



Non-successive Occurrence of Digit 5 in Prime Numbers till 1 Trillion

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Abstract: Number 5 doesn't occur as a digit in unit's place of primes except in itself so. But it occurs quite often in other places in higher primes. This work analyzes non-successive occurrences of digit 5 in prime numbers till 1,000,000,000,000. All possible multiple non-successive occurrences of 5 in primes are jotted down. While doing so, the very first and the very last such multiple non-successive occurrences of digit 5 in primes are presented in various ranges of increasing base powers.

Keywords: Prime numbers, digit 5, non-successive occurrences

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I. Introduction

Some number play peculiar role in some situations. Particularly small numbers 0 through 9 are special as they have two roles; they are numbers as well as digits occurring in other numbers. Of these some enjoy interesting properties. Like 0, which is the first counting number, signifying absence of anything, gives an assurance that when present in units place of any number, that number cannot be prime number. Same is the case of 4, 6 and 8. While 2 and 5 take second position in this case as number with these as digits in units place cannot be prime, with the exception of 2 and 5 themselves as numbers.

In above discussion, the focus was on digits in prime numbers. In the realm of numbers, most of the times, primes are candidates of analysis as they still await precise formulation. Lack of simple formula to give n^{th} prime for every positive integer n has kept prime numbers in forefront of symmetry hunt [1]. Numbers have been known to humans from ancient times [2] and primes are also not too new discovery in history of mathematics. But the aforementioned lacking formula forces to analyse primes in general way [1], [3] or in specific ways [4], [5], [6], [7]. While different types of occurrences of non-zero digits in all natural numbers show specific patterns [8], [9], [10], yet there is no success in observing such things in case of primes; which has led to intensive range-wise analysis for different digits' occurrences in primes [11] – [27].

II. Single Non-successive Digit 5 in Prime Numbers

Single occurrence of digit 5 in prime numbers is discussed in [26] and [27]. Single occurrence is liable to be treated as successive and hence there is no question of occurrence of single non-successive occurrence of 5 in any numbers and hence primes also!

III. Multiple Non-successive 5s in Prime Numbers

Multiple non-successive occurrences of 1, covering all non-zero digits including 5, in all natural numbers in the ranges of $1 - 10^n$, $\forall n = 1, 2, 3, \dots$, have been formulated in [10]. On the same line, 5's presence in prime numbers is the theme of this work.

In any selected range of $1 - 10^n$, $\forall n = 4, 5, 6, \dots$, there can occur at most $n - 1$ non-successive 5's. There exact counts have been determined to be as follows.

Table 1 :Number of Primes in Various Ranges with Multiple Non-successive 5's in Their Digits

Sr. No.	Number Range <	Number of Primes with			
		2 Non-successive 5's	3 Non-successive 5's	4 Non-successive 5's	5 Non-successive 5's
1	10^4	11	0	0	0
2	10^5	232	13	0	0
3	10^6	3,344	434	19	0
4	10^7	42,953	7,560	647	24
5	10^8	505,716	112,921	12,567	849
6	10^9	5,641,446	1,489,660	214,371	18,989
7	10^{10}	60,701,221	18,497,483	3,199,014	357,535
8	10^{11}	636,320,534	219,512,376	44,126,576	5,930,315
9	10^{12}	6,542,386,357	2,515,674,558	575,998,440	90,273,825

Table 1 :Continued ...

Sr. No.	Number Range <	Number of Primes with					
		6	7	8	9	10	11
		Non-successive 5's					
1	10^8	23	0	0	0	0	0
2	10^9	1,057	24	0	0	0	0
3	10^{10}	26,202	1,184	24	0	0	0
4	10^{11}	548,536	34,132	1,361	24	0	0
5	10^{12}	10,027,560	792,146	43,477	1,555	24	0

Owing to rise their count, we preferred plotting of their graph with logarithmic vertical axis.

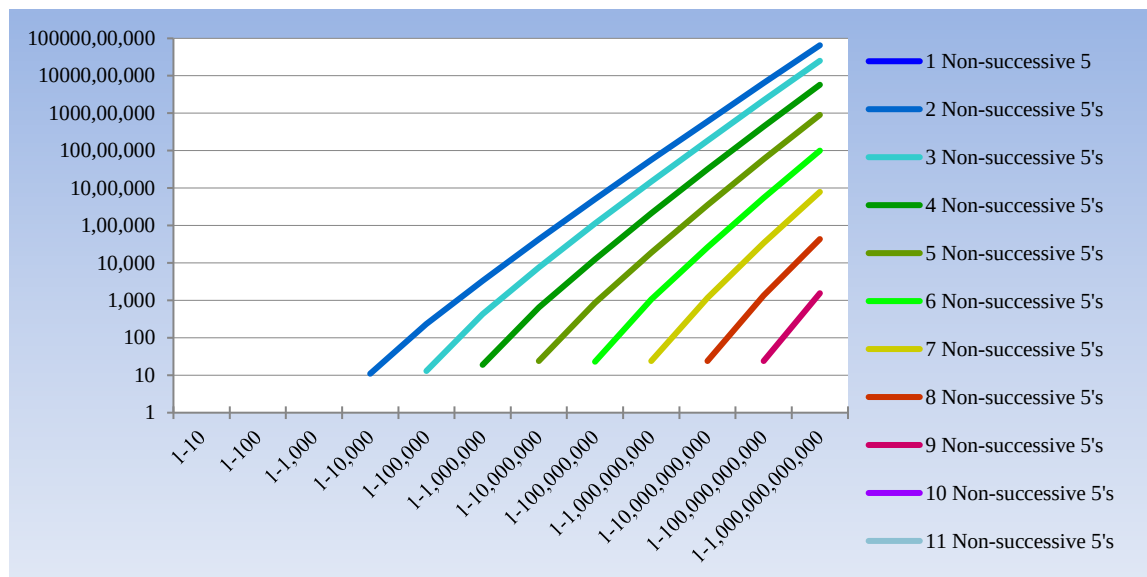


Figure 1 :Number of Primes in Various Ranges with Multiple Non-successive 5's in Their Digits

The comparison of primes with all positive integers presents a density picture. Hence, within different ranges, the percentage of primes having different number of non-successive 5's in them with respect to all such integers is considered.

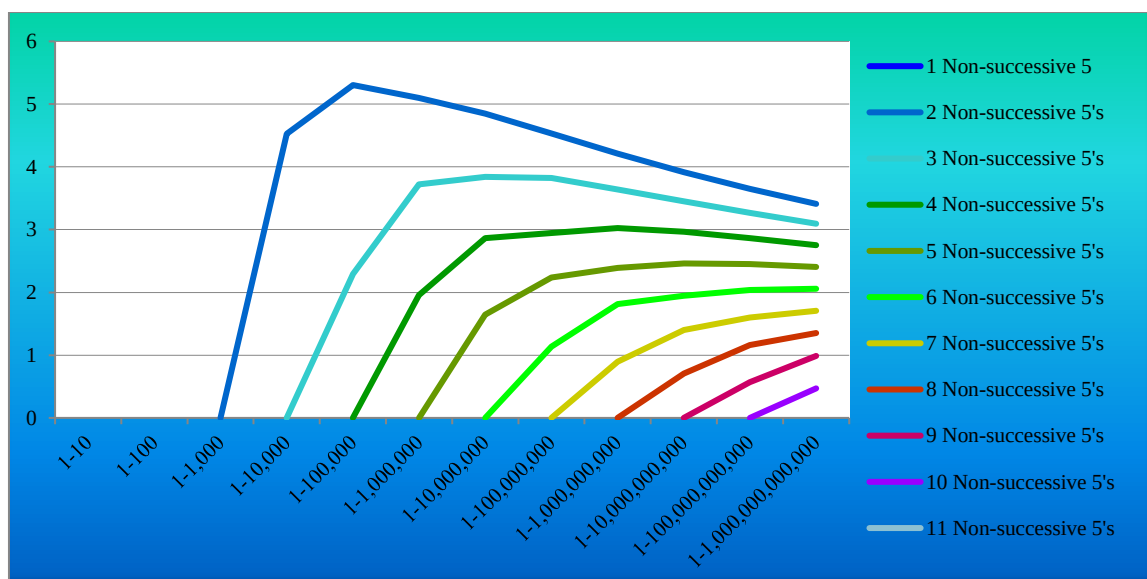


Figure 2 :Percentage of Primes in Different Ranges with Multiple Non-successive 5's in Their Digits With Respect To All Such Positive Integers in Respective Ranges

IV. First Occurrence of Non-successive Digit 5's in Prime Numbers

In case of all positive integers in specific ranges, the first or the smallest of them containing different number of non-successive digit 5's comes in a formula obtained from little variation of the one given in [10].

Formula 1 : If n and r are natural numbers, then the first occurrence of r number of non-successive 5's in integers

$$\text{in range } 1 \leq m < 10^n \text{ is } f = \begin{cases} - & , \text{ if } r < 2 \text{ or } r \geq n \\ \sum_{\substack{j=0 \\ j \neq r-1}}^r (5 \times 10^j) & , \text{ if } r \geq 2 \text{ and } r < n \end{cases}$$

Again non-availability of simple formula for primes forces to scan all primes till first such occurrences are noticed. Exhaustive search within them gave following results.

Table 2 : First Primes in Various Ranges with Multiple Non-successive 5's in Their Digits

Sr. No.	Number Range	First Prime Number in Range with				
		2	3	4	5	6
		Non-successive 5's				
1	$1 - 10^1$	-	-	-	-	-
2	$1 - 10^2$	-	-	-	-	-
3	$1 - 10^3$	-	-	-	-	-
4	$1 - 10^4$	5,051	-	-	-	-
5	$1 - 10^5$	5,051	50,551	-	-	-
6	$1 - 10^6$	5,051	50,551	505,559	-	-
7	$1 - 10^7$	5,051	50,551	505,559	5,055,551	-
8	$1 - 10^8$	5,051	50,551	505,559	5,055,551	54,555,551
9	$1 - 10^9$	5,051	50,551	505,559	5,055,551	54,555,551
10	$1 - 10^{10}$	5,051	50,551	505,559	5,055,551	54,555,551
11	$1 - 10^{11}$	5,051	50,551	505,559	5,055,551	54,555,551
12	$1 - 10^{12}$	5,051	50,551	505,559	5,055,551	54,555,551

Table 2 : Continued ...

Sr. No.	Number Range	First Prime Number in Range with				
		7	8	9	10	11
		Non-successive 5's				
1	$1 - 10^1$	-	-	-	-	-
2	$1 - 10^2$	-	-	-	-	-
3	$1 - 10^3$	-	-	-	-	-
4	$1 - 10^4$	-	-	-	-	-
5	$1 - 10^5$	-	-	-	-	-
6	$1 - 10^6$	-	-	-	-	-
7	$1 - 10^7$	-	-	-	-	-
8	$1 - 10^8$	-	-	-	-	-
9	$1 - 10^9$	525,555,557	-	-	-	-
10	$1 - 10^{10}$	525,555,557	5,055,555,559	-	-	-
11	$1 - 10^{11}$	525,555,557	5,055,555,559	51,555,555,559	-	-
12	$1 - 10^{12}$	525,555,557	5,055,555,559	51,555,555,559	525,555,555,557	-

Comparing this information with that obtained in [25], it is seen that except the case of 10 occurrences of 5's, none is really the very first occurrence of those many number of 5's in primes in concerned ranges as equal number of successive 5's occur first before non-successive ones.

V. Last Occurrence of Non-successive Digit 5's in Prime Numbers

For positive integers in specific ranges, the last or the largest of them containing different number of non-successive digit 5's comes in another formula also obtained from little variation of the one given in [10].

Formula 2 : If n and r are natural numbers, then the first occurrence of r number of non-successive 5's in integers

$$\text{in range } 1 \leq m < 10^n \text{ is } l = \begin{cases} - & , \text{ if } r < 2 \text{ or } r \geq n \\ \sum_{\substack{j=0 \\ j \neq r-1}}^r (5 \times 10^j) + \sum_{\substack{j=r-1 \\ j \neq r}}^{n-1} (9 \times 10^j) & , \text{ if } r \geq 2 \text{ and } r < n \end{cases}$$

Same non-availability of simple formula for primes again forces to scan all primes till range end for last such occurrences.

Table 3 : Last Primes in Various Ranges with Multiple Non-successive 5's in Their Digits

Sr. No.	Number of Non-Successive 5's	Last Prime Number in Range 1 -							
		10^1	10^2	10^3	10^4	10^5	10^6	10^7	10^8
1	1	-	-	-	-	-	-	-	-
2	2	-	-	-	5,953	95,959	995,959	9,995,659	99,995,957
3	3	-	-	-	-	59,557	958,553	9,955,853	99,958,553

Sr. No.	Number of Non-Successive 5's	Last Prime Number in Range 1 –							
		10^1	10^2	10^3	10^4	10^5	10^6	10^7	10^8
4	4	-	-	-	-	-	585,551	9,595,559	99,585,553
5	5	-	-	-	-	-	-	5,955,553	95,955,551
6	6	-	-	-	-	-	-	-	57,555,559
7	7	-	-	-	-	-	-	-	-
8	8	-	-	-	-	-	-	-	-
9	9	-	-	-	-	-	-	-	-
10	10	-	-	-	-	-	-	-	-
11	11	-	-	-	-	-	-	-	-

Table 3 :Continued ...

Sr. No.	Number of Non-Successive 5's	Last Prime Number in Range 1 –		
		10^9	10^{10}	10^{11}
1	1	-	-	-
2	2	999,995,959	9,999,995,953	99,999,995,657
3	3	999,959,557	9,999,959,551	99,999,955,057
4	4	999,585,551	9,999,575,557	99,999,559,559
5	5	995,595,553	9,995,955,559	99,995,955,557
6	6	959,555,551	9,957,555,557	99,958,555,559
7	7	595,555,559	9,558,555,551	99,595,555,559
8	8	-	5,955,555,553	95,655,555,551
9	9	-	-	59,555,555,557
10	10	-	-	-
11	11	-	-	-

Remark : The highest number of non-successive digits 5's in any prime number in the range $1 - 10^n$ is less than or equal to $n - 2$.

All integer arrays appearing in this work give rise to different integer sequences and are worth separate treatments.

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