



F1: A New Method for Transportation Problem Initial Solution

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Abstract: Transportation is a crucial factor in supply chain management. In this field transportation means distributing a certain quantity of products from a set of sources to a set of destinations to fulfill the need. Transportation decision can be considered as a tactical decision which means the decision to transportation problem is performed within short to medium period of time. In transportation problem, the problem is solved by first determining the initial solution. In this paper, new methods F1 that based on minimizing weighted cost, is proposed. The proposed method than is compared with three popular methods, i.e. Northwest Corner Method (NWC), Least Cost Method (LC), and Vogel Approximation Method (VAM). Calculation results showed that the method is better than NWC and LC. The result is comparable with VAM.

Keywords: Transportation problem; initial solution; distribution; algorithm; VAM, LC, NWC

I. Introduction

In supply chain management (SCM), transportation means moving product(s) from one location to another location along its supply chain network. Transportation shifts raw materials from suppliers to manufacturers and hauls finish products from manufacturers to distributors, retailers and to end consumers. Transportation is an important part in SCM since it is rarely that products are produced and consumed at the same area. In the case where there occurs a miss match unit, transportation will also flow at reverse direction from end customers to manufacturers. According to [1], in America transportation constitutes for more than 10 % of its GDP in 2002 and employs almost 20 million people. This number consistently remains stable to the recent year [2]. In New Zealand, a country with a less land than America, transportation contribution to GDP is around 4.25%. This number is consistently the same for 5 consecutive years of 2012 to 2016 [3]. Almost the same percentage as New Zealand, on 2011 Canada transportation sector contributes of 4.2% to her GDP. This value is comparable to \$53 billion [4].

One popular method to solve transportation problem is modeling the problem as a linear programming model and then solved by Simplex algorithm. A generic LP model for transportation problem is formulated as follows:

$$\text{Min} \quad Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \quad (1)$$

$$\text{ST} \quad \sum_{j=1}^n x_{ij} \leq a_i \quad \text{for } \forall i = 1, 2, \dots, m \quad (2)$$

$$\sum_{i=1}^m x_{ij} \geq b_j \quad \text{for } \forall j = 1, 2, \dots, n \quad (3)$$

$$x_{ij} \geq 0 \quad \text{for } \forall i, j \quad (4)$$

x_{ij} is the problem decision variable which indicates the number of units sent from source i to fulfill the need at destination j . The model basically seeks to find how many units from supplier i should be sent to fulfill demand

at destination j in order to find the least cost hauling them. As the number of supplier m and/or the number of destination n increases, the size of the problem increases tremendously. This makes the model extremely difficult to solve. Based on its complexity, transportation problem belongs to NP problem [5], [6], [7] which could not be solved in polynomial time. Fortunately, there exists a special structure (called as for transportation algorithm) that enables to solve the problem more efficient without sacrificing the accuracy of the result. Transportation algorithm, in general, consists of two parts namely initial solution and optimal solution. In this matter, in general, any initial solutions that do not violate the constraints can be used to reach the optimal solution. However, a better initial solution will indeed improve the efficiency in searching the optimal solution.

Solving transportation problem thru transportation algorithm is using a special format, i.e. Transportation Table. The table is a matrix that displays systematically all sources and destinations and their related demand and supply. It also includes costs to haul item(s) from each source to each corresponding destination. A generic transportation table is shown at Figure 1 below. Figure 1 shows that the table is for a problem with m sources (A, B, C, ..., m) with their corresponding supplies (S_A , S_B , S_C , ..., S_m) to n destinations (1, 2, 3, ..., n) with their correspondence demands (D_1 , D_2 , D_3 , ..., D_n). From each source to each destination there is a cost C_{SD} .

Figure 1 Generic transportation table

		Destination j					Supply
		1	2	3		N	
Source i	A	C_{A1}	C_{A2}	C_{A3}	...	C_{An}	S_A
	B	C_{B1}	C_{B2}	C_{B3}	...	C_{Bn}	S_B
	C	C_{C1}	C_{C2}	C_{C3}	...	...	S_C
	...	...	...	...	...	...	
	m	C_{m1}	C_{m2}	C_{m3}	...	C_{mn}	S_m
Demand		D_1	D_2	D_3	...	D_n	

Many methods to find transportation problem initial solution have been proposed. To name some, they are North West Corner Method, Least Cost Method, Column Minimum Method, and Vogel's Approximation Method [8, 9, 10, 11]. Among these four methods, North West Corner Method (NWC) is the simplest method to used. NWC is started by fulfilling demand at destination 1(column 1) from supply 1 at row 1 regardless the cost C_{A1} is big (this resembles the direction of North West). The rest demands are fulfilled according to the available supply accordingly. This weakness leads to the NWC's initial solution result most of the time is not that good. Utilizing NWC to get the initial solution will end up with many iteration to get optimal solution. Least Cost Method (LC) starts to fill demand with the least C_{SD} . LC will give a better initial solution than NWC but with cost of sacrificing the simplicity of NWC. Column Minimum Method is a variant of LC that assigns demand with the least cost on a certain column. Vogel's Approximation Method (VAM) so far in most cases is the best method for finding the initial solution. But, this method suffers with a drawback. VAM is in fact the most complex method to used [12]. It also takes the longest solution time compare to NWC and LC. VAM is basically a penalty method that calculated by subtracting the lowest cost of each row (column) to the next smallest cost at that row (column). Some variants of VAM have been developed. [10] used penalty as the different between the maximum and the minimum costs of each corresponding row and column. A more recent method has been proposed by [13].

II. Proposed F1 Method

Existing initial solution transportation methods basically manipulate costs c_{ij} one way or another to find the total cost. The proposed F1 method was developed based on the fact that the objective function of the problem is to minimize the total weighted cost as written mathematically on Equation (1). The equation says that the objective function is the product of two components, namely cost to send a supply from source i to destination j (C_{ij}) and its correspondence quantity hauled (x_{ij}). The cost C_{ij} is being weighted by the quantity x_{ij} . This observation leads to the idea that to reach the minimum objective function, it is more appealing if both components are taken into consideration simultaneously instead of just considering the cost C_{ij} .

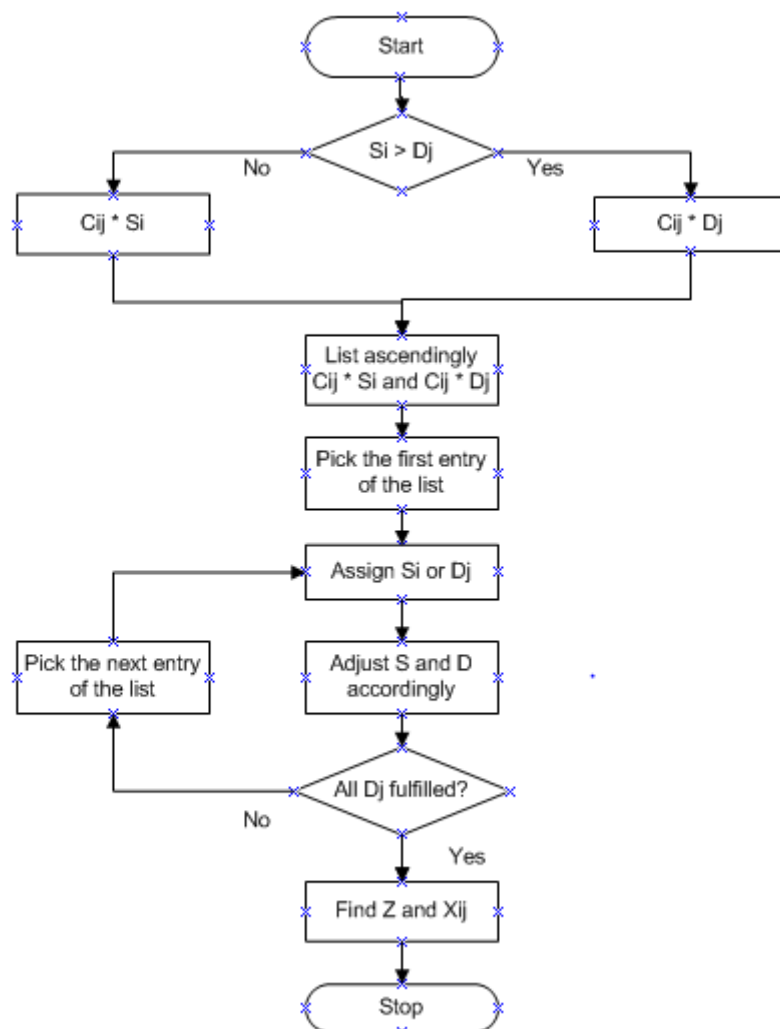
Second observation, in transportation problem, the item to be carried is not just from one source to one destination, but from m sources to n destinations. It makes more sense to choose a route with cost a bit higher to haul a less item to a route with a less cost to move a more item. In this case, if $c_{ij} < c_{kl}$ and $x_{ij} > x_{kl}$, but if $c_{kl} * x_{kl}$ is bigger than $c_{ij} * x_{ij}$ it is preferable to choose $c_{kl} * x_{kl}$ route than $c_{ij} * x_{ij}$. In many cases in transportation, the item x_{ij} is more valuable than the cost c_{ij} . These instances also support the idea to include x_{ij} to the calculation too.

Based on those two insights, proposed F1 method is as follows:

1. Calculate weighted cost by multiplying cost and its corresponding supply or demand
2. List ascendingly the calculation results
3. Pick the first entry on the list, assign as much as possible available supply to fulfill demand
4. Adjust the supply and demand accordingly
5. Use the second entry on the list to fill the next demand
6. Repeat step 4 until all demands are fulfilled
7. Calculate the total cost
8. Stop

Figure 2 shows the F1 method in flow chart format:

Figure 2 F1 method flow chart



A. Numerical Example

Consider a typical transportation problem as follow: MG Auto has three production facilities located at Los Angeles, Detroit, and New Orleans and four distribution centers at Denver, Des Moines, Pittsburgh, and Miami. The capacity of those three plants during the next quarter is 5, 2, and 3 cars, respectively. While demand at the four centers is 3, 3, 2, and 2 cars, respectively. Transportation cost per car from Los Angeles to Denver, Des

Moines, Pittsburgh, and Miami, from Detroit to Denver, Des Moines, Pittsburgh, and Miami, and from New Orleans to Denver, Des Moines, Pittsburgh, and Miami is 3, 7, 6, 4, 2, 4, 3, 2, 4, 3, 8, and 5, respectively. The questions to answer are what is the minimum cost to allocate products of those three plants to fulfill the demands and what are the allocations?

B. Solution with Proposed F1 Method

Applying step 1 and 2 of the F1 end ups with ascending list as: 4, 4, 6, 8, 8, 9, 9, 10, 12, 12, 16, 21. Applying step 3 to 6 of the algorithm, the initial solution of the problem shown as transportation table format displayed on Figure 3.

Figure 3 F1 transportation table depicts initial solution results

		Destination								Supply
		Denver		Des Moines		Pittsburgh		Miami		
Source	LA	9		21		12		8		5
			3				2			
	DTW	4		8		6		4		2
								2		
	NO	12		9		16		10		3
					3					
Demand		3		3		2		2		

From Figure 3 above, LA plant will send its three products to fill Denver demand and the rest 2 are sent to Pittsburgh. Detroit plant is solely to serve Miami while all New Orleans products will be shipped to Des Moines. The total cost Z of fulfilling all demands is calculated using equation 1. In this case the value is the summation of weighted cost of 9, 12, 4, and 9. The value is 34. Initial solution cost using NWC, LC, and VAM is 118, 36, and 32, respectively. The result shows that the proposed F1 is better than NWC and LC while it gives comparable result with VAM.

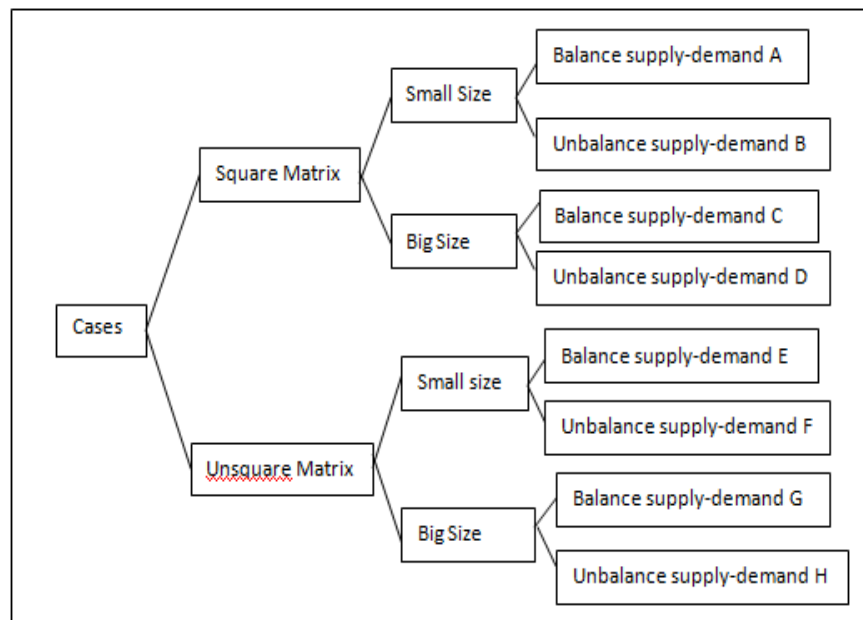
C. Simulation and Result

To compare F1 to NWC, LC, and VAM, all the algorithms were coded with C++ and tested for various transportation problems with different size and setting. Three factors, i.e. number of suppliers and destinations, size of supply and demand, and total number of items supplied and requested, were considered. Square matrix is the case when the number of suppliers is the same as the number of destinations. Small size matrix is representing the case with less than 6 suppliers or destinations. While unbalance supply – demand is the case where the total item provided by all suppliers is less or more than the total item requested by all destinations. Figure 4 shows tree diagram for cases being tested. For all cases, cost to send an item from supplier i to destination j (c_{ij}) is set randomly. All cases were run on personal laptop. Table 1 is summarizing the results

Since transportation problem objective is to minimize costs the best value is the smallest one. Simulation results displayed on Table 1 clearly show that among the four algorithms being tested, NWC is the method that gives the least optimum result. The results also show that there is no algorithm that guarantees giving the best results for all settings.

Comparing F1 and LC, among 32 cases being run there are 16 cases (50%) where F1 beats LC, 8 cases (25%) where F1 and LC give the same solutions and the rest 8 cases (25%) LC provides better solution than F1. These results show that F1 is most of the time gives better solution. This finding is supporting the idea of F1 that say to find the minimum total cost, the number of item to be transported x_{ij} should also be considered not just the transportation cost c_{ij} .

Figure 4 Tree diagram of tested cases



Comparing F1 with VAM, 6 cases where F1 are better than VAM, 10 cases end up with the same or close by and the rest 16 cases (50%) where VAM is better than F1. These results show that VAM is so for better than F1. But, the results also open up a room for improvement

Table 1 Simulation results

Case	No	Objective Function Z				Z Ranking*		
		NWC	LC	VAM	F1	NWC vs F1	LC vs F1	VAM vs F1
A	1	96,750	83,250	83,000	83,250	F1	LC = F1	F1 ≈ VAM
	2	3,260	2,100	1,890	2,020	F1	F1	VAM
	3	5,925	4,550	5,125	4,525	F1	F1	F1
	4	2,165	1,682	1,458	1,458	F1	F1	F1 = VAM
B	1	96,750	81,500	81,500	81,500	F1	F1=LC	F1 = VAM
	2	1,090	940	880	880	F1	F1	F1 = VAM
	3	9,425	7,300	6,450	7300	F1	F1=LC	VAM
	4	38,050	35,800	34,600	35,800	F1	F1=LC	VAM
C	1	482,075	331,525	317,875	329,700	F1	F1=LC	VAM
	2	29,155	20,690	16,650	20,055	F1	F1	VAM
	3	89,425	39,525	39,350	39,525	F1	F1=LC	F1 ≈ VAM
	4	4,070	2,504	2,480	2,455	F1	F1	F1
D	1	314,875	232,500	225,850	250,200	F1	LC	VAM
	2	307,750	189,350	184,625	197,975	F1	LC	VAM
	3	6,457	4,778	4,514	4,737	F1	F1	VAM
	4	10,926	6,346	5,888	6,818	F1	LC	VAM
E	1	93,000	82,750	82,500	82,750	F1	F1=LC	F1 ≈ VAM
	2	760	600	540	580	F1	F1	VAM
	3	1,090	965	1,015	950	F1	F1	F1
	4	20,200	18,200	18,200	18,200	F1	F1=LC	F1 = VAM
F	1	93,000	81,000	81,000	55,500	F1	F1	F1
	2	1,390	1,015	1,015	1,045	F1	LC	F1 ≈ VAM
	3	1,534,970	1,317,580	1,308,380	1,282,168	F1	F1	F1
	4	87,875	75,750	75,750	78,000	F1	LC	VAM
G	1	83,160	58,540	45,000	47,580	F1	F1	VAM
	2	92,700	78,900	68,200	72,700	F1	F1	VAM
	3	86,200	62,875	66,975	65,425	F1	LC	F1
	4	85,450	64,925	65,025	66,325	F1	LC	F1 ≈ VAM
H	1	220,200	180,725	159,450	173,650	F1	F1	VAM
	2	103,730	104,450	66,865	90,785	F1	F1	VAM
	3	110,375	22,950	72,025	73,100	F1	LC	F1 ≈ VAM
	4	101,600	100,000	59,350	93,100	F1	F1	VAM

*The smaller the better

III. Conclusion

In this paper a new F1 method, based on weighted cost principle, to find transportation problem initial solution is proposed. The method was tested with 32 study cases. The results show that F1 is better than NWC and LC while slightly below VAM.

IV. References

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V. Acknowledgments

This work has been supported partially by PITTA 2018 Grant funded by DRPM Universitas Indonesia under contract No. 2436/UN2.R3.1/HKP.05.00/2018.