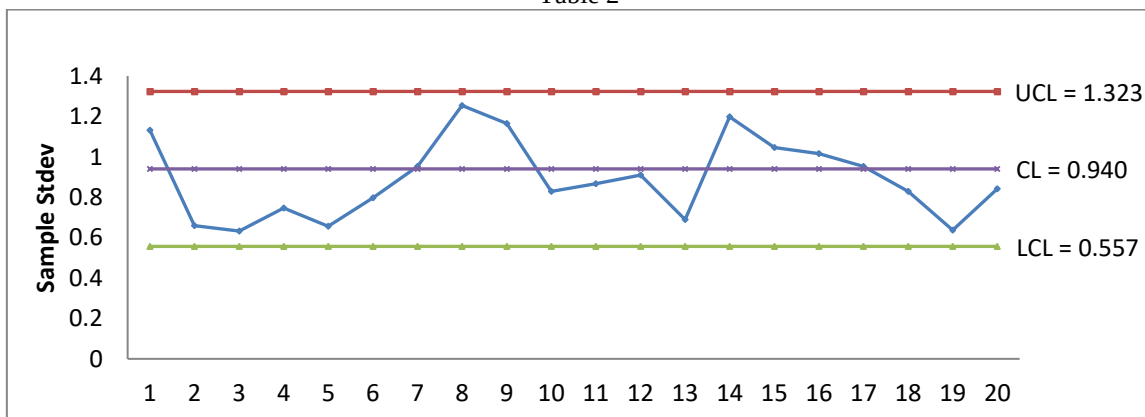


Figure 4 Modified S chart after removal of out of control points (C_p (expected) = 1.00)

Process Capability Analysis for C1 - C5

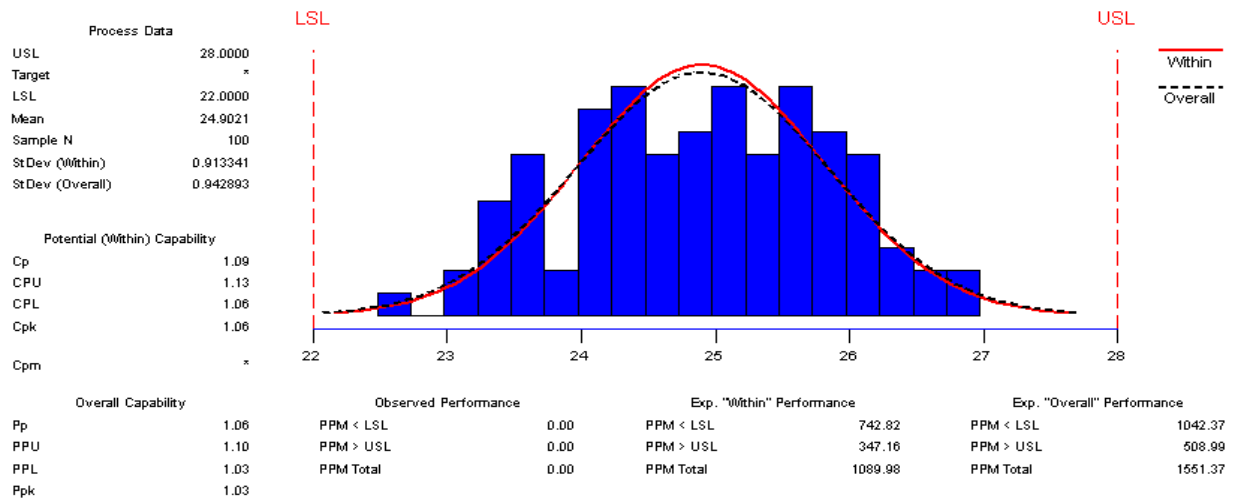


Figure 5 Capability analysis of the simulated process after removal of out-of control points using Modified S - Chart for Specified Cp value of 1.00

Figure 4 represents Modified S chart ($C_p(\text{expected}) = 1.00$) after removing out of control points. The $UCL=1.323$, $LCL= 0.557$ and $CL= 0.940$. It is clear that all the points fall within the control limits and also from figure 5, the capability index C_p value has increased to 1.09.

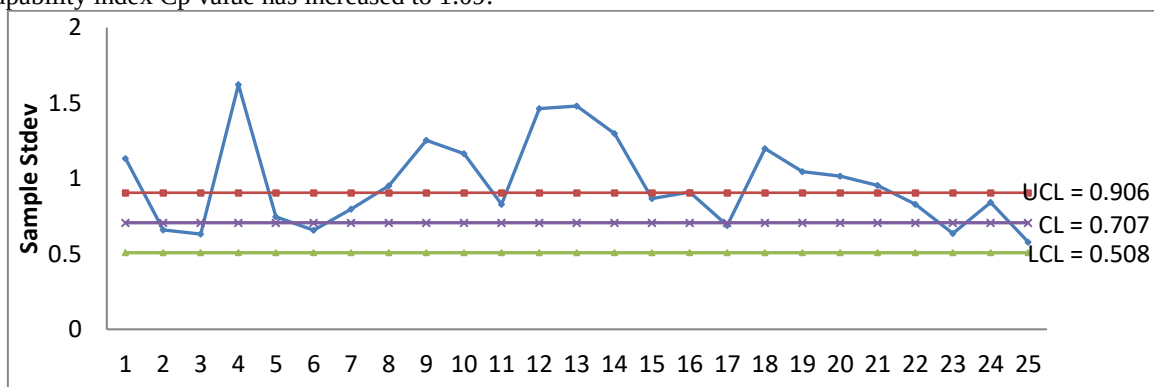


Figure 6 Modified S - Chart for Specified Cp value of 1.33

Cp	Control Limits for Modified S – Chart after removal of out of control points			
	UCL	LCL	CL	Range of Control Limits
1.33	0.971	0.443	0.707	0.528421335

Table 3

Figure 6 represents the Modified S chart with $C_p(\text{expected}) = 1.33$ and it is observed that thirteen points 1, 4, 8, 9, 10, 12, 13, 14, 16, 18, 19, 20 and 21 lies outside the upper and lower control limits. After removal of out of control points, the control limits of Modified S chart is calculated and presented in table 3.

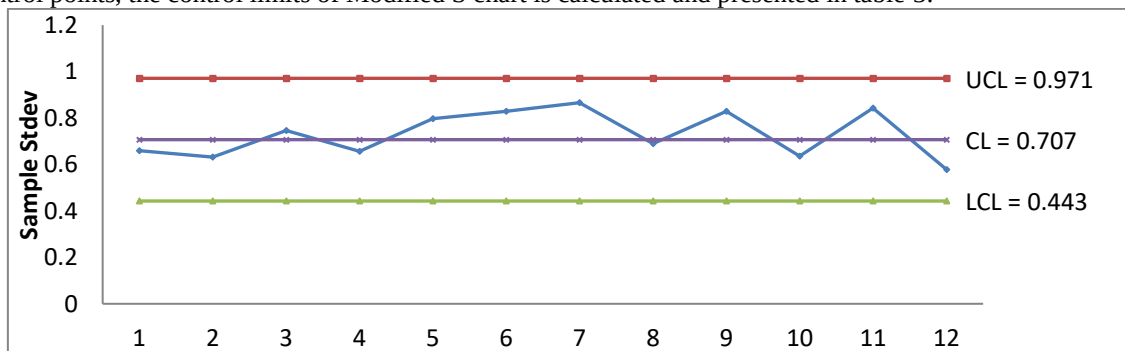


Figure 7 Modified S chart after removal of out of control points ($C_p(\text{expected}) = 1.33$)

Process Capability Analysis for C1 - C5

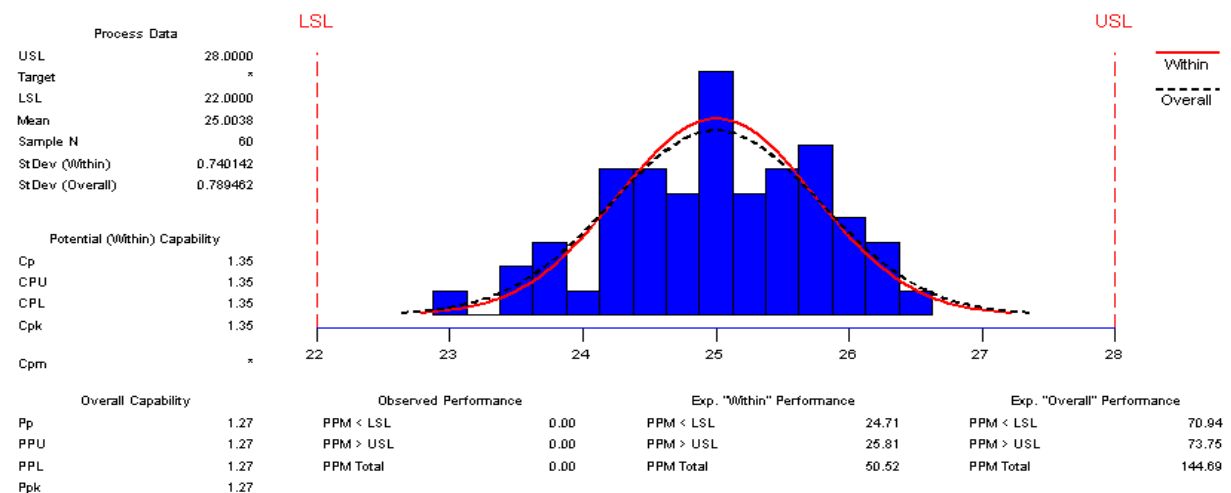


Figure 8 Capability analysis of the simulated process after removal of out-of control points using Modified S - Chart for Specified Cp value of 1.33

Figure 7 represents Modified S chart ($C_p(\text{expected}) = 1.33$) after removing out of control points. The $UCL = 0.971$, $LCL = 0.443$ and $CL = 0.707$. It is clear that all the points fall within the control limits and also from figure 8, the capability index C_p value has increased to 1.35.

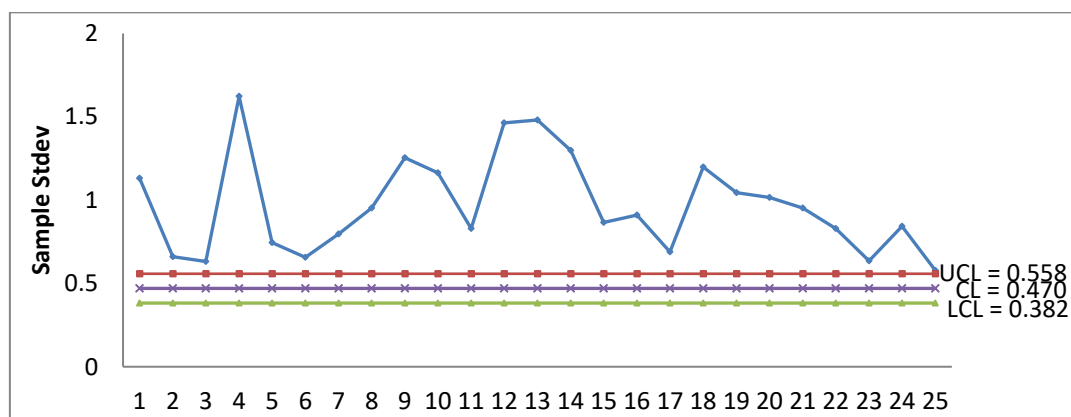


Figure 9 Modified S - Chart for Specified Cp value of 2.00

Figure 9 represents the Modified S chart with $C_p(\text{expected}) = 2.00$ and it is observed that all the points lies outside the upper and lower control limits.

V. Conclusion

Control charts are the tools used to assess the statistical control of the process. Process capability indices are used to measure the ability of the process to meet the customer specifications. Subramaniam J et. al (2012) proposed a control chart that combines the two stage mechanism namely, control charts and the process capability indices into a single stage controlling mechanism. Souha Ben Amara et. al (2016), presented Control chart limits based on true process capability with consideration of measurement system error. Modified S control chart for specified value of C_p presented in this paper unifies the two stage mechanism namely, control charts and the process capability indices into a single stage controlling mechanism and also by using this proposed control chart in industry the process can be optimized for requires capability index value C_p . The proposed chart is illustrated through simulated normally distributed data which produces the expected C_p value after applying the proposed control chart. Also it is observed that the range of control limits of various specified values capability index values decreases when C_p value increases.

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