



Determination of Top Flow Line in an Embankment Using Closed Form Solution

Arun K. Chopra

Civil Department, Datta Meghe College of Engineering, Airoli Navi Mumbai 400708
Mumbai University, Maharashtra, India.

Dr. S K Ukarande,

Principal, K J Somaiya Institute of Engineering & Information Technology, Sion, Mumbai,
Mumbai University, Maharashtra, India.

Abstract: The seepage pressure is one of the most important causes that affects the stability of an embankment. The graphical properties of a flow net are used to obtain the solutions to many seepage problems such as estimation of seepage through an embankment, checking the possibility of piping beneath dams and determination of uplift pressures below dams etc. In order to construct a flow net, the foremost step is to find the location and shape of the phreatic line or the top flow line. In the present study, an attempt is further made to develop the new methods for the construction of a top flow line. For this purpose, the equation of cubic parabola and Laplace's equation are used. Further, using the Laplace's equation, a closed-form solution is derived to compute the potential at various points in the seepage zone of an embankment.

Keywords: seepage pressure, phreatic line

I. Introduction

A flow net represents a graphical solution to various equations of steady ground water flow. To arrive at a solution to various seepage problems, the properties of flow net are used, such as estimation of seepage through an embankment, checking the possibility of piping beneath dams and determination of uplift pressures below dams etc. To draw a flow net, the first step is to locate the position and shape of phreatic line. A phreatic line is defined as a line below which the soil is saturated. Also it has been established that pressure on the Phreatic line is atmospheric and established different methods such as analytical method, graphical method and experimental method to establish the top flow line. In the present study, an attempt is made to develop new methods for establishing the top flow line. And attempt has been made in closed form solution to get the co-ordinates of several points on the top flow line. The proposed results are also compared with those given by Casagrande (1937).

II. Methodology: – Using the Laplace's Equation and Assuming the Function for Potential, $\phi(x, y)$

In Fig.1.0 is shown a schematic diagram of an embankment; the other details for which are as given below. 1) TW – top width, 2) BW – bottom width, 3) α – angle made by the u/s surface with the horizontal, 4) β – angle made by the d/s surface with the horizontal, 4) H = water level and 5) $H_1 = H + \text{freeboard}$.

Laplace's Equation is given as

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \quad (1.0)$$

The potential is given by

$$\phi = a_0 + a_1x + a_2y + a_3xy \quad (\text{Satisfies the Laplace Equation}) \quad (2.0)$$

The constants, $a_{i(i=0-3)}$ in Eq. (2.0) are determined using the boundary conditions as given below

$$1. \quad \text{At } x = 0, y = H, \phi = H$$

$$\therefore H = a_0 + a_2H \quad (3.0)$$

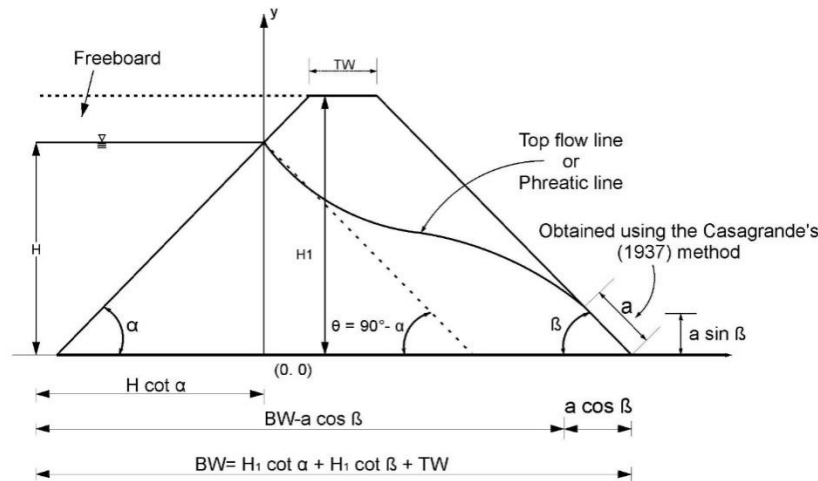


Figure 1.0 Determination of top flow line for seepage through an embankment

2. At $x = -H \cdot \cot \alpha, y = 0, \phi = H$

$$\therefore H = a_0 - a_1 H \cdot \cot \alpha \quad (4.0)$$

$$x_2 = BW - H \cdot \cot \alpha$$

3. At $x = x_2, y = 0, \phi = 0$

$$\therefore 0 = a_0 + a_1 x_2 \quad (5.0)$$

4. At $x = x_1, y = a \cdot \sin \beta, \phi = a \cdot \sin \beta$

$$a \cdot \sin \beta = a_0 + a_1 x_1 + a_2 \cdot a \cdot \sin \beta + a_3 x_1 \cdot a \cdot \sin \beta \quad (6.0)$$

Subtracting Eq. (5.0) from Eq.(4.0), the constant, a_1 is obtained as

$$H = -a_1 H \cdot \cot \alpha - a_1 x_2$$

$$\therefore a_1 = \frac{-H}{H \cdot \cot \alpha + x_2} \quad (7.0)$$

From Eq. (5.0), the constant a_0 is obtained as

$$a_0 = -a_1 x_2$$

$$\therefore a_0 = \frac{H \cdot x_2}{H \cdot \cot \alpha + x_2} \quad (8.0)$$

From Eq. (3.0), the constant a_2 is obtained as

$$a_2 H = H - a_0$$

$$\therefore a_2 = 1 - \frac{a_0}{H}$$

$$a_2 = 1 - \frac{\left\{ \frac{H \cdot x_2}{H \cdot \cot \alpha + x_2} \right\}}{H}$$

$$(9.0)$$

$$\therefore a_2 = \frac{H \cdot \cot \alpha}{H \cdot \cot \alpha + x_2}$$

Now, rewriting Eq. (2.0) using all the constants, $a_i (i = 0-3)$ as computed in the earlier steps,

$$a \cdot \sin \beta = a_0 + a_1 x_1 + a_2 \cdot a \cdot \sin \beta + a_3 x_1 \cdot a \cdot \sin \beta$$

$$a \cdot \sin \beta = \frac{H \cdot x_2}{H \cdot \cot \alpha + x_2} + \frac{-H}{H \cdot \cot \alpha + x_2} x_1 + \frac{H \cdot \cot \alpha \cdot a \cdot \sin \beta}{H \cdot \cot \alpha + x_2} + a_3 x_1 \cdot a \cdot \sin \beta$$

$$a \cdot \sin \beta \cdot x_1 \cdot a_3 = a \cdot \sin \beta - \frac{(Hx_2 - Hx_1 + H \cot \alpha \cdot a \cdot \sin \beta)}{H \cot \alpha + x_2} \quad (10.0)$$

Using Eq. (10.0), the constant, a_3 is obtained as

$$\therefore a_3 = \frac{1}{x_1} - \frac{(Hx_2 - Hx_1 + H \cot \alpha \cdot a \cdot \sin \beta)}{(H \cot \alpha + x_2) \cdot x_1 \cdot a \cdot \sin \beta} \quad (11.0)$$

Finally, the equation for the potential function is given as

$$\therefore \phi(x, y) = \frac{Hx_2}{H \cot \alpha + x_2} - \frac{Hx}{H \cot \alpha + x_2} + \frac{H \cot \alpha \cdot y}{H \cot \alpha + x_2} + \left\{ \frac{1}{x_1} - \frac{(Hx_2 - Hx_1 + H \cot \alpha \cdot a \cdot \sin \beta)}{(H \cot \alpha + x_2) \cdot x_1 \cdot a \cdot \sin \beta} \right\} \cdot xy \quad (12.0)$$

Using Eqs. (12.0), it is possible to obtain the potential, $\phi(x, y)$ at various coordinates, which further facilitates the construction of a flow net.

III. RESULTS AND DISCUSSIONS

The following is an example that demonstrates the procedure for establishing the top flow line for an embankment using the methods developed in the present study.

Given Data:

Water head (u/s) $H = 10$ m

Freeboard = 2 m

$H_1 = 10 + 2 = 12$ m

$\alpha = 45^\circ$

$\beta = 45^\circ$

Top width (TW) = 3 m

Bottom width (BW) = $H_1 \cot \alpha + H_1 \cot \beta + TW = 27$ m

Solution: Based on the computational steps as shown earlier, the following results are obtained

Table 1.0 Coordinates of several points of the phreatic line

x (m)	y (m) Casagrande's (1937) Method	$\phi(x, y)$ (m) [Proposed Method, Eq.(12.0)]
0	9.2648	10.0000
1	9.0064	9.4788
2	8.7403	9.0055
3	8.4659	8.5768
4	8.1823	8.1863
5	7.8885	7.8249
6	7.5834	7.4808
7	7.2654	7.1392
8	6.9329	6.7827
9	6.5836	6.3911
10	6.2147	5.9412
11	5.8225	5.4074
12	5.4018	4.7610
13	4.9455	3.9706

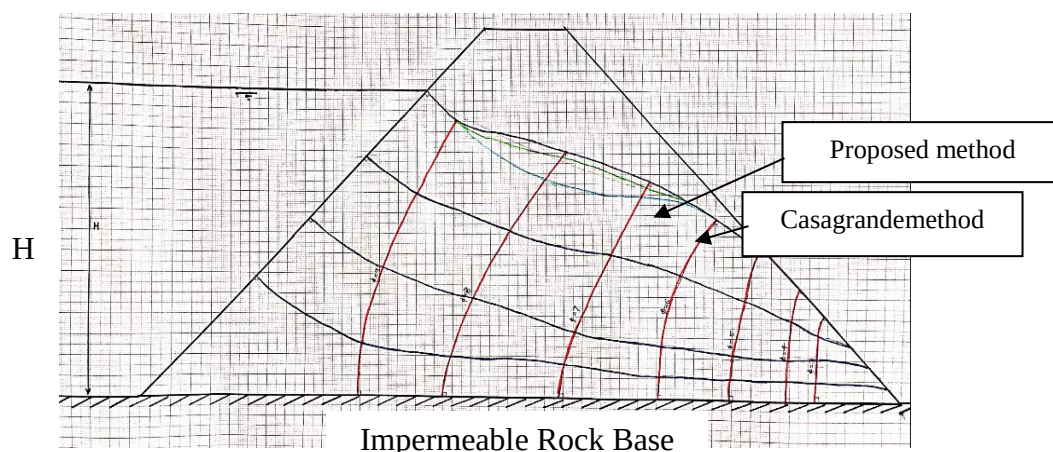


Figure 2.0 Generation of top flow lines using $\Phi = 9$ to $\Phi = 0$ and equipotential lines $\Phi = 9$ to $\Phi = 0$

In Fig. 2.0 is shown a plot of the top flow line established using proposed method and Casagrande's (1937) method. As seen from this figure, the top flow line established compares well with the one plotted using the Casagrande's (1937) method. Also, the proposed closed-form solution facilitates the construction of a very accurate flow net, which otherwise would not be possible using the available graphical methods.

IV. Summary

In the present study, closed-form solutions are obtained for establishing the top flow line in an embankment. For this purpose, new method is developed using the Laplace's equation and assuming the function for potential, $\phi(x, y)$. The proposed result is observed to be in good agreement reported by Casagrande (1937)

V. CONCLUSIONS

The conclusions that are drawn from the present study are as follows.

1. New method is proposed to establish the top flow line in the seepage zone of an embankment. For this purpose, Laplace's equation are used.
2. The closed-form solution derived using the Laplace's equation facilitates the construction of a very accurate flow net, which otherwise would not be possible using the available graphical methods

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