



EVALUATION OF CHARACTERISTICS OF ANTI FUZZY GRAPH

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Abstract: In this paper, we present the characteristics of an Anti Fuzzy Graph $G_A(\sigma, \mu)$, which is obtained from the fuzzy graph $G = (S, \sigma, \mu)$. Some types of an Anti fuzzy graphs are discussed with an example. The main purpose of this paper is to evaluate the results of isomorphism, weak isomorphism and co-weak isomorphism of an Anti fuzzy graphs.

Keywords: FuzzyGraph, AntiFuzzyGraphs (AFG), Isomorphism, Weak Isomorphism, Co-Weak Isomorphism.

I. Introduction

Fuzzy logic has very powerful tool to represent many real world problems. Since the world is full of uncertainty, so the fuzzy graphs occurs in many real life situations which is applied in Decision Making, Networking, Communication, Traffic Flows and Planning Schedule etc.

The idea of fuzzy logic was propounded by Lotfi.Zadeh [1] in 1965. Although, Rosenfeld [2] introduced the fuzzy vertex, fuzzy edge and the notion of fuzzy graphs. Bhutani K.R[3] introduced the notion of isomorphism of fuzzy graphs. [4] R.Muthuraj and A.Sasireka was introduced the concept of an Anti Fuzzy Graphs. [5]A. Nagoor Gani and J. Malarvizhi discuss some properties of weak and co weak isomorphism of fuzzy graphs. In this paper we find the homomorphism, isomorphism, weak isomorphism and co-weak isomorphism of anti fuzzy graphs and also discussed some related properties with an example.

II. Preliminaries

A. Fuzzy set[1]

Let X be an universal set. Then fuzzy subset A of X is defined by its membership function

$$\mu_A: X \rightarrow [0, 1] \text{ or } 0 \leq \mu_A(x) \leq 1.$$

B. Fuzzy Graph[2]

A fuzzy graph with S as the underlying set is a pair of $G(\sigma, \mu)$ where $\sigma: S \rightarrow [0,1]$ is a fuzzy subset $\mu: S \times S \rightarrow [0,1]$ is a fuzzy relation on the fuzzy subset σ , such that $\mu(x, y) \leq \sigma(x) \wedge \sigma(y) \forall x, y$ in S .

C. Anti Fuzzy Graph[4]

A fuzzy graph $G = (\sigma, \mu)$ is said to be an Anti Fuzzy Graph, with a pair of functions $\sigma: S \rightarrow [0,1]$ and $\mu: V \times V \rightarrow [0,1]$, where for all u, v in V We have, $\mu(u, v) \geq \sigma(u) \vee \sigma(v) \forall u, v \in V$. It's denoted by $G_A(\sigma, \mu)$.

D. Homomorphism of Fuzzy Graphs[5]

A homomorphism of fuzzy graphs of fuzzy $h: G \rightarrow G'$ is a map $h: S \rightarrow S'$, satisfying

$$\sigma(x) \leq \sigma'(h(x)) \quad \forall x \in S \text{ and} \\ \mu(x, y) \leq \sigma(x) \wedge \mu'(h(x), h(y)) \quad \forall x, y \in S.$$

E. Weak Isomorphism of Fuzzy Graphs[4]

A weak isomorphism $h: G \rightarrow G'$ is a map, $h: S \rightarrow S'$ which is a bijective homomorphism that satisfies, $\sigma(x) = \sigma'(h(x)) \forall x \in S$.

F. Co-Weak Isomorphism of Fuzzy Graphs[4]

A co-weak isomorphism $h: G \rightarrow G'$ is a map, $h: S \rightarrow S'$ which is a bijective homomorphism that satisfies, $\mu(x, y) = \mu'(h(x), h(y)) \forall x, y \in S$.

III. Analysis of Anti Fuzzy Graphs

This section investigates the characteristics of an Anti Fuzzy Graphs when they become homomorphism, weak isomorphism and co weak isomorphism. Moreover the result of Strong Anti Fuzzy Graphs derives with an example.

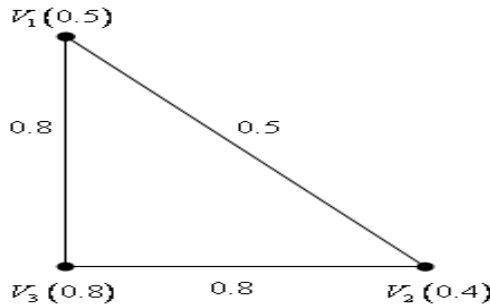
A. Strong Anti Fuzzy Graph

A fuzzy graph $G_A(\sigma, \mu)$ is said to be strong anti fuzzy graph.

If $\mu(u, v) = \sigma(u) \vee \sigma(v)$ or

$\mu(u, v) = \max(\sigma(u), \sigma(v)) \forall u, v \in \mu^*$.

Example



Fig(1) Strong Anti Fuzzy Graph $G_A(\sigma, \mu)$

From the figure, $\sigma(v_1)=0.5$, $\sigma(v_2)=0.4$, $\sigma(v_3)=0.8$ and $\mu(v_1, v_2) = 0.5$, $\mu(v_2, v_3) = 0.8$, $\mu(v_3, v_1) = 0.8$. Here the vertices and edges satisfies the condition, $\mu(u, v) = \sigma(u) \vee \sigma(v)$.

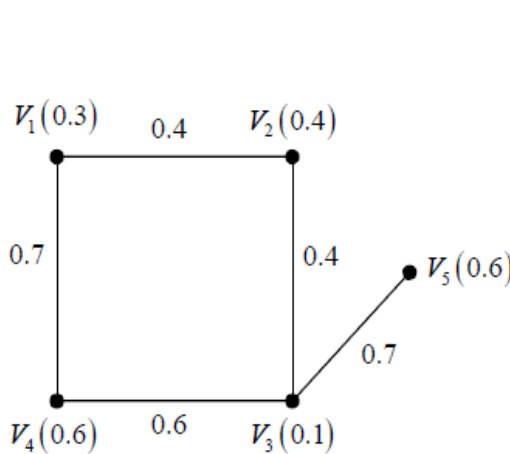
B. Homomorphism of Anti Fuzzy Graph

Consider any two anti fuzzy graphs $G_A(\sigma, \mu)$ and $G'_A(\sigma', \mu')$. If they are said to satisfies the homomorphism if the following conditions hold.

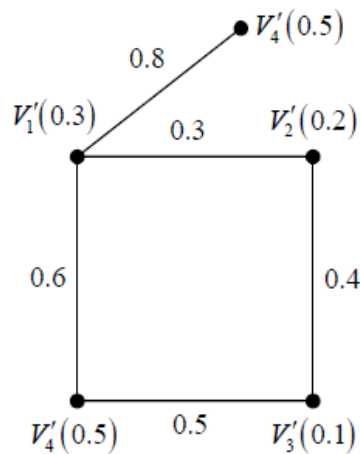
Homomorphism of Anti fuzzy graph $h: G \rightarrow G'$ is a map $h: V \rightarrow V'$ which satisfies,

$$\sigma(u) \geq \sigma'(h(u)) \forall u \in V$$

$$\mu(u, v) \geq \mu'(h(u), h(v)) \forall u, v \in V$$



Fig(a) Anti Fuzzy Graph $G_A(\sigma, \mu)$



Fig(b) Anti Fuzzy Graph $G'_A(\sigma', \mu')$

Fig(2) Homomorphism of Anti Fuzzy Graphs

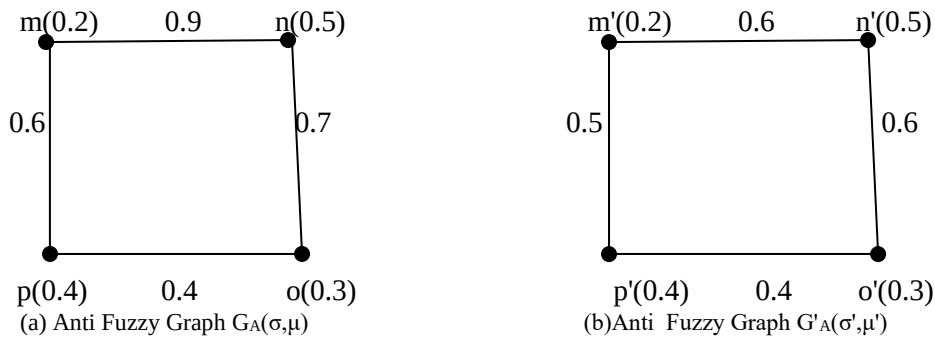
C. Weak Isomorphism of Anti Fuzzy Graph

Let $G_A(\sigma, \mu)$ and $G'_A(\sigma', \mu')$ be two anti fuzzy graphs with underlying sets $V = \{a, b, c, d\}$ and $V' = \{a', b', c', d'\}$ respectively. A weak isomorphism of an Anti fuzzy graph with $h: G_A \rightarrow G'_A$ is a map $h: V \rightarrow V'$ which is a bijective homomorphism graph that satisfies the condition,

$$\sigma(a) = \sigma'(h(a)) \forall a \in \sigma$$

$$\mu(a, b) \geq \mu'(h(a), h(b)) \forall (a, b) \in \mu$$

Example



Fig(3) Weak Isomorphism Graphs

From the fig(3),

$\sigma(m)=0.2, \sigma(n)=0.5, \sigma(o)=0.3, \sigma(p)=0.4,$
 $\sigma'(m')=0.2, \sigma'(n')=0.5, \sigma'(o')=0.3, \sigma'(p')=0.4$
 $\mu(m,n)=0.9, \mu(n,o)=0.7, \mu(o,p)=0.4, \mu(p,m)=0.6,$ and
 $\mu'(m',n')=0.6, \mu'(n',o')=0.6, \mu'(o',p')=0.4,$ and $\mu'(p',m')=0.5$

Clearly, the vertices and edges are satisfies the homomorphism conditions.

Result:

- Anti Fuzzy Graphs also satisfies the characteristic of weak isomorphism.
- If there exist a Weak isomorphism of Anti fuzzy graphs, then their vertices have the same degree.
- That is the order of both Anti fuzzy graphs are same. $\sum \sigma(G) = \sum \sigma(G') = 0.2+0.5+0.3+0.4=1.4$

D. Co - Weak Isomorphism of Anti Fuzzy Graph

Let G_A and G'_A be a two anti fuzzy graphs with sets $\sigma=(x,y,z,w)$ and $\sigma'=(x',y',z',w')$ respectively.
 A Co -Weak isomorphism of Anti fuzzy graph $h:G_A \rightarrow G'_A$ which is a map $h:V \rightarrow V'$ which is a bijective homomorphism of graph that satisfies,

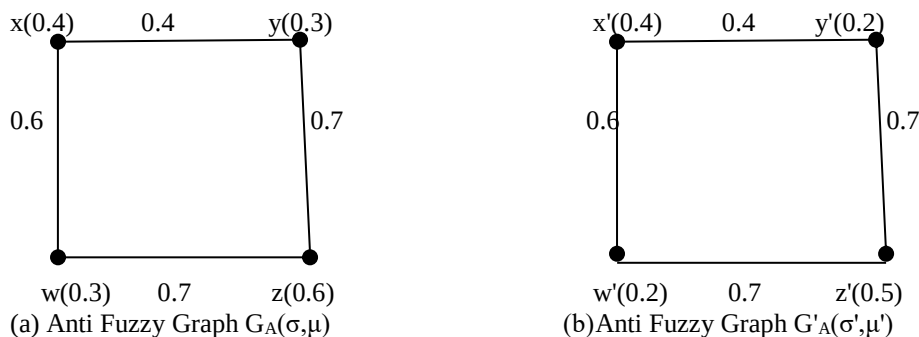
$$\sigma(x) \geq \sigma'(h(x)) \quad \forall x \in S$$

$$\mu(x,y) = \mu'(h(x), h(y)) \quad \forall x, y \in S$$

Example

From the following graphs,

$\sigma(x)=0.4, \sigma(y)=0.3, \sigma(z)=0.6, \sigma(w)=0.3$
 $\sigma'(x')=0.4, \sigma'(y')=0.2, \sigma'(z')=0.5, \sigma'(w')=0.2$
 $\mu(x,y)=0.4, \mu(y,z)=0.7, \mu(z,w)=0.7,$ and $\mu(w,x)=0.6,$
 $\mu'(x',y')=0.4, \mu'(y',z')=0.7, \mu'(z',w')=0.7, \mu'(w',x')=0.6.$



Fig(4) co- weak isomorphism

Hence, $h(x)=h(x'), h(y)=h(y'), h(z)=h(z')$ and $h(w)=h(w')$ satisfying,

$$\sigma(x) \geq \sigma'(h(x)) \quad \forall x \in S$$

$$\mu(x,y) = \mu'(h(x), h(y)) \quad \forall (x,y) \in S$$

Result:

- Anti Fuzzy Graphs also satisfies the property of co weak isomorphism.
- If there exist a Co-Weak isomorphism of Anti fuzzy graphs, then their edges have the same size.

- That is the size of both Anti fuzzy graphs are same. $\sum \mu(G) = \sum \mu(G') = 0.4 + 0.7 + 0.7 + 0.6 = 2.4$

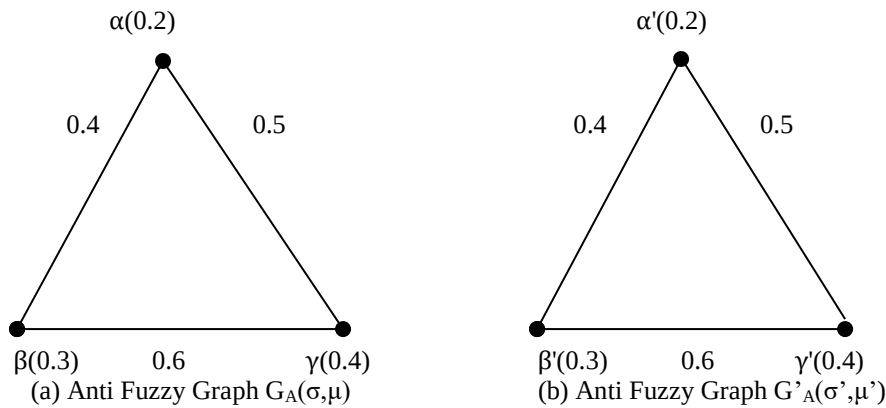
E. Isomorphism of Anti Fuzzy Graph

Consider G_A and G'_A be a two Anti fuzzy graphs which are isomorphism if there exist a bijective map $h: S \rightarrow S'$. Which satisfies,

$$\sigma(a) = \sigma'(h(a)) \quad \forall a \in S$$

$$\mu(a,b) = \mu'(h(a), h(b)) \quad \forall (a,b) \in S$$

Example



Fig(5) Isomorphism of Anti Fuzzy Graphs

From the following figures we get,

$$\sigma(\alpha) = \sigma'(\alpha') = 0.2$$

$$\sigma(\beta) = \sigma'(\beta') = 0.3$$

$$\sigma(\gamma) = \sigma'(\gamma') = 0.4$$

Therefore, $G_A \cong G'_A$

and

$$\mu(\alpha, \beta) = \mu'(\alpha', \beta') = 0.4$$

$$\mu(\beta, \gamma) = \mu'(\beta', \gamma') = 0.6$$

$$\mu(\gamma, \alpha) = \mu'(\gamma', \alpha') = 0.5$$

Result:

- Anti Fuzzy Graphs can also be isomorphic.
- If any two anti fuzzy graphs are isomorphism then their order and size are same. That is, the weight of the edges and vertices are preserved.

IV. Conclusion

This paper conclude the following results

- 1) If there exist a Strong Anti Fuzzy Graph then the edge satisfies $\mu(u, v) = \sigma(u) \vee \sigma(v) \quad \forall u, v \in \mu^*$
- 2) A Homomorphic Anti Fuzzy Graph satisfies the same conditions of homomorphism of fuzzy graph with $\mu(u, v) = \sigma(u) \vee \sigma(v)$.
- 3) In a Weak Isomorphic Anti Fuzzy Graph, the weights of the vertices are preserved and in Co-Weak Isomorphic Anti Fuzzy Graph, the weights of the edges are preserved.
- 4) If there exist a Isomorphic Anti Fuzzy Graphs then the order and size are same since it preserves the weight of the nodes as well as the edges.

This concluding remark also helps us to find out more results in the field of Anti Fuzzy Graph.

V. References

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