



Performance of Short Journal Bearings with MR Fluid under the Effect of Small Rotation

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Abstract: The present paper analyses the pressure distribution and load carrying capacity of short journal bearing with magnetorheologic fluid under the effect of second order rotatory theory of hydrodynamic lubrication. There are the derivations of the new equations for pressure and load capacity under the effects of second order rotatory theory with MR fluid having high density and much more viscosity in the presence of magnetic field. The viscosity of the fluid varies with the intensity of magnetic flux. The comparative studies of the results with the analysis of short bearing with nano fluid and normal fluid give extraordinary results. The load carrying capacity is of order of 10^{13} as compared to 10^9 in the previous solutions.

Keywords: MR fluid, Load Capacity, Pressure, Rotation number, Reynolds equation, Viscosity, Film thickness.

2010 Mathematics Subject Classification: 76D08

I. Introduction

A magnetorheologic fluid (MR fluid, or MRF) could be a form of smart fluid within a carrier fluid, sometimes a sort of oil. Once subjected to a flux, the fluid greatly will increase its apparent viscosity, to the purpose of changing into an elastic solid. Significantly, the yield stress of the fluid once in its active state will be controlled terribly accurately by variable the flux intensity. The event is that the fluid's ability to transmit force will be controlled with magnet, which supplies raise to its several potential control-based applications [1].

To understand and predict the behavior of the MR fluid it's necessary to model the fluid mathematically, a task slightly sophisticated by the variable material properties such as yield stress etc. As mentioned earlier the smart fluids are such they need an occasional body within the absence of applied flux, however become quasi-solid with the appliance of such a field. In the case of smart fluids and ER, the fluid truly assumes properties adore a solid once within the activated state, up till some extent of yield i.e., the shear stress on that occurs. This yield stress i.e., commonly noted as apparent yield stress, relies on the flux applied to the fluid, however can reach a most conversion that will increase in magnetic density don't have any additional result, because the fluid is then magnetically saturated. The behavior of a smart fluid will therefore be thought-about kind of like a Bingham plastic, a model that has been well-investigated [2].

However, a MR fluid doesn't precisely follow the characteristics of a Bingham plastic. As an example, below the yield stress in the activated state, the fluid behaves as a elastic material, with a fancy modulus that's additionally renowned to be addicted to the flux intensity. MR fluids are renowned to be subject to shear dilution, whereby the viscosity of yield decreases with enhanced shear rate. What is more, the behavior of MR fluids once within the "off" state is additionally non-Newtonian and temperature dependent, but it deviates very little enough for the fluid to be ultimately thought-about as a Bingham plastic for a straightforward analysis [3].

MR fluid is completely different from a ferrofluid that has smaller particles. MR fluid particles are totally on the micrometre-scale and are too dense for Brownian motion to stay them suspended in the lower density carrier fluid. Ferrofluid particles are primarily nanoparticles that are suspended by Brownian motion and usually won't settle underneath traditional conditions. As a result, these fluids have terribly completely different applications. Fluids with magnetic properties could also be shaped by a solution of solid magnetic particles like iron ore in a very parent liquid. The body of the fluid in a field of force is foreseen by dimensional analysis to operate of the quantitative relation of fluid mechanics stress to magnetic stress. The prediction is verified through an experiment and from the correlative information it's determined that for values of this ratio bigger than 10^{-4} there's no modification of body in an applied field. For values between 10^{-6} and 10^{-4} the body could be operate of each field of force and rate of shear. In theory, for prime values of applied field, the body becomes independent of field or shear rate [4]. Analysis of infinitely long bearing within the presence of field of force for the case of electrically conducting fluid was given by Kuzma [5]. N S Patel et al. [6] have created a trial to check and analyze the performance of a magnetic- fluid - based mostly fluid mechanics short bearing. Tze-Chi and Hsin [7] given the performance of lubrication of short bearing by considering surface roughness and field of force and showed that load carrying capability will increase and reduces the friction coefficient. Bujurke and kudenetti [8] studied the

surface roughness result in MHD lubrication flow between the oblong plates and located that the bearing performances are improved. Aught and Osman [9] investigated the matter on finite fluid mechanics journal bearings lubricated by magnetic fluid considering the result of couple stress fluid. Gururajan et al. [10] have investigated the influence of magneto hydrodynamics (MHD) fluid on rough short porous journals bearing, operational beneath standard fluid mechanics regime.

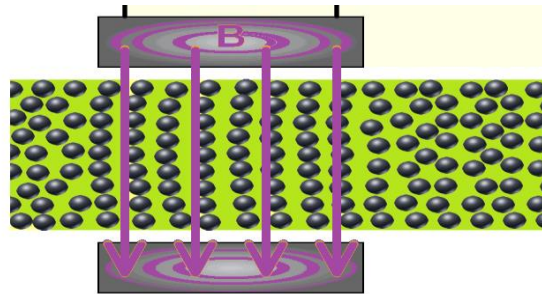


Figure-1 Application of magnetic field polarizes and aligns magnetic particles.

The two dimensional classical theory of hydrodynamic lubrication was first given by Osborne Reynolds [11]. Osborne Reynolds himself derived “Generalized Reynolds Equation” [11], which depends on density viscosity, film thickness, surface and transverse velocities. The differential equation originally derived by Reynolds was restricted to incompressible fluids, so it was formulated broadly enough to include the effects of compressibility and dynamic loading and was said to be Generalized Reynolds Equation. The extended version of “Generalized Reynolds Equation” is said to be “Extended Generalized Reynolds Equation” given by Banerjee et al. [12], [13], which takes into account of the effects of the uniform rotation about an axis that lies across the fluid film and depends on the rotation number M , i.e. the square root of the conventional Taylor’s number. This generalization of the classical theory is known as the “Rotatory Theory of Hydrodynamic Lubrication” [14].

II. Formulation of Problem

The Banerjee et al. had given the derivation of the extended generalized Reynolds equation that is given as follows:

$$\begin{aligned} \frac{\partial}{\partial x} \left[-\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \rho \frac{\partial P}{\partial x} \right] + \frac{\partial}{\partial y} \left[-\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \rho \frac{\partial P}{\partial y} \right] \\ + \frac{\partial}{\partial x} \left[-\frac{M\rho^2h^5}{120\mu^2} \left(1 - \frac{31M^2\rho^2h^4}{3024\mu^2} \right) \frac{\partial P}{\partial y} \right] - \frac{\partial}{\partial y} \left[-\frac{M\rho^2h^5}{120\mu^2} \left(1 - \frac{31M^2\rho^2h^4}{3024\mu^2} \right) \frac{\partial P}{\partial x} \right] \\ = -\frac{\partial}{\partial x} \left[\frac{\rho U}{2} \left\{ h - \frac{M^2\rho^2h^5}{120\mu^2} \left(1 - \frac{31M^2\rho^2h^4}{3024\mu^2} \right) \right\} \right] \\ - \frac{\partial}{\partial y} \left[\frac{M\rho^2U}{2} \left\{ -\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \right\} \right] \quad (2.1) \end{aligned}$$

Where; ρ = fluid density, μ = viscosity, h = film thickness of fluid film, U = sliding velocity, x, y, z = coordinates, P = pressure, M = Rotation number (Square root of conventional Taylor’s number).

If the bearing is infinitely short, then the pressure gradient in x -direction is much smaller than the pressure gradient in y -direction. In y -direction the gradient $\partial P / \partial y$ is of the order of (P/L) and in the x -direction, and is of order of (P/B) . If $L \ll B$, Where; L = Length of bearing, B = Breadth of the bearing, then

$$\frac{P}{L} > \frac{P}{B} \Rightarrow \frac{\partial P}{\partial x} < \frac{\partial P}{\partial y} \quad (2.2)$$

Then the terms containing $\frac{\partial P}{\partial x}$ can be neglected as compared to the terms $\frac{\partial P}{\partial y}$ containing in the expanded form of Generalized Reynolds Equation. Then in view of (2.2) the equation (2.1) takes the form:

$$\begin{aligned} \frac{\partial}{\partial y} \left[-\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \rho \frac{\partial P}{\partial y} \right] + \frac{\partial}{\partial x} \left[-\frac{M\rho^2h^5}{120\mu^2} \left(1 - \frac{31M^2\rho^2h^4}{3024\mu^2} \right) \frac{\partial P}{\partial y} \right] \\ = -\frac{\partial}{\partial x} \left[\frac{\rho U}{2} \left\{ h - \frac{M^2\rho^2h^5}{120\mu^2} \left(1 - \frac{31M^2\rho^2h^4}{3024\mu^2} \right) \right\} \right] \\ - \frac{\partial}{\partial y} \left[\frac{M\rho^2U}{2} \left\{ -\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \right\} \right] \quad (2.3) \end{aligned}$$

In the present paper there is analysis for the performance of short bearing with MR fluid as compared to normal fluid, so on taking

$$h = h(x), U = -U, P = P(y) \quad (2.4)$$

The film thickness ‘ h ’ and ‘ x ’ can be taken as:

$$h=C(1+e\cos\theta), x=R\theta \quad (2.5)$$

Where, C =Radial clearance, e =Eccentricity ratio, R = Radius of the bearing, θ =Angular coordinate measured from x-direction;

For the determination of pressure the boundary conditions are as follows:

$$P=0, y = \pm \frac{L}{2} \quad (2.6)$$

III. Results and Discussion

The solution of the differential equation (2.3) under the boundary condition (2.6) gives the pressure for infinite short journal bearing under the effects of second order rotatory theory of hydrodynamic lubrication as follows:

$$P = A(\theta)\mu + \frac{B(\theta)}{\mu} + C(\theta) \quad (3.1)$$

Where;

$$A(\theta) = \frac{(3Ue\sin\theta)(L^2 - 4y^2)}{4C^2(1 + e\cos\theta)^3 R}$$

$$B(\theta) = \frac{Ue\sin\theta\rho^2 C^2}{16R(1 + e\cos\theta)} \left\{ \frac{e^2 \sin^2\theta}{R^2} \left(y^4 - \frac{L^2 y^2}{2} + \frac{L^2}{16} \right) - \frac{53}{35} (1 + e\cos\theta)^2 \left(y^2 - \frac{L^2}{4} \right) \right\} M^2$$

$$C(\theta) = \frac{\rho U e \sin\theta (L^2 y - 4y^3)}{8CR(1 + e\cos\theta)^2} M$$

The equation for the difference in the pressure due to MR fluid as compared to normal fluid, i.e., Δp^* is given as:

$$\Delta p^* = \Delta\mu A(\theta) - \frac{\Delta\mu}{\mu\mu^*} B(\theta)$$

(3.2)

$$\Delta\mu = \mu^* - \mu$$

μ = Viscosity of normal fluid;

μ^* = Viscosity of MR fluid

The load capacity for short bearing is given by

$$W = (W_x^2 + W_y^2)^{1/2} \quad (3.3)$$

Where W_x and W_y are the components of the load capacity in x-direction and y-direction respectively.

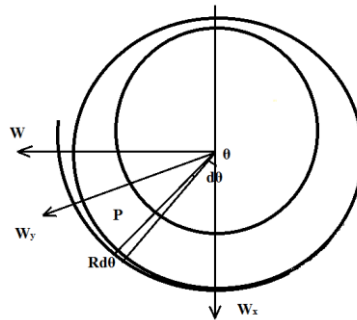


Figure-2 Geometry for the components of Load Capacity

$$W_x = -2 \int_0^\pi \int_0^{L/2} P \cos\theta R d\theta dy \quad (3.4)$$

$$W_y = 2 \int_0^\pi \int_0^{L/2} P \sin\theta R d\theta dy \quad (3.5)$$

The W_x and W_y in the increasing values of M are given by

$$W_x = F_1\mu + \frac{F_2}{\mu} + F_3; W_y = F_4\mu + \frac{F_5}{\mu} + F_6 \quad (3.6)$$

$$F_1 = \frac{Ue^2 L^3}{C^2(1 - e^2)^2}$$

$$F_2 = \frac{Ue^3 \rho^2 C^2 L^5 M^2}{480R^2} \left\{ \frac{\log(1 + e)}{e} + \frac{2}{e^3} (2e + \log(1 - e) - \log(1 + e)) + \frac{\pi}{e\sqrt{(1 - e^2)}} - \frac{\log(1 - e)}{e} - \frac{4}{3} \right\} - \frac{53Ue^2 \rho^2 C^2 L^3 M^2}{5040}$$

$$F_3 = \frac{\rho UL^4}{64Ce} \left\{ \log \frac{1 + e}{1 - e} + \frac{2e}{1 - e^2} \right\} M$$

$$F_4 = \frac{\pi U e L^3}{4 C^2 (1 - e^2)^{3/2}}$$

$$F_5 = \frac{U e^3 \rho^2 C^2 L^5 M^2}{480 R^2} \left\{ \frac{(e^2 - 1)}{e^3} - \frac{1}{e^3} - \frac{2}{3e} - \frac{\pi}{2e^2} - \frac{2\pi}{e^3} - \frac{4}{e^2} \right\} + \frac{53 U e^2 \rho^2 C^2 L^3 M^2}{5040} + \frac{53 \pi U e \rho^2 C^2 L^3 M^2}{6720}$$

$$F_6 = \frac{\rho U L^4 M}{64 C} \left\{ \frac{\pi}{e \sqrt{(1 - e^2)}} - \frac{\pi}{e} \right\} + \frac{53 \pi U e \rho^2 C^2 L^3 M^2}{6720}$$

The equation for the difference in the load capacity due to MR fluid as compared to normal fluid, i.e., ΔW^* is given as:

$$\Delta W^* = [\Delta \mu^{*2} (F_1^2 + F_4^2) + \frac{\Delta \mu^{*2}}{\mu^2 \mu^{*2}} (F_2^2 + F_5^2) - 2 \frac{\Delta \mu^{*2}}{\mu \mu^*} (F_1 F_2 + F_4 F_5) - 2 \frac{\Delta \mu^*}{\mu \mu^*} (F_2 F_3 + F_5 F_6) + 2 \Delta \mu^* (F_1 F_3 + F_4 F_6)]^{1/2} \quad (3.7)$$

By taking the values of different mathematical terms in C.G.S. system as follows:

$\theta=30^\circ$, $M=0.1$, $C=0.0067$ cm, $\rho=3.818$ gm/ cm³, $U=100$ cm/sec, $h=0.00786$ cm, $y=50$ cm, $R=3.35$ cm; the calculated values of the pressure and load capacity with respect to μ , are given by table 1.

Table-1

S.NO.	μ	P(X 10 ⁹)	W(X 10 ¹¹)
1.	5	4.65595	155.796
2.	6	5.58251	186.514
3.	7	6.50907	217.237
4.	8	7.43563	247.963
5.	9	8.36218	278.691
6.	10	9.28874	309.420
7.	11	10.21529	340.151
8.	12	11.14185	370.882
9.	13	12.06841	401.614

The graph show that the pressure distribution and load carrying capacity both vary with viscosity and density of fluid and increase for MR fluid with respect to viscosity of fluid. The logarithmic variation of pressure and load capacity is given by

$$P = (3.368 \log \mu + 3.57) 10^9$$

$$W = (1.117 \log \mu + 1.197) 10^{13}$$

The logarithmic variation of pressure and load capacity for the classical fluid was given by $P = 3.828 e^{0.298\mu} \times 10^7$ and for nano fluid was $P = 4.773 e^{0.259\mu} \times 10^7$ [15] and for nano fluid has load carrying capacity with the polynomial trend line was $W = (-4.009 \mu^2 + 39.19 \mu - 28.69) 10^8$, [16] i.e., of order of 10^7 and 10^{11} respectively. In the case of MR fluid the pressure and load capacity are of order of 10^9 and 10^{13} respectively. Hence by using MR fluid the journal has much more load carrying capacity.

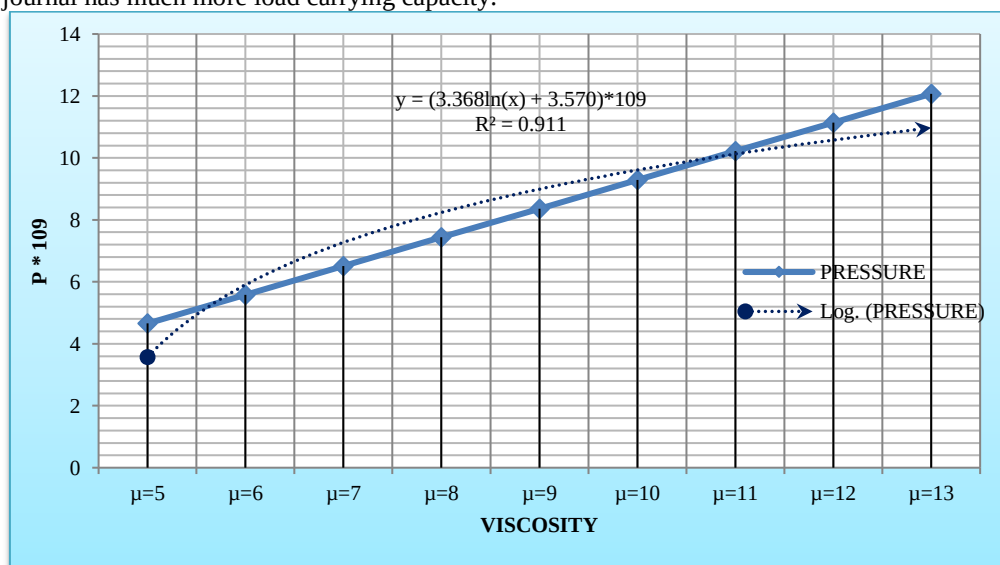


Figure-3 Pressure Distribution with respect to Viscosity

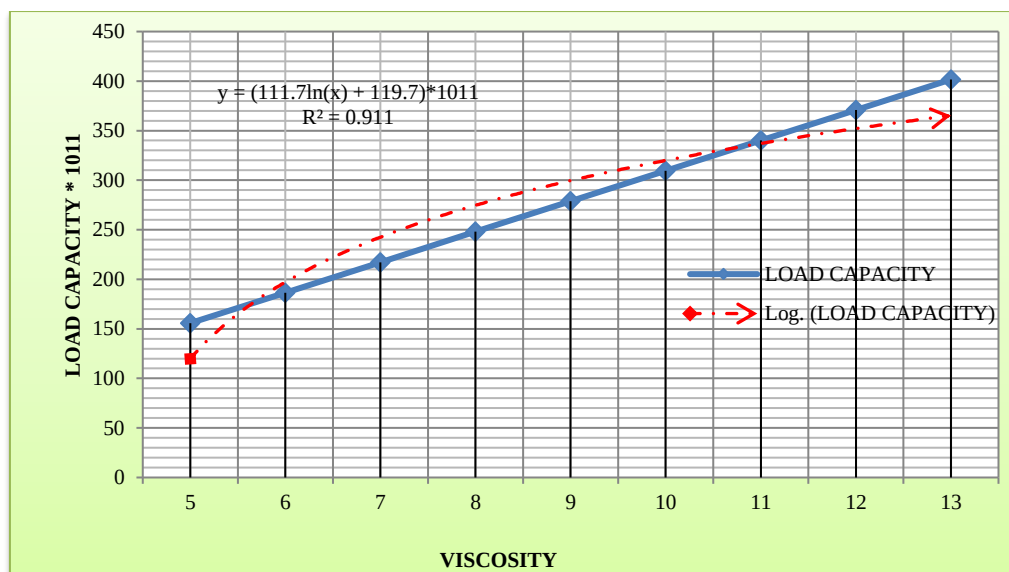


Figure-4 Load Capacity with respect to Viscosity

IV. Conclusion

The variation of the pressure and load capacity is obtained by integrating the differential equation with boundary conditions. The graph show that the pressure distribution and load carrying capacity both vary with viscosity and density of fluid and increase for MR fluid with respect to viscosity of fluid. The logarithmic variation of pressure and load capacity is given by the terms: $P = (3.368 \log \mu + 3.57)10^9$ and $W = (1.117 \log \mu + 1.197)10^{13}$. The logarithmic variation of pressure and load capacity for the classical fluid was given by $P = 3.828 e^{0.298\mu} \times 10^7$ and for nano fluid was $P = 4.773 e^{0.259\mu} \times 10^7$ (Miyán, 2018) and for nano fluid has load carrying capacity with the polynomial trend line was $W = (-4.009 \mu^2 + 39.19 \mu - 28.69)10^8$, (Miyán, 2018) i.e., of order of 10^7 and 10^{11} respectively. In the case of MR fluid the pressure and load capacity are of order of 10^9 and 10^{13} respectively. Hence by using MR fluid the journal has much more load carrying capacity.

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