



Digit 2's Occurrence in First 10 Billion Digits of π after Decimal Point in Decimal Representation

Neeraj Anant Pande

Associate Professor

Department of Mathematics & Statistics,

Yeshwant Mahavidyalaya, Nanded – 431602, Maharashtra, INDIA

Abstract: π is one of the first few irrational numbers that we come across in mathematics. Geometry uses it heavily in computation of areas and volumes as also other branches of subject like analysis. Infinite succession of non-zero digits in non-recurring patterns in the decimal representation after the decimal point makes π and other irrational numbers worth demanding attention. In this work, for decimal representation of π , appearance of digit 2 in its digits after the decimal point is at focus, for both consecutive and non-consecutive patterns.

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I. Introduction

Some entities are hard to know to perfection but can be harnessed through use and employment to work regardless of former character. Irrational numbers fully fit in this category. The reason for classifying them as an entity hard to know fully is that the exact value of any irrational number can seldom be known owing to its nature that it contains infinite number of non-zero digits in its decimal expansion after the decimal point and the pattern of these digits is also not repeating in any specific manner. So, given an irrational number, if one wants to know the n^{th} digit after the decimal point, specifically for sufficiently large n , its difficult as it lacks a formula for this. But, of course, this characteristic of irrational numbers is not a hurdle in their usefulness. Rather, many mathematicians, particularly cryptologists, believe that it is this property that makes irrational numbers more applicable than not. π is one of the earliest known, and of course also the earliest applied, irrational number. Johann Heinrich Lambert has proved irrationality of π in 18th century, but it has been in use quite before that. Now there are multiple, including very simple, proofs that it is irrational [1].

II. Digit 2 in π

So due to the aforementioned reasons, inspecting of digits individually occurring at every place after the decimal point of any irrational number becomes necessary. π has already been subject to this rigorous analysis. But for such work, first it becomes necessary to have knowledge of the digits in π , within huge ranges. Ways to achieve this have been paved out [2]. Recent two works have undertaken investigation of occurrence pattern of digits 0 [3] and 1 [4] within first 10 billion digits of π after decimal point.

In this work we take up next decimal digit, viz., 2, for analysis of its occurrence in decimal digits of π after the decimal point. The digit ranges are also kept in powers of base 10.

Table 1: Occurrences of 2 in Digits of π in Blocks of 10 Powers

Sr. No.	Digit Numbers' Range $1 - x$	Range as Ten Power 10^x	Number of Occurrences of 2	First Occurrence of 2 at Digit Number	Last Occurrence of 2 at Digit Number
1.	1 – 10	10^1	1	6	6
2.	1 – 100	10^2	12	6	93
3.	1 – 1,000	10^3	103	6	995
4.	1 – 10,000	10^4	1,021	6	9,992
5.	1 – 100,000	10^5	9,908	6	99,996
6.	1 – 1,000,000	10^6	100,026	6	999,982
7.	1 – 10,000,000	10^7	1,000,306	6	9,999,954
8.	1 – 100,000,000	10^8	10,001,092	6	100,000,000
9.	1 – 1,000,000,000	10^9	100,002,410	6	999,999,992
10.	1 – 10,000,000,000	10^{10}	1,000,017,271	6	9,999,999,999

(Seeming) Randomness in occurrence and density being two independent things, expectations of even density distribution of all digits in decimal expansion of π are not prohibited. To inspect this, setting out the expected occurrence density of each digit, including the one under investigation 2, to be $1/10^{\text{th}}$ of whole digit range in consideration, deviation of 2's density from this expectation is shown below graphically.

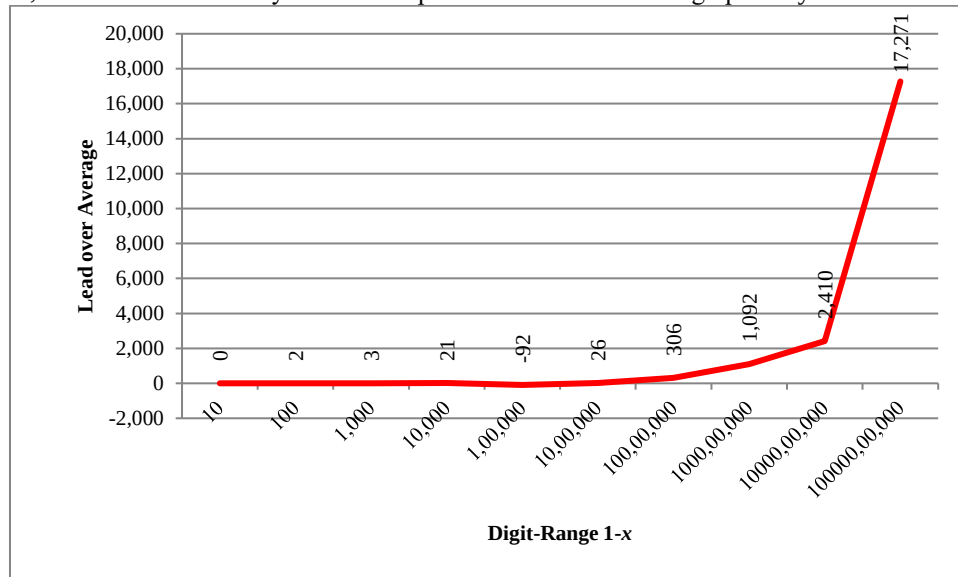


Figure 1: Deviation of Occurrence Density of Digit 2 in π from Average

As far as the above 10 discrete digit range-values are considered, 2 is seen always above average except for block of $1 - 10^5$. Whether this is consistent behavior for all higher ranges is a subject matter of further investigation. The last occurrence of digit 2 falls short to reach the last digit in the range by following amounts.

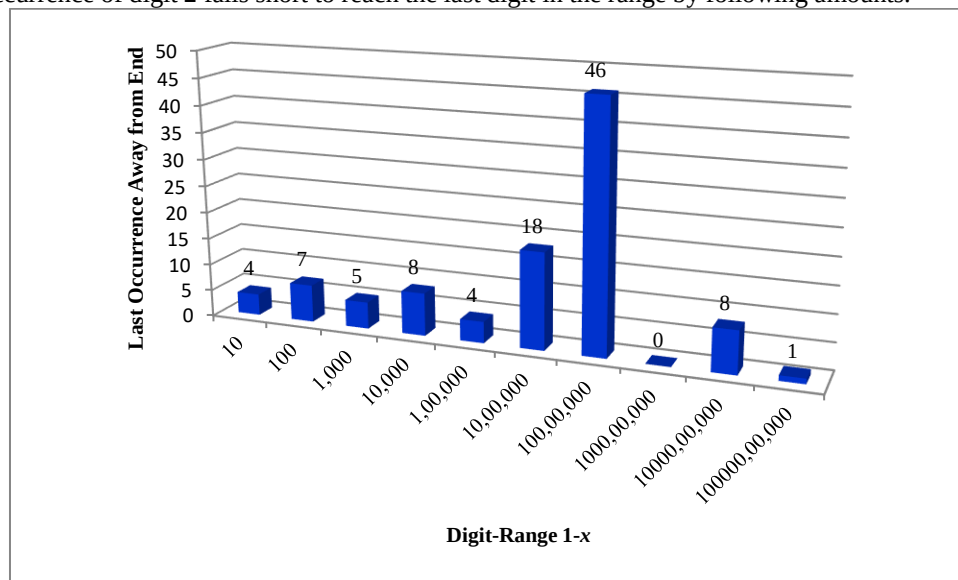


Figure 2: The Extents by which the Last Appearance of Digit 2 in π is Away from End

Quantitatively, 2 is farthest from end in block of $1 - 10^7$ and nearest in block of $1 - 10^8$, in fact it is right at the end of the block there.

III. Successive Occurrence of Digit 2 in π

The occurrences of 2 in digits of π analysed in former section were general, including successive and non-successive. Now we focus only on successive occurrences.

Table 2: Successive Occurrences of 2 in Digits of π in Blocks of 10 Powers

Sr. No.	Digit Numbers' Range 1 - x	Range as Ten Power 10^x	Number of Successive Occurrences of 2	First Successive Occurrence of 2 at Digit Number	Last Successive Occurrence of 2 at Digit Number
1.	1 - 10	10^1	0	-	-
2.	1 - 100	10^2	0	-	-
3.	1 - 1,000	10^3	8	135	964

Sr. No.	Digit Numbers' Range 1 – x	Range as Ten Power 10^x	Number of Successive Occurrences of 2	First Successive Occurrence of 2 at Digit Number	Last Successive Occurrence of 2 at Digit Number
4.	1 – 10,000	10^4	110	135	9,967
5.	1 – 100,000	10^5	971	135	99,844
6.	1 – 1,000,000	10^6	10,062	135	999,981
7.	1 – 10,000,000	10^7	99,931	135	9,999,843
8.	1 – 100,000,000	10^8	1,001,323	135	99,999,954
9.	1 – 1,000,000,000	10^9	10,005,081	135	999,999,737
10.	1 – 10,000,000,000	10^{10}	100,003,043	135	9,999,999,998

There is no question of successive occurrence of 2 in the first digit range 1 – 10, as it occurs only once there. In the second 10 power blocks 1 – 100, digit 2 doesn't occur consecutively even once. It starts occurring such from digit number 135.

The percentage of occurrence of successive 2's in respective blocks has following trend.

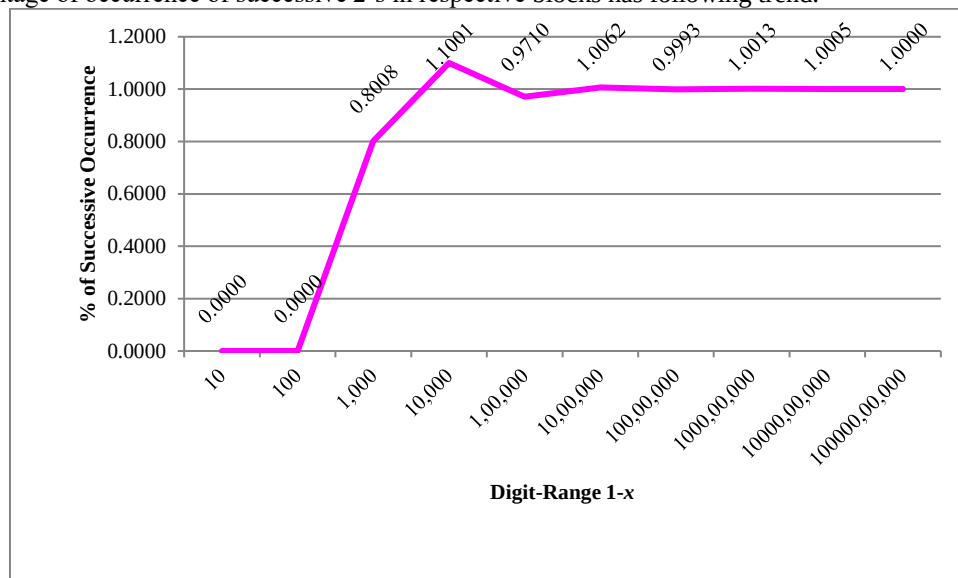


Figure 3: Percentages of Successive Occurrences of 2 in Digits of π after Decimal Point.

While the first successive occurrence of 2 comes in late; more than 22 times of solo occurrence; this percentage smoothly reaches unity till the end of our range of investigation.

Barring the first two blocks of 1 – 10 & 1 – 100 of absence of successive 2's, the last successive occurrence of digit 2's stops before last digit in range by keeping following spaces.

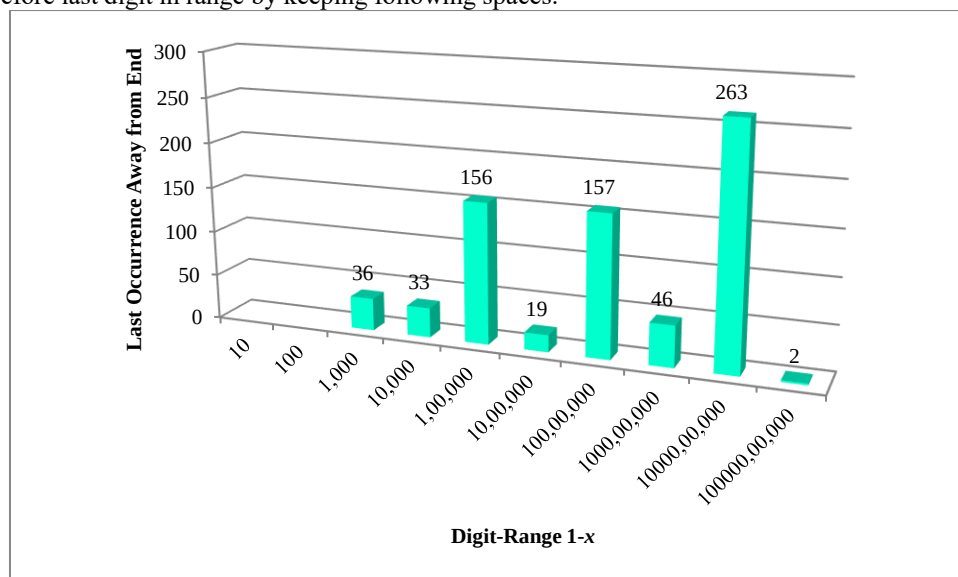


Figure 4: Extents by which the Last Successive Occurrences of 2 in Digit of π are Away from Ends

No specific regularity could be marked.

The above analysis was for 2 consecutive 2's. As many as 9 successive occurrences of 2's are found in these digit ranges.

Table 3: Multiple Successive Occurrences of 2 in Digits of π in Blocks of 10 Powers

Digit Range	Number of Successive 2's									
	1	2	3	4	5	6	7	8	9	10
$1-10^1$	1	0	0	0	0	0	0	0	0	0
$1-10^2$	12	0	0	0	0	0	0	0	0	0
$1-10^3$	103	8	0	0	0	0	0	0	0	0
$1-10^4$	1,021	110	13	2	0	0	0	0	0	0
$1-10^5$	9,908	971	102	13	1	0	0	0	0	0
$1-10^6$	100,026	10,062	1,010	101	9	1	0	0	0	0
$1-10^7$	1,000,306	99,931	9,983	980	89	9	0	0	0	0
$1-10^8$	10,001,092	1,001,323	100,367	10,054	1,028	87	2	0	0	0
$1-10^9$	100,002,410	10,005,080	999,612	99,983	9,953	999	90	11	0	0
$1-10^{10}$	1,000,017,271	100,003,041	9,999,602	999,496	100,394	10,167	980	94	6	0

In each digit block of $1 - 10^x$, after first existence, the rate of recurrence of increasing sequence of 2's drops out roughly by order of magnitude 10.

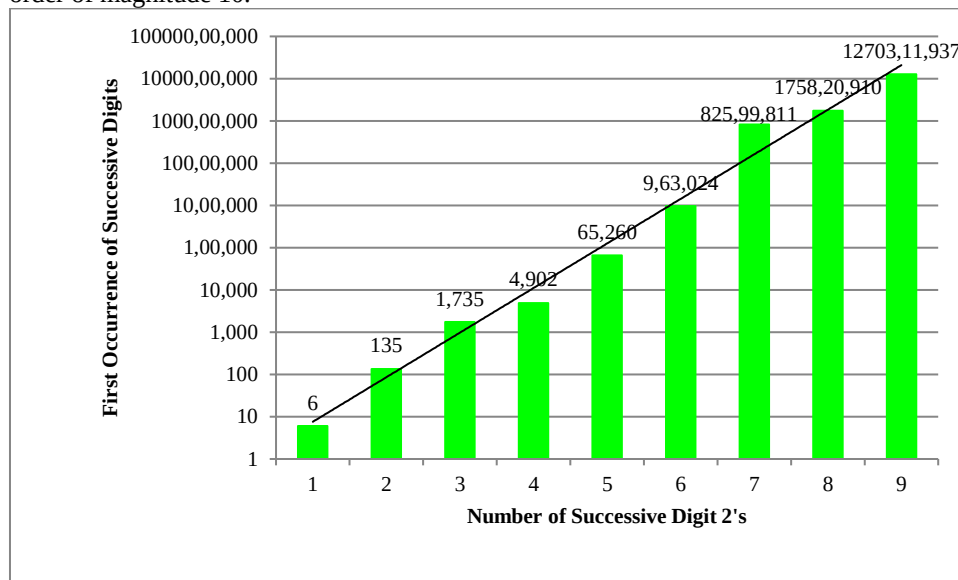


Figure 5: First Occurrences of Multiple Successive 2's in Digits of π .

The last occurrences of successive 2's in the blocks of 10^x are determined to be following.

Table 4: Last Multiple Successive Occurrences of 2 in Digits of π in Blocks of 10 Powers

2's ↓	Digit Range and Last Occurrence									
	10^1	10^2	10^3	10^4	10^5	10^6	10^7	10^8	10^9	10^{10}
1	6	93	995	9,992	99,996	999,982	9,999,954	100,000,000	999,999,992	9,999,999,999
2	-	-	964	9,967	99,844	999,981	9,999,843	99,999,954	999,999,737	9,999,999,998
3	-	-	-	9,334	98,731	999,271	9,999,842	99,999,806	999,999,351	9,999,998,320
4	-	-	-	7,964	98,423	993,920	9,999,627	99,992,428	999,999,350	9,999,984,884
5	-	-	-	-	65,260	993,919	9,750,578	99,976,780	999,906,055	9,999,706,409
6	-	-	-	-	-	963,024	9,742,967	91,604,473	998,799,215	9,996,544,541
7	-	-	-	-	-	-	-	88,301,507	984,558,912	9,996,544,540
8	-	-	-	-	-	-	-	-	933,237,784	9,786,344,533
9	-	-	-	-	-	-	-	-	-	7,819,346,553
10	-	-	-	-	-	-	-	-	-	-

Non-successive Occurrence of Digit 2 in π

Treating the occurrence of digit 2 with other digit(s) in between itself and next 2 as non-successive, it has also been analyzed. Such an occurrence is counted as belonging to a specific block of 10^n only if the next 2 falls in the same block otherwise it is not credited for that block, neither for the next block.

Table 5: Non-successive Occurrences of 2 in Digits of π in Blocks of 10 Powers

Sr. No.	Digit Numbers' Range $1 - x$	Range as Ten Power 10^x	Number of Non-Consecutive Occurrences of 2	First Non-Consecutive Occurrence of 2 at Digit Number	Last Non-Consecutive Occurrence of 2 at Digit Number
1.	$1 - 10$	10^1	0	-	-
2.	$1 - 100$	10^2	11	6	89
3.	$1 - 1,000$	10^3	94	6	991
4.	$1 - 10,000$	10^4	910	6	9,980
5.	$1 - 100,000$	10^5	8,936	6	99,984

Sr. No.	Digit Numbers' Range $1 - x$	Range as Ten Power 10^x	Number of Non-Consecutive Occurrences of 2	First Non-Consecutive Occurrence of 2 at Digit Number	Last Non-Consecutive Occurrence of 2 at Digit Number
6.	1 – 1,000,000	10^6	89,963	6	999,968
7.	1 – 10,000,000	10^7	900,374	6	9,999,940
8.	1 – 100,000,000	10^8	8,999,768	6	99,999,990
9.	1 – 1,000,000,000	10^9	89,997,328	6	999,999,976
10.	1 – 10,000,000,000	10^{10}	900,014,227	6	9,999,999,996

In the first 10 power block 1 – 10, Digit 2 doesn't occur non-consecutively even once. It starts occurring such from digit number 6, but that gets counted for higher block size as next 2 is after 10. The non-consecutive occurrences of 2 dominate those of consecutive occurrences.

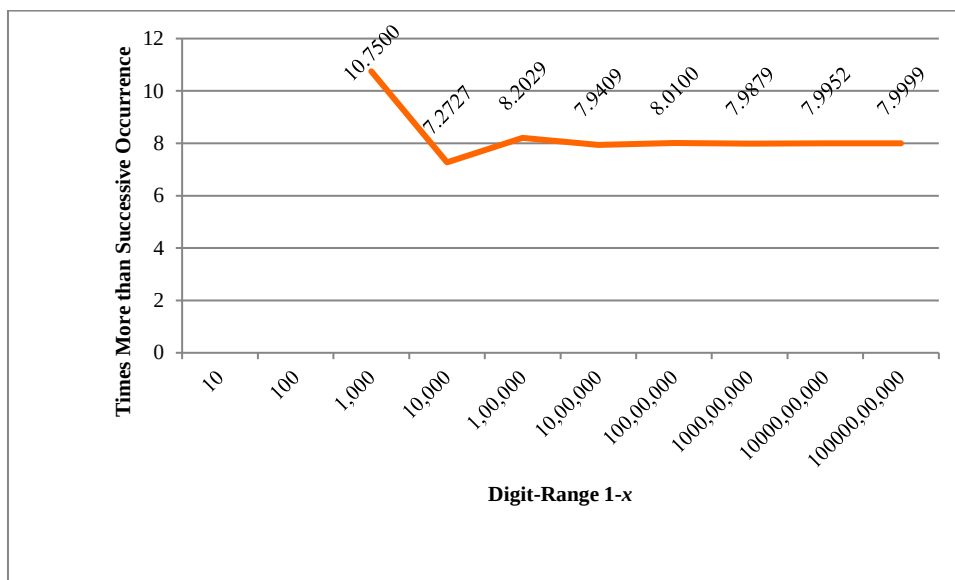


Figure 6: Number of Times Non-successive 2's are more than Successive 2's in Digits of π

Except first block of 1 – 10 of absence of occurrence, the last non-successive occurrence of digit 2 halts prior to last digit in range by following quantities.

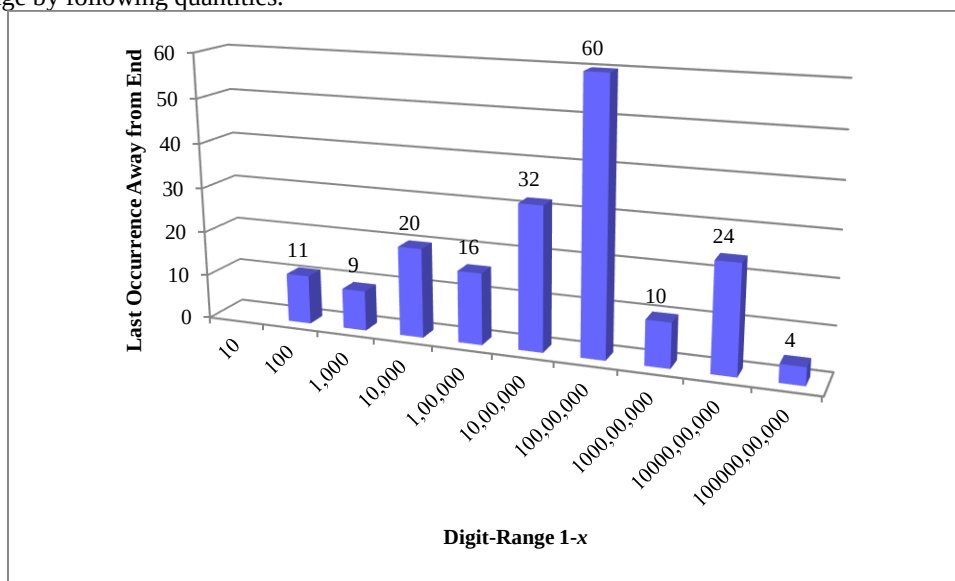


Figure 7: Extents by which the Last Non-successive Occurrence of 2 in Digits of π is Away from End

Higher frequency of non-consecutive occurrences of digit 2 results in pull down of the heights of these bars as compared to their counterparts for successive ones.

The present analysis has been in continuation with attempts [3], [4] to identify possibly existing patterns and/or regularities in decimal digits of π through occurrence of various digits. The scope is open for other remaining digits.

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