



Inverse Split and Inverse Non-Split Two-Out Degree Equitable Domination Number of the Line graphs of Some Special Classes of Graphs

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Abstract: A dominating set D' of G is said to be an inverse two-out degree equitable dominating set if for any two vertices $u, v \in D'$ such that $|od_{D'}(u) - od_{D'}(v)| \leq 2$ where $od_{D'}(u) = |N(v) \cap V - D'|$. The minimum cardinality of a minimal inverse two-out degree equitable dominating set is called the inverse two-out degree equitable domination number and it is denoted by $\gamma'_{2oe}(G)$. The purpose of this paper is to introduce the concept of inverse split (non-split) two-out degree equitable domination numbers of the line graphs of $S_n, P_n, C_n, G_n, H_n, W_n$, and $K_{1,n}$ respectively.

Keywords: Two-out degree equitable domination number, Inverse two-out degree equitable number, Inverse Split (Non-Split) two-out degree equitable number, Line graph.

I. Introduction

Let $G = (V, E)$ be a simple, finite, connected and undirected graph. A dominating set D of a graph $G = (V, E)$ is a split (non-split) dominating set if $\langle V - D \rangle$ is disconnected (connected). The split (non-split) domination number of G is the minimum cardinality of a minimal split (non-split) dominating set in G . Let D be the minimum dominating set in G . If a non-empty subset D' of $V - D$ is a dominating set in G , then D' is called an inverse dominating set with respect to D . Furthermore, if $\langle V - D' \rangle$ is disconnected (connected) then D' is called an inverse split (non-split) dominating set. The inverse split (non-split) domination number of G is the minimum cardinality of a minimal inverse split (non-split) dominating set in G .

V.R Kulliet all introduced the concept of split domination non-split domination and inverse domination in graphs. Let $G = (V, E)$ be a simple, finite, undirected and connected graph. Any undefined term in this paper may be found in Haynes T.W. et al (1998). A non-empty sub set $D \subseteq V$ of a graph G is a dominating set of G if every vertex in $V - D$ is adjacent to some vertex in D . The domination number $\gamma(G)$ is the minimum cardinality taken over all the minimal dominating sets of G . Let D be the minimum dominating sets of G . If $V - D$ contains a dominating set D' then D' is called the inverse dominating set of G with respect to D . The inverse dominating number $\gamma'(G)$ is the minimum cardinality taken over all the minimal inverse dominating sets of G . A dominating set $D \subseteq V$ of a graph G is a split (non-split) dominating set if the induced sub graph $\langle V - D \rangle$ is disconnected (connected). The split (non-split) domination number $\gamma_s(G)$ ($\gamma_{ns}(G)$) is the minimum cardinality of a minimal split (non-split) dominating set of G .

K. Ameen Bibi and R. Selvakumar (2008) introduced the concept of inverse split and non-split domination numbers in graphs. Let D' be the minimum inverse dominating set of G with respect to D . Then D' is called an inverse split (non-split) dominating set of G if the induced subgraph $\langle V - D' \rangle$ is disconnected (connected). The inverse split (non-split) domination number is denoted by $\gamma'_s(G)$ ($\gamma'_{ns}(G)$) and it is the minimum cardinality taken over all the minimal inverse split (non-split) dominating sets of G .

Let $G = (V, E)$ be a simple graph. Let n and m denote the order and size of the graph G respectively. For any vertex $v \in V$, the open neighbourhood of v is the set $N(v) = \{u \in V, uv \in E\}$. Anitha .A, Arumugam . S and Chellai.M introduced the out degree as the outdegree of v with respect to D is denoted by $od_D(u) = |N(v) \cap V - D|$. Ali Sahal and V.Mathad introduced the concept of two-out degree equitable domination in graphs. A dominating set D of G is said to two-out degree equitable dominating set if for any two vertices $u, v \in D$ such that $|od_D(u) - od_D(v)| \leq 2$. The minimum cardinality of a minimal two-out degree equitable dominating set is called two-out degree equitable domination number and is denoted by $\gamma_{2oe}(G)$.

The line graph $L(G)$ of a graph $G = (V, E)$ is the graph with vertex set $E(G)$ in which two vertices are adjacent if and only if the corresponding edges in G are adjacent [14]. In this paper, we initiated the concept of inverse split and inverse non-split two-out degree equitable domination numbers in the line graphs of the graphs

namely $S_n, P_n, C_n, G_n, H_n, W_n$ and $K_{1,n}$. Here $[x]$ is the greatest integer not greater than x and $\lfloor x \rfloor$ is the least integer not less than x .

1.1 Gear Graph [12]

The gear graph is a wheel graph with vertices added between pair of vertices of the outer cycle. The gear graph G_n has $2n + 1$ vertices and $3n$ edges. Let $V(G_n) = \{v_i: 1 \leq i \leq n\} \cup \{u_i: 1 \leq i \leq n\} \cup \{v\}$ and $E(G_n) = \{e_i = v_i u_i, 1 \leq i \leq n\} \cup \{e'_i = v_i v, 1 \leq i \leq n\} \cup \{e''_i = u_i v_{i+1}, 1 \leq i \leq n, \text{subscripts modulo } n\}$ where v is an external vertex adjacent to every other vertex v_i for $1 \leq i \leq n$.

1.2 Helm Graph [12]

The helm graph H_n is the graph obtained from a n -wheel graph by adjoining a pendent edge at each vertex of the cycle. The helm graph H_n has $2n + 1$ vertices and $3n$ edges and $V(G_n) = \{v\} \cup \{v_i: 1 \leq i \leq n\} \cup \{u_i: 1 \leq i \leq n\}$ and $E(G_n) = \{e_i = v_i v_{i+1}, 1 \leq i \leq n - 1\} \cup \{e'_i = v_i v, 1 \leq i \leq n - 1\} \cup \{e_i = v_i u_i, 1 \leq i \leq n - 1\}$ where v is an external vertex adjacent to every other vertex v_i for $1 \leq i \leq n$.

1.3 Wheel Graph [12]

The wheel graph W_n on $n + 1$ vertices is defined as $W_n = C_n + K_1$ where C_n is n -cycle. Let $V(W_n) = \{v_i: 1 \leq i \leq n\} \cup \{v\}$ and $E(W_n) = \{e_i = v_i v_{i+1}, 1 \leq i \leq n, \text{subscripts modulo } n\} \cup \{e'_i = v v_i, 1 \leq i \leq n\}$ where v is an external vertex adjacent to every other vertex.

1.4 Sunlet Graph [15]

The Sunlet graph on $2n$ vertices is defined as $V(S_n) = \{v_1, v_2 \dots v_n\} \cup \{u_1, u_2 \dots u_n\}$ where v_i 's are the vertices of cycles taken in cyclic order and u_i 's are pendent edges.

1.5 Line Graph [14]

A Line graph $L(G)$ of a graph G is a graph on the set of edges of G such that in $L(G)$ there is an edge uv if and only if the vertices u and v are adjacent in G .

II .Main Results on $\gamma'_{s2oe}(G)$:

Inverse Split Two-Out Degree Equitable Domination Number of the Line graphs of Some Special Graphs

In this section, we obtained the inverse split two-out degree equitable domination number of the line graphs of Sunlet, Path and Cycle graphs.

Definition 2.1

Whenever D is a dominating set of G , $V - D$ is also a dominating set of G . Let $D' \subseteq V - D$ is dominating set of G . Then D' is called an Inverse dominating set of G . The inverse dominating set D' of a graph G is called an inverse split two-out degree equitable dominating set if the induced sub graph $\langle V - D' \rangle$ is disconnected and if for any two vertices $u, v \in D'$ such that $|od_{D'}(u) - od_{D'}(v)| \leq 2$ where $od_{D'}(u) = |N(v) \cap V - D'|$. The minimum cardinality of a minimal inverse split two-out degree equitable dominating set is called an inverse split two-out degree equitable domination number of G and is denoted by $\gamma'_{s2oe}(G)$.

Theorem 2.2

For the Line graph of Sunlet graph S_n , the inverse split two-out degree equitable domination number

$$\text{is: } \gamma'_{s2oe}[L(S_n)] = \left\lceil \frac{n}{2} \right\rceil \text{ for } n \geq 4.$$

Proof

By the definition of line graph, $V(L(S_n)) = E(S_n) = \{u'_i: (1 \leq i \leq n)\} \cup \{v'_i: \{e_i: 1 \leq i \leq n - 1\} \cup \{v'_n\}\}$ where v'_i and u'_i represents the edge e_i and e'_i ($1 \leq i \leq n$) respectively.

Let $D' = \left\{ \begin{array}{l} v_{2i} \text{ for } 1 \leq i \leq \frac{n}{2} \text{ if } L(S_n) \text{ is even} \\ v_{2i} \text{ for } 1 \leq i \leq \frac{n-1}{2} \text{ if } L(S_n) \text{ is odd} \end{array} \right\}$ be an inverse dominating set of $L(S_n)$ and

$$V - D' = \left\{ \begin{array}{l} v_{2i-1} \cup \{u_i: 1 \leq i \leq n\} \text{ for } 1 \leq i \leq \frac{n}{2} \text{ if } L(S_n) \text{ is even} \\ v_{2i-1} \cup \{u_i: 1 \leq i \leq n\} \text{ for } 1 \leq i \leq \frac{n+1}{2} \text{ if } L(S_n) \text{ is odd} \end{array} \right\}$$

Now, $v_{2i} \in D$ then $od_{D'}(v_{2i}) = |N(v_{2i}) \cap V - D'|$

If $L(S_n)$ is even, for $i = 1$

$$\begin{aligned} od_{D'}(v_2) &= |N(v_2) \cap V - D'| \\ &= |(v'_1, v'_3, u'_2, u'_3) \cap (v'_1, v'_3 \dots v'_{n-1}) \cap (u'_1, u'_2, \dots u'_n)| \\ &= |(v'_1, v'_3, u'_2, u'_3)| \\ &= 4 \end{aligned}$$

$$\begin{aligned} \text{For } i = 2, od_{D'}(v_4) &= |N(v_4) \cap V - D'| \\ &= |(v'_3, v'_5, u'_4, u'_5) \cap \{(v'_1, v'_3, v'_5 \dots v'_{n-1}) \cap (u'_1, u'_2, \dots u'_n)\}| \\ &= |(v'_3, v'_5, u'_4, u'_5)| \\ &= 4 \end{aligned}$$

$$\begin{aligned} \text{For } i = \frac{n}{2}, od_{D'}(v_n) &= |N(v_n) \cap V - D'| \\ &= |(v'_1, v'_{n-1}, u'_1, u'_n) \cap \{(v'_1, v'_3 \dots v'_{n-1}) \cap (u'_1, u'_2, \dots u'_n)\}| \\ &= |(v'_1, v'_{n-1}, u'_1, u'_n)| \\ &= 4 \end{aligned}$$

If $L(S_n)$ is odd, for $i = 1$

$$\begin{aligned} od_{D'}(v_2) &= |N(v_2) \cap V - D'| \\ &= |(v'_1, v'_3, u'_2, u'_3) \cap \{(v'_1, v'_3, v'_5, \dots v'_n) \cap (u'_1, u'_2, \dots u'_n)\}| \\ &= |(v'_1, v'_3, u'_2, u'_3)| \\ &= 4 \end{aligned}$$

$$\begin{aligned} \text{For } i = \frac{n-1}{2}, od_{D'}(v_{n-1}) &= |N(v_{n-1}) \cap V - D'| \\ &= |(v'_{n-2}, v'_n, u'_{n-1}, u'_n) \cap \{(v'_1, v'_3 \dots v'_n, v'_{n-1}, v'_n) \cap (u'_1, u'_2, \dots u'_n)\}| \\ &= |(v'_{n-2}, v'_n, u'_{n-1}, u'_n)| \\ &= 4 \end{aligned}$$

Then $|od_{D'}(u) - od_{D'}(v)| \leq 2$ for any $u, v \in D'$. So D' is the inverse two-out degree equitable dominating set and $\langle V - D' \rangle$ is disconnected. Therefore D' is the inverse split two-out degree equitable dominating set. Hence,

$$\gamma'_{s2oe}[L(S_n)] = \left\lceil \frac{n}{2} \right\rceil \quad \text{for } n \geq 4.$$

Observation 2.3

For the Line graph of S_n , $\gamma_{s2oe}[L(S_n)] \geq \gamma'_{s2oe}[L(S_n)]$.

Theorem 2.4

For the Line graph of P_n , the inverse split two-out degree equitable domination number is

$$\gamma'_{s2oe}[L(P_n)] = \left\lceil \frac{n-2}{2} \right\rceil - 1 \quad \text{for } n \geq 5.$$

Proof

The line graph of P_n has $n - 1$ vertices and $n - 2$ edges, since the degree of any vertex in $L(P_n)$ is 2 except the initial and terminal vertices.

Here, D' is the inverse two-out degree equitable dominating set. It is found that the induced sub graph $\langle V - D' \rangle$ is disconnected. Therefore D' is the minimum inverse split two-out degree equitable dominating set.

$$\text{Hence, } \gamma'_{s2oe}[L(P_n)] = \left\lceil \frac{n-2}{2} \right\rceil - 1 \quad \text{for } n \geq 5.$$

Observation 2.5

For the Line graph of P_n , $\gamma_{s2oe}[L(P_n)] > \gamma'_{s2oe}[L(P_n)]$.

Theorem 2.6

For the Line graph of the Cycle C_n , the inverse split two-out degree equitable domination number is

$$\gamma'_{s2oe}[L(C_n)] = \left\lceil \frac{n}{3} \right\rceil \quad \text{for } n \geq 3.$$

Proof

The line graph of C_n has n vertices and n edges. Each vertex is of degree 2. Here, the induced sub graph $\langle V - D' \rangle$ is disconnected. Therefore D' is the minimum inverse split two-out degree equitable dominating set.

$$\text{Hence, } \gamma'_{s2oe}[L(C_n)] = \left\lceil \frac{n}{3} \right\rceil \quad \text{for } n \geq 3.$$

Observation 2.7

For the Line graph of C_n , $\gamma_{s2oe}[L(C_n)] \geq \gamma'_{s2oe}[L(C_n)]$.

III. Main Results on $\gamma'_{ns2oe}(G)$:

Inverse Non-Split Two-Out Degree Equitable Domination Numbers of the Line graphs of Some Special Graphs

In this section, we obtained the inverse non-split two-out degree equitable domination number of the line graphs of Gear graph, Helm graph, Wheel graph and Star graph.

Definition 3.1

Whenever D is a dominating set of G , $V - D$ is also a dominating set of G . Let $D' \subseteq V - D$ is dominating set of G . Then D' is called an Inverse dominating set of G . The inverse dominating set D' of a graph G is called an inverse non-split two-out degree equitable dominating set if the induced sub graph $\langle V - D' \rangle$ is connected and if for any two vertices $u, v \in D'$ such that $|od_{D'}(u) - od_{D'}(v)| \leq 2$ where $od_{D'}(u) = |N(u) \cap V - D'|$. The minimum cardinality of a minimal inverse non-split two-out degree equitable dominating set is called an inverse non-split two-out degree equitable domination number of G and is denoted by $\gamma'_{ns2oe}(G)$.

Theorem 3.2

For the Line graph of Gear graph G_n , the inverse non-split two -out degree equitable domination number is : $\gamma'_{ns2oe}[L(G_n)] = n$.

Proof

Let $L(G_n)$ be the line graph of G_n of order $3n$. By the definition of line graph, $V(L(G_n)) = E(G_n)$ in which the set of vertices of $L(G_n)$, $\{e'_i : 1 \leq i \leq n\}$ induces a clique of order n .

Let $\{(e_1, e_2, \dots, e_n) \cup (e'_1, e'_2, \dots, e'_n) \cup (e''_1, e''_2, \dots, e''_n)\}$ be the vertices in $L(G_n)$.

Let $D' = \{e_i : 1 \leq i \leq n\}$ be the inverse dominating set of $L(G_n)$ and $V - D' = \{(e'_i \cup e''_i) : 1 \leq i \leq n\}$

Now, $e_i \in D'$ then $od_{D'}(e_i) = |N(e_i) \cap V - D'|$

$$\begin{aligned} \text{for } i = 1, e_1 \in D' \text{ then } od_{D'}(e_1) &= |N(e_1) \cap V - D'| \\ &= |(e_n'', e_1'', e_1') \cap \{(e'_i \cup e''_i) : 1 \leq i \leq n\}| \\ &= |(e_n'', e_1'', e_1')| \\ &= 3 \end{aligned}$$

$$\begin{aligned} \text{If } e_i \in D' \text{ then } od_{D'}(e_i) &= |N(e_i) \cap V - D'| \text{ for } 2 \leq i \leq n \\ &= |(e_{i-1}'', e_i'', e_i') \cap \{(e'_i \cup e''_i) : 1 \leq i \leq n\}| \\ &= |(e_{i-1}'', e_i'', e_i')| \\ &= 3 \end{aligned}$$

Then $|od_{D'}(e_i) - od_{D'}(e_j)| \leq 2$, for any $e_i, e_j \in D'$. Therefore D' is an inverse two-out degree equitable dominating set. So, $\gamma'_{ns2oe}[L(G_n)] = n$ and $\langle V - D' \rangle$ is connected.

Then D' is the minimum inverse non-split two-out degree equitable dominating set. Hence, $\gamma'_{ns2oe}[L(G_n)] = n$.

3.3 Observation on $L(G_n)$:

- (i) $\gamma_{ns2oe}[L(G_n)] = \gamma'_{ns2oe}[L(G_n)] = n$
- (ii) $\gamma_{ns2oe}[L(G_n)] + \gamma'_{ns2oe}[L(G_n)] = 2n$
- (iii) $\gamma_{ns2oe}[L(G_n)] \cdot \gamma'_{ns2oe}[L(G_n)] = n^2$
- (iv) $\gamma_{ns2oe}[L(G_n(V - D)))] = 2n$
- (v) $\gamma'_{ns2oe}[L(G_n(V - D')))] = 2n$
- (vi) $\gamma_{ns2oe}[L(G_n(V - D)))] + \gamma'_{ns2oe}[L(G_n(V - D')))] = 4n$
- (vii) $\gamma_{ns2oe}[L(G_n(V - D)))] \cdot \gamma'_{ns2oe}[L(G_n(V - D')))] = 4n^2$

Theorem 3.4

For the Line graph of Helm graph H_n , the inverse non-split two -out degree equitable domination number is : $\gamma'_{ns2oe}[L(H_n)] = n$

Proof

Let $L(H_n)$ be the line graph of H_n of order $3n$. Let $\{e'_i \cup e_i \cup e''_i : 1 \leq i \leq n\}$ be the vertex set of $L(H_n)$.

Let $D' = \{e_i'' : 1 \leq i \leq n\}$ be an inverse dominating set of $L(H_n)$ and $V - D' = \{(e'_i \cup e_i) : 1 \leq i \leq n\}$

Now, $e_i'' \in D'$ then $od_{D'}(e_i'') = |N(e_i'') \cap V - D'|$

$$\begin{aligned} \text{for } i = 1, e_1'' \in D' \text{ then } od_{D'}(e_1'') &= |N(e_1'') \cap V - D'| \\ &= |(e_n, e_1, e_1') \cap \{(e'_i \cup e_i) : 1 \leq i \leq n\}| \\ &= |(e_n, e_1, e_1')| \\ &= 3 \end{aligned}$$

$$\begin{aligned} \text{If } e_i'' \in D' \text{ then } od_{D'}(e_i'') &= |N(e_i'') \cap V - D'| \text{ for } 2 \leq i \leq n \\ &= |(e_{i-1}, e_i, e_i') \cap \{(e'_i \cup e_i) : 1 \leq i \leq n\}| \\ &= |(e_{i-1}, e_i, e_i')| \\ &= 3 \end{aligned}$$

Then $|od_{D'}(e_i) - od_{D'}(e_j)| \leq 2$, for any $e_i, e_j \in D'$. Therefore D' is an inverse two-out degree equitable dominating set. So, $\gamma'_{ns2oe}[L(H_n)] = n$ and the induced sub graph $\langle V - D' \rangle$ is connected.

Then D' is a minimum inverse non-split two-out degree equitable dominating set. Hence, $\gamma'_{ns2oe}[L(H_n)] = n$.

3.5 Observation on $L(H_n)$:

- (i) $\gamma_{ns2oe}[L(H_n)] = \gamma'_{ns2oe}[L(H_n)] = n$
- (ii) $\gamma_{ns2oe}[L(H_n)] + \gamma'_{ns2oe}[L(H_n)] = 2n$
- (iii) $\gamma_{ns2oe}[L(H_n)] \cdot \gamma'_{ns2oe}[L(H_n)] = n^2$
- (iv) $\gamma_{ns2oe}[L(H_n(V - D))] = 2n$
- (v) $\gamma'_{ns2oe}[L(H_n(V - D'))] = 2n$
- (vi) $\gamma_{ns2oe}[L(H_n(V - D))] + \gamma'_{ns2oe}[L(H_n(V - D'))] = 4n$
- (vii) $\gamma_{ns2oe}[L(H_n(V - D))] \cdot \gamma'_{ns2oe}[L(H_n(V - D'))] = 4n^2$

Theorem 3.6

For the Line graph of Wheel graph W_n , the inverse non-split two-out degree equitable domination number is :

$$\gamma'_{ns2oe}[L(W_n)] = \begin{cases} \left\lceil \frac{n}{2} \right\rceil & \text{if } n \text{ is even} \\ \text{does not exist} & \text{if } n \text{ is odd for } n > 5 \end{cases}$$

Proof

Case (i) if n is even

Let $L(W_n)$ be the line graph of W_n of order $2n$. By the definition of the line graph, $V(L(W_n)) = \{e_i = v_i v_{i+1}, 1 \leq i \leq n, \text{subscripts modulo } n\} \cup \{e'_i = v v_i, 1 \leq i \leq n\}$.

Also, $V(L(W_n)) = 2n$

Let $D' = \{e_{2i} : 1 \leq i \leq \frac{n}{2}\}$ be an inverse dominating set of $L(W_n)$ and $V - D' = \{(e_{2i-1} : 1 \leq i \leq \frac{n}{2}) \cup (e'_i : 1 \leq i \leq n)\}$

Now, $e_{2i} \in D'$ then $od_{D'}(e_{2i}) = |N(e_{2i}) \cap V - D'|$

$$\begin{aligned} \text{for } i = 1, e_2 \in D' \text{ then } od_{D'}(e_2) &= |N(e_2) \cap V - D'| \\ &= |\{e_1, e_3, e'_2, e'_3\} \cap \{(e_1, e_3, \dots, e_{n-1}) \cup (e'_1, e'_2, \dots, e'_n)\}| \\ &= |\{e_1, e_3, e'_2, e'_3\}| \\ &= 4. \end{aligned}$$

$$\begin{aligned} \text{If } i = 2, e_4 \in D' \text{ then } od_{D'}(e_4) &= |N(e_4) \cap V - D'| \\ &= |\{e_3, e_5, e'_4, e'_5\} \cap \{(e_1, e_3, \dots, e_{n-1}) \cup (e'_1, e'_2, \dots, e'_n)\}| \\ &= |\{e_3, e_5, e'_4, e'_5\}| \\ &= 4. \end{aligned}$$

$$\begin{aligned} \text{If } i = \frac{n}{2}, e_n \in D' \text{ then } od_{D'}(e_n) &= |N(e_n) \cap V - D'| \\ &= |\{e_1, e_{n-1}, e'_n, e'_1\} \cap \{(e_1, e_3, \dots, e_{n-1}) \cup (e'_1, e'_2, \dots, e'_n)\}| \\ &= |\{e_1, e_{n-1}, e'_n, e'_1\}| \\ &= 4. \end{aligned}$$

Then $|od_{D'}(e_{2i}) - od_{D'}(e_{2j})| \leq 2$, for any $e_{2i}, e_{2j} \in D'$. So D' is an inverse two-out degree equitable dominating set. It is found that the induced sub graph $\langle V - D' \rangle$ is connected. Then D' is the minimum inverse

non-split two-out degree equitable dominating set. Hence, $\gamma'_{ns2oe}[L(W_n)] = \left\lceil \frac{n}{2} \right\rceil$ if n is even.

Case (i) if n is odd, with $n > 5$

Let D' be an inverse dominating set of $L(W_n)$, if for any two vertices $u, v \in D'$ such that $|od_{D'}(u) - od_{D'}(v)| \leq 2$,

$$\text{Here, } od_{D'}(u) = |N(u) \cap V - D'| = 4$$

$$od_{D'}(v) = |N(v) \cap V - D'| = n + 1$$

(i.e) $|od_{D'}(u) - od_{D'}(v)| \neq 2$. This shows that $L(W_n)$ is a graph with $n \geq 5$ (if n is odd) does not satisfy the two-out degree equitable domination condition.

Observation 3.7

$$\text{For the Line graph of } W_n, \gamma_{ns2oe}[L(W_n)] = \gamma'_{ns2oe}[L(W_n)] = \left\lceil \frac{n}{2} \right\rceil$$

Theorem 3.8

For the line graph of Star graph $K_{1,n}$, the inverse non-split two-out degree equitable domination number is : $\gamma'_{ns2oe}[L(K_{1,n})] = 2$ for $n > 4$.

Proof

The line graph of $K_{1,n}$ is of order n and $L(K_{1,n})$ is a complete graph. Let $D' = \{v_3, v_4\}$ be an inverse dominating set of $L(K_{1,n})$ and $V - D' = \{v_1, v_2, v_5, v_6 \dots v_n\}$

$$\begin{aligned} \text{Now, } v_3 \in D' \text{ then } od_{D'}(v_3) &= |N(v_3) \cap V - D'| \\ &= |\{v_1, v_2, v_5 \dots v_n\} \cap \{v_1, v_2, v_5, v_6 \dots v_n\}| \\ &= |\{v_1, v_2, v_5 \dots v_n\}| \\ &= n - 2 \end{aligned}$$

Similarly, $od_{D'}(v_4) = n - 2$. Then $|od_{D'}(v_3) - od_{D'}(v_4)| \leq 2$ for any $v_3, v_4 \in D'$. So D' is an inverse two-out degree equitable dominating set and the induced sub graph $< V - D' >$ is connected. Then D' is the minimum inverse non-split two-out degree equitable dominating set. Hence, $\gamma'_{ns2oe}[L(K_{1,n})] = 2$ for $n > 4$.

Observation 3.9

For the Line graph of $K_{1,n}$, $\gamma_{ns2oe}[L(K_{1,n})] = \gamma'_{ns2oe}[L(K_{1,n})]$

IV. Conclusion

In this paper, we introduced the concept of inverse split (non-split) two-out degree equitable domination number of the line graphs of some standard graphs. Also their relationships with other domination parameters are discussed. We further extended this study on the Line graphs of some more special classes of graphs.

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