



The possibility of merging for two 0-regular graphs or null regular graphs

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Abstract: As we know that 0-regular graph is an only isolated vertex and the edge set of every 0-regular graph is empty. Let us consider N_n be the smallest 0-regular graph, where n denotes natural numbers and for smallest 0-regular graph put $n=1$. Here first we are working on the merging on two smallest similar 0-regular graphs and find out the possibility of merging for two smallest similar 0-regular graphs. Later we are working on the two important ways of the merging of two similar 0-regular graphs. In first way we are find out the possibility of merging for two similar 0-regular graphs, when n is finite natural numbers and another way when n is non terminated natural numbers.

Here first we discussed the possibility of merging for two smallest similar 0-regular graphs. Also we are proving some important results based on the merging of two 0-regular graphs. Here we are using some definitions like merging technique, operator, special vertex and some previous results for the final completion of the merging of the 0-regular graphs.

Keywords: 0-regular graph, vertex, special vertex, operator, smallest, similar.

I. Introduction

As we know that the merging is a process, which is used for converting two or more than two regular graphs into a single regular graph[1]. Basically, 0-regular graph is a smallest regular graph in the world. Simply, the set of vertices has only one vertex according to the definition of 0-regular graphs for smallest 0-regular graph N_1 and the set of the edge is empty[2]. Here first we are discussing about possibility of merging for two smallest similar 0-regular graphs and after that we prove result based on the possibility of merging for two smallest similar 0-regular graphs.

Also we are working on the N_n 0-regular graph, where n denotes the natural numbers. As we know that the set of natural numbers has non terminated numbers[3]. Then first we take n is finite and begin our task for two similar 0-regular graphs. Here we prove result for the merging of two similar 0-regular graphs for finitely many natural numbers. After that we prove the merging of two similar 0-regular graphs for the rest natural numbers. Conclusions part of this research paper show that everything about the possibility of merging of two smallest similar 0-regular graphs and rest 0-regular graphs.

II. Definition

Regular graphs: A graph in which each vertex has the same degree is a regular graph. If each vertex has degree r , the graph is regular of degree r or r -regular. Note that the null graph N_n is regular of degree 0, the cycle graph C_n is regular of degree 2, and the complete graph K_n is regular of degree $n-1$ [4].

Here, we are working on the null regular graph N_n . That means we are working on the 0-regular graph or regular graph with degree zero.

III. Descriptions

Now, we are select two smallest similar null regular graphs or smallest similar 0-regular graphs and applying all the definitions of merging for the proof. Those definitions are necessary required for the find out the possibility of merging for two regular graphs[1]. After the complete analysis, we are finding out the final solution of merging for two smallest similar 0-regular graphs or smallest similar null regular graphs.

Let $N_1 = (V_1, E_1)$ and $N_1' = (V_1', E_1')$ are two 0-regular graphs (null regular graphs), where

Table 1	
0-regular graph: $N_1 = (V_1, E_1)$	0-regular graph: $N_1' = (V_1', E_1')$
Set of vertex $V_1 = \{v_1\}$	Set of vertex $V_1' = \{v_1'\}$
Set of edge $E_1 = \phi$	Set of edge $E_1' = \phi$

Now we are drawing the figures of N_1 and N_1' :



Figure 1: $N_1 = (V_1, E_1)$ and $N_1' = (V_1', E_1')$

After these we use our definitions of the process of merging for the complete the description of the merging of two 0 -regular graphs.

Here we use two operators α and β for the store/remove the edge for the merging process[2].

According to above table and figure, it is clear that there is no edge in the both 0 -regular graphs. Therefore, we are not eligible to apply the definition of operators because $E_1 = \emptyset$ and $E_1' = \emptyset$ and also there is no special vertex in the both graphs[1].

The work of operators and special vertices are necessarily required for the final completion of the merging for two 0 -regular graphs.

But here there is no operator are work and there is on special vertices in the both graphs.

Thus the process of the merging is stop implicitly for two smallest similar 0 -regular graphs and the merging is not possible for two smallest similar 0 -regular graphs.

IV. Result

Theorem 1: Let N_1 and N_1' are two smallest similar 0 -regular graphs. Then show that $N_1 \not\ll N_1'$. Or

Show that merging is not possible for two smallest similar 0 -regular graphs.

Proof: Here we use the description part for proof the result.

According to **Table 1**, N_1 and N_1' are two smallest similar 0 -regular graphs. Since $V_1 = \{v_1\}$ is vertex of 0 -regular graph N_1 and $V_1' = \{v_1'\}$ is vertex of smallest similar 0 -regular graph N_1' . Also we know from the **Table 1**, both the smallest similar 0 -regular graphs have no edges.

Now we use **figure 1** for extend the proof of theorem.

Let α and β are two operators, where operator α is used for store/remove edge of 0 -regular graph N_1 at the time of merging and similarly operator β is used for store/remove edge of 0 -regular graph N_1' at the time of merging[2].

Now we are ready for the merging of 0 -regular graphs N_1 and N_1' .

As we know that from the definition of operator, since operators are used to store/remove the edges of given regular graphs for the merging of two regular graphs and each operator is store/remove at most one edge[2].

Here for the smallest similar 0 -regular graph N_1 , edge set $E_1 = \emptyset$. That means the edge storing operator α is not working and we are not get any special vertex from smallest similar 0 -regular graph N_1 [2]. Also remained process (when the stored edge is leaved/removed from the operator, then that edge is again come in the regular graph and one part of edge is joining to the special vertex of first regular graph[2]. Another part of edge is joining to the special vertex of second regular graph. This is half merging of two regular graphs.) is not working for smallest similar 0 -regular graph N_1 [1].

Similarly, there is no edge in smallest similar 0 -regular graph N_1' . That means $E_1' = \emptyset$. Therefore, edge storing operator β is not working[2].

Thus we are not getting any special vertex form both smallest similar 0 -regular graphs[1].

This is a contradiction. Therefore, the merging of two smallest similar 0 -regular graphs is stopped implicitly.

Thus the merging is not possible for two smallest similar 0 -regular graphs or smallest similar null regular graphs. i.e. $N_1 \not\ll N_1'$. $\square$

Note: Continuous merging is not possible for null regular graphs/ 0 -regular graphs because merging is not possible for null regular graphs/ 0 -regular graphs from the **Theorem 1**[2].

Corollary1: If $N_n = (V_n, E_n)$ and $N_n' = (V_n', E_n')$ are two similar 0 -regular graphs. Where n is finite natural numbers [3]. Then show that merging is not possible.

Proof: Let $N_n = (V_n, E_n)$ is similar 0 -regular graph, where n is finite natural numbers[3]. Then set of vertices $V_n = \{v_1, v_2, v_3, \dots, v_{n-1}, v_n\}$ and the set of edge $E_n = \emptyset$ from definition of 0 -regular graph[4].

Similarly, $N_n' = (V_n', E_n')$ is similar 0 -regular graph, where n is finite natural numbers[3]. Then set of vertices $V_n' = \{v_1', v_2', v_3', \dots, v_{n-1}', v_n'\}$ and the set of edge $E_n' = \emptyset$ from definition of 0 -regular graph[4].

Since both similar 0 -regular graphs N_n and N_n' having finitely many vertices but unfortunately edge sets of both similar 0 -regular graphs are empty. So, there is no possibility of merging for similar 0 -regular graphs N_n and N_n' from the previous **Theorem 1**.

Thus $N_n \not\ll N_n'$, where n is finite natural numbers[3]. $\square$

Corollary 2: Let $N_n = (V_n, E_n)$ and $N_n' = (V_n', E_n')$ are two similar 0 -regular graphs. Where n is non terminated natural numbers[3]. Then show that merging is not possible.

Proof: This is the extended proof of the **Corollary 1**. Here we are working on the non terminated natural numbers[3]. That means both similar 0 -regular graphs N_n and N_n' having non terminated vertices but for both graphs set of edge $E_n = \emptyset$ and set of edge $E_n' = \emptyset$. Therefore, merging is not possible for both similar 0 -regular graphs N_n and N_n' .

Thus $N_n \not\sim N_n'$, where n is non terminated natural numbers[3]. $\square$

V. Conclusion

After complete the research paper we are getting following conclusion:

1. Merging is not possible for two smallest similar **0**-regular graphs N_1 and N_1' .
2. Merging is not possible for two similar **0**-regular graphs N_n and N_n' , where n is finitely many natural numbers[3].
3. Merging is not possible for two similar **0**-regular graphs N_n and N_n' , where n is non terminated natural numbers[3].

Thus merging is not possible for **0**-regular graphs or null regular graphs.

References

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