



One Raised Sum Prime Labeling of Some Tree Graphs

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Abstract: One raised sum prime labeling of a graph is the labeling of the vertices with $\{1, 2, \dots, p\}$ and the edges with sum of the labels of the incident vertices plus 1. The greatest common incidence number of a vertex (**gcin**) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the **gcin** of each vertex of degree greater than one is one, then the graph admits one raised sum prime labeling. Here we investigate some tree graphs for one raised sum prime labeling.

Keywords: Graph labeling, sum, prime labeling, prime graphs, tree.

I. Introduction

All graphs in this paper are simple, finite and undirected. The symbol $V(G)$ and $E(G)$ denotes the vertex set and edge set of a graph G . The graph whose cardinality of the vertex set is called the order of G , denoted by p and the cardinality of the edge set is called the size of the graph G , denoted by q . A graph with p vertices and q edges is called a (p, q) - graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1],[2],[3], [4], [5], [6] and [7]. Some basic concepts are taken from Frank Harary [2]. In this paper we investigated one raised sum prime labeling of some tree graphs.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (**gcin**) of a vertex of degree greater than or equal to 2, is the greatest common divisor (**gcd**) of the labels of the incident edges.

II. Main Results

Definition 2.1 Let $G = (V(G), E(G))$ be a graph with p vertices and q edges. Define a bijection $f: V(G) \rightarrow \{1, 2, \dots, p\}$ by $f(v_i) = i$, for every i from 1 to p and define a 1-1 mapping $f_{or\text{sp}}^*: E(G) \rightarrow$ set of natural numbers N by $f_{or\text{sp}}^*(uv) = f(u) + f(v) + 1$. The induced function $f_{or\text{sp}}^*$ is said to admit one raised sum prime labeling, if for each vertex of degree at least 2, the **gcin** of the labels of the incident edges is 1.

Definition 2.2 A graph which admits one raised sum prime labeling is called one raised sum prime graph.

Theorem 2.1 Let G be the graph obtained by joining pendant edges to each vertex of a path P_n ($n > 2$). G is called comb graph and is denoted by $P_n \odot K_1$. G admits one raised sum prime labeling.

Proof: Let $G = P_n \odot K_1$ and let $v_1, v_2, \dots, v_{2n}$ are the vertices of G .

Here $|V(G)| = 2n$ and $|E(G)| = 2n-1$.

Define a function $f: V \rightarrow \{1, 2, \dots, 2n\}$ by

$$f(v_i) = i, \quad i = 1, 2, \dots, 2n$$

Clearly f is a bijection.

For the vertex labeling f , the induced edge labeling $f_{or\text{sp}}^*$ is defined as follows:

$$\begin{aligned} f_{or\text{sp}}^*(v_{2i-1}v_{2i}) &= 4i, & i &= 1, 2, \dots, n. \\ f_{or\text{sp}}^*(v_{2i}v_{2i+2}) &= 4i+3, & i &= 1, 2, \dots, n-1. \end{aligned}$$

Clearly $f_{or\text{sp}}^*$ is an injection.

$$\begin{aligned} \text{gcin of } (v_{2i+2}) &= \text{gcd of } \{f_{or\text{sp}}^*(v_{2i}v_{2i+2}), f_{or\text{sp}}^*(v_{2i+1}v_{2i+2})\} \\ &= \text{gcd of } \{4i+3, 4i+4\} \\ &= 1, & i &= 1, 2, \dots, n-1. \\ \text{gcin of } (v_2) &= \text{gcd of } \{f_{or\text{sp}}^*(v_1v_2), f_{or\text{sp}}^*(v_2v_4)\} \\ &= \text{gcd of } \{4, 7\} \\ &= 1. \end{aligned}$$

So, **gcin** of each vertex of degree greater than one is 1.

Hence, $P_n \odot K_1$ admits one raised sum prime labeling.

Example 2.1 $G = P_4 \odot K_1$

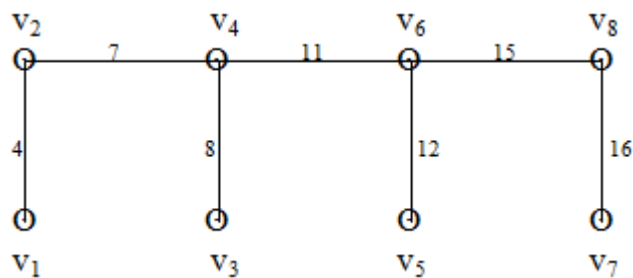


fig.2.1

Theorem 2.2 Let G be the graph obtained by joining two pendant edges to each vertex of a path P_n ($n > 2$). G is called centipede graph and is denoted by $P_n \odot 2K_1$. G admits one raised sum prime labeling.

Proof: Let $G = P_n \odot 2K_1$ and let $v_1, v_2, \dots, v_{3n}$ are the vertices of G .

Here $|V(G)| = 3n$ and $|E(G)| = 3n-1$.

Define a function $f : V \rightarrow \{1, 2, \dots, 3n\}$ by

$$f(v_i) = i, \quad i = 1, 2, \dots, 3n$$

Clearly f is a bijection.

For the vertex labeling f , the induced edge labeling $f_{or\text{sp}}^*$ is defined as follows:

$$f_{or\text{sp}}^*(v_{3i-2}v_{3i-1}) = 6i-2, \quad i = 1, 2, \dots, n.$$

$$f_{or\text{sp}}^*(v_{3i-2}v_{3i}) = 6i-1, \quad i = 1, 2, \dots, n.$$

$$f_{or\text{sp}}^*(v_{3i-2}v_{3i+1}) = 6i, \quad i = 1, 2, \dots, n-1.$$

Clearly $f_{or\text{sp}}^*$ is an injection.

$$\begin{aligned} \text{gcin of } (v_{3i-2}) &= \text{gcd of } \{f_{or\text{sp}}^*(v_{3i-2}v_{3i-1}), f_{or\text{sp}}^*(v_{3i-2}v_{3i})\} \\ &= \text{gcd of } \{6i-2, 6i-1\} \\ &= 1, \quad i = 1, 2, \dots, n. \end{aligned}$$

So, **gcin** of each vertex of degree greater than one is 1.

Hence, $P_n \odot 2K_1$ admits one raised sum prime labeling.

Example 2.2 $G = P_4 \odot 2K_1$

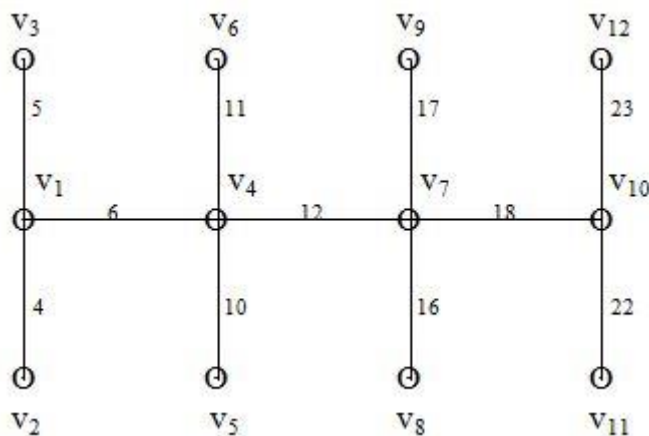


fig.2.2

Theorem 2.3 Let G be the graph obtained by joining two pendant edges to each internal vertex of a path P_n ($n > 2$). G is called twig graph and is denoted by $T_w(n)$. G admits one raised sum prime labeling.

Proof: Let $G = T_w(n)$ and let $v_1, v_2, \dots, v_{3n-4}$ are the vertices of G .

Here $|V(G)| = 3n-4$ and $|E(G)| = 3n-5$.

Define a function $f : V \rightarrow \{1, 2, \dots, 3n-4\}$ by

$$f(v_i) = i, \quad i = 1, 2, \dots, 3n-4$$

Clearly f is a bijection.

For the vertex labeling f , the induced edge labeling $f_{or\text{sp}}^*$ is defined as follows:

$$f_{or\text{sp}}^*(v_1v_2) = 4.$$

$$f_{or\text{sp}}^*(v_{3i-1}v_{3i}) = 6i, \quad i = 1, 2, \dots, n-2.$$

$$f_{or\text{sp}}^*(v_{3i-1}v_{3i+1}) = 6i+1, \quad i = 1, 2, \dots, n-2.$$

$$f_{or\text{sp}}^*(v_{3i-1}v_{3i+2}) = 6i+2, \quad i = 1, 2, \dots, n-3.$$

$$\begin{aligned}
 f_{orsp}^*(v_{3n-7}v_{3n-4}) &= 6n-10. \\
 \text{Clearly } f_{orsp}^* &\text{ is an injection.} \\
 \text{gcin of } (v_{3i-1}) &= \gcd \text{ of } \{f_{orsp}^*(v_{3i-1}v_{3i}), f_{orsp}^*(v_{3i-1}v_{3i+1})\} \\
 &= \gcd \text{ of } \{6i, 6i+1\} \\
 &= 1, \quad i = 1, 2, \dots, n-2.
 \end{aligned}$$

So, **gcin** of each vertex of degree greater than one is 1.

Hence, $T_w(n)$ admits one raised sum prime labeling.

Example 2.3 $G = T_w(4)$

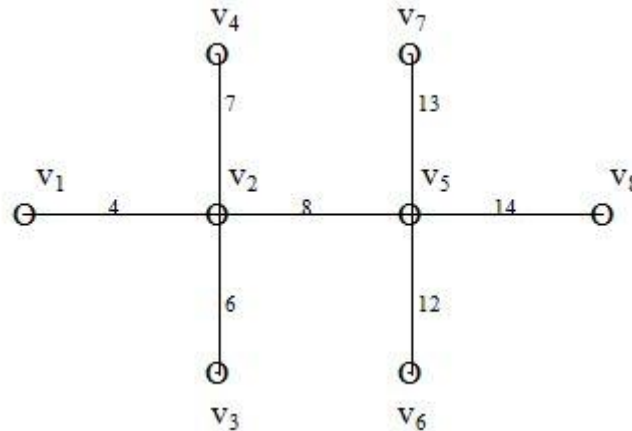


fig.2.3

Theorem 2.4 Let G be the graph obtained by joining pendant edges to each internal vertex of a path P_n ($n > 2$). G is called hurdle graph and is denoted by $Hd(n)$. G admits one raised sum prime labeling.

Proof: Let $G = Hd(n)$ and let $v_1, v_2, \dots, v_{2n-2}$ are the vertices of G .

Here $|V(G)| = 2n-2$ and $|E(G)| = 2n-3$.

Define a function $f : V \rightarrow \{1, 2, \dots, 2n-2\}$ by

$$f(v_i) = i, \quad i = 1, 2, \dots, 2n-2$$

Clearly f is a bijection.

For the vertex labeling f , the induced edge labeling f_{orsp}^* is defined as follows:

$$\begin{aligned}
 f_{orsp}^*(v_1v_2) &= 4, \\
 f_{orsp}^*(v_{2i}v_{2i+1}) &= 4i+2, \quad i = 1, 2, \dots, n-2. \\
 f_{orsp}^*(v_{2i}v_{2i+2}) &= 4i+3, \quad i = 1, 2, \dots, n-2.
 \end{aligned}$$

Clearly f_{orsp}^* is an injection.

$$\begin{aligned}
 \text{gcin of } (v_{2i}) &= \gcd \text{ of } \{f_{orsp}^*(v_{2i}v_{2i+1}), f_{orsp}^*(v_{2i}v_{2i+2})\} \\
 &= \gcd \text{ of } \{4i+2, 4i+3\} \\
 &= 1, \quad i = 1, 2, \dots, n-2.
 \end{aligned}$$

So, **gcin** of each vertex of degree greater than one is 1.

Hence, $Hd(n)$ admits one raised sum prime labeling.

Example 2.4 $G = Hd(4)$

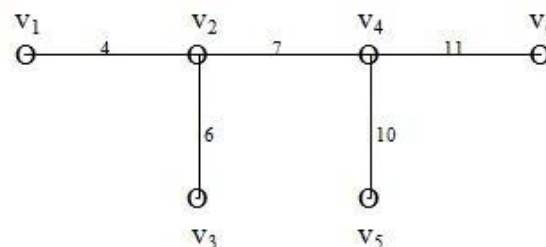


fig.2.4

Theorem 2.5 $BistarB(m, n)$ admits one raised sum prime labeling.

Proof: Let G be the graph and let $v_1, v_2, \dots, v_{m+n+2}$ are the vertices of G .

Here $|V(G)| = m+n+2$ and $|E(G)| = m+n+1$.

Define a function $f : V \rightarrow \{1, 2, \dots, m+n+2\}$ by

$$f(v_i) = i, \quad i = 1, 2, \dots, m+n+2$$

Clearly f is a bijection.

For the vertex labeling f , the induced edge labeling f_{orsp}^* is defined as follows:

$$f_{orsp}^*(v_{m+1}v_i) = m+i+2, \quad i = 1, 2, \dots, m.$$

$$\begin{aligned}
 f_{or\text{sp}}^*(v_{m+1}v_{m+2}) &= 2m+4. \\
 f_{or\text{sp}}^*(v_{m+2}v_{m+i+2}) &= 2m+i+5, & i = 1, 2, \dots, n. \\
 \text{Clearly } f_{or\text{sp}}^* \text{ is an injection.} \\
 \text{gcin of } (v_{m+1}) &= \gcd \text{ of } \{f_{or\text{sp}}^*(v_1v_{m+1}), f_{or\text{sp}}^*(v_2v_{m+1})\} \\
 &= \gcd \text{ of } \{m+3, m+4\} \\
 &= 1. \\
 \text{gcin of } (v_{m+2}) &= \gcd \text{ of } \{f_{or\text{sp}}^*(v_{m+2}v_{m+3}), f_{or\text{sp}}^*(v_{m+2}v_{m+4})\} \\
 &= \gcd \text{ of } \{2m+6, 2m+7\} \\
 &= 1.
 \end{aligned}$$

So, **gcin** of each vertex of degree greater than one is 1.

Hence, G admits one raised sum prime labeling.

Example 2.5 $G = B(3,4)$.

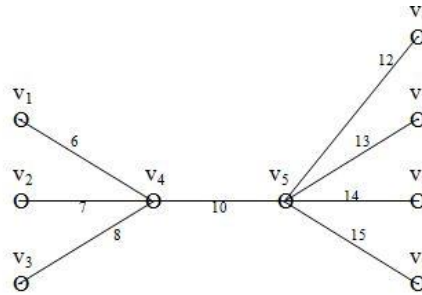


fig.2.5

Theorem 2.6 H graph of path P_n ($n > 2$) admits one raised sum prime labeling.

Proof: Let $G = H(P_n)$ be the graph and let $v_1, v_2, \dots, v_{2n}$ are the vertices of G .

Here $|V(G)| = 2n$ and $|E(G)| = 2n-1$.

Define a function $f : V \rightarrow \{1, 2, \dots, 2n\}$ by

$$f(v_i) = i, \quad i = 1, 2, \dots, 2n$$

Clearly f is a bijection.

For the vertex labeling f , the induced edge labeling $f_{or\text{sp}}^*$ is defined as follows:

$$\begin{aligned}
 f_{or\text{sp}}^*(v_{2i-1}v_{2i+1}) &= 4i+1 & i = 1, 2, \dots, n-1. \\
 f_{or\text{sp}}^*(v_{2i}v_{2i+2}) &= 4i+3, & i = 1, 2, \dots, n-1. \\
 f_{or\text{sp}}^*(v_n v_{n+1}) &= 2n+2.
 \end{aligned}$$

Clearly $f_{or\text{sp}}^*$ is an injection.

$$\begin{aligned}
 \text{gcin of } (v_{2i+2}) &= \gcd \text{ of } \{f_{or\text{sp}}^*(v_{2i}v_{2i+2}), f_{or\text{sp}}^*(v_{2i+2}v_{2i+4})\} \\
 &= \gcd \text{ of } \{4i+3, 4i+7\} \\
 &= \gcd \text{ of } \{4, 4i+3\} \\
 &= 1, & i = 1, 2, \dots, n-2. \\
 \text{gcin of } (v_{2i+1}) &= \gcd \text{ of } \{f_{or\text{sp}}^*(v_{2i-1}v_{2i+1}), f_{or\text{sp}}^*(v_{2i+1}v_{2i+3})\} \\
 &= \gcd \text{ of } \{4i+1, 4i+5\} \\
 &= \gcd \text{ of } \{4, 4i+1\} \\
 &= 1, & i = 1, 2, \dots, n-2.
 \end{aligned}$$

So, **gcin** of each vertex of degree greater than one is 1.

Hence, $H(P_n)$ admits one raised sum prime labeling.

Example 2.6 $G = H(P_5)$

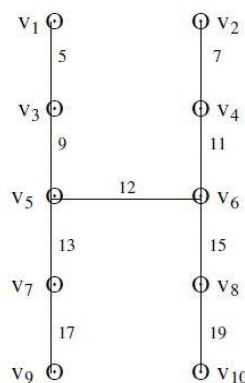


fig.2.6

Theorem 2.7 Coconut tree graph $CT(m,n)$ admits one raised sum prime labeling, if $(m+n) \not\equiv 0 \pmod{3}$.

Proof: Let G be the graph and let $v_1, v_2, \dots, v_{m+n}$ are the vertices of G .

Here $|V(G)| = m+n$ and $|E(G)| = m+n-1$.

Define a function $f: V \rightarrow \{1, 2, \dots, m+n\}$ by

$$f(v_i) = i, \quad i = 1, 2, \dots, m+n$$

Clearly f is a bijection.

For the vertex labeling f , the induced edge labeling f_{or}^* is defined as follows:

$$\begin{aligned} f_{or}^*(v_i v_{m+1}) &= m+i+2, & i &= 1, 2, \dots, m. \\ f_{or}^*(v_{m+2i-1} v_{m+2i+1}) &= 2m+4i+1, & i &= 1, 2, \dots, \frac{n-1}{2}, \text{ if } n \text{ is odd.} \\ & & i &= 1, 2, \dots, \frac{n-2}{2}, \text{ if } n \text{ is even.} \\ f_{or}^*(v_{m+2i} v_{m+2i+2}) &= 2m+4i+3, & i &= 1, 2, \dots, \frac{n-3}{2}, \text{ if } n \text{ is odd.} \\ & & i &= 1, 2, \dots, \frac{n-2}{2}, \text{ if } n \text{ is even.} \end{aligned}$$

$$f_{or}^*(v_{n+m-1} v_{n+m}) = 2n+2m.$$

Clearly f_{or}^* is an injection.

$$\begin{aligned} \text{gcin of } (v_{m+1}) &= \gcd \{f_{or}^*(v_1 v_{m+1}), f_{or}^*(v_2 v_{m+1})\} \\ &= \gcd \{m+3, m+4\} \\ &= 1. \\ \text{gcin of } (v_{m+2i+1}) &= \gcd \{f_{or}^*(v_{m+2i-1} v_{m+2i+1}), f_{or}^*(v_{m+2i+1} v_{m+2i+3})\} \\ &= \gcd \{2m+4i+1, 2m+4i+5\} \\ &= \gcd \{4, 2m+4i+1\} \\ &= 1, & i &= 1, 2, \dots, \frac{n-3}{2}, \text{ if } n \text{ is odd.} \\ & & i &= 1, 2, \dots, \frac{n-4}{2}, \text{ if } n \text{ is even.} \\ \text{gcin of } (v_{m+2i+2}) &= \gcd \{f_{or}^*(v_{m+2i} v_{m+2i+2}), f_{or}^*(v_{m+2i+2} v_{m+2i+4})\} \\ &= \gcd \{2m+4i+3, 2m+4i+7\} \\ &= \gcd \{4, 2m+4i+3\} \\ &= 1, & i &= 1, 2, \dots, \frac{n-5}{2}, \text{ if } n \text{ is odd.} \\ & & i &= 1, 2, \dots, \frac{n-4}{2}, \text{ if } n \text{ is even.} \\ \text{gcin of } (v_{m+n-1}) &= \gcd \{f_{or}^*(v_{m+n-3} v_{m+n-1}), f_{or}^*(v_{m+n-1} v_{m+n})\} \\ &= \gcd \{2m+2n-3, 2m+2n\} \\ &= \gcd \{3, 2m+2n-3\} \\ &= 1. \\ \text{gcin of } (v_{m+n}) &= \gcd \{f_{or}^*(v_{m+n-2} v_{m+n}), f_{or}^*(v_{m+n-1} v_{m+n})\} \\ &= \gcd \{2m+2n-1, 2m+2n\} = 1. \end{aligned}$$

So, **gcin** of each vertex of degree greater than one is 1.

Hence, G admits one raised sum prime labeling.

Example 2.7 $G = B(3,5)$.

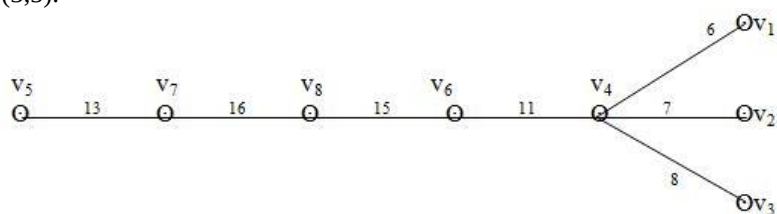


fig.2.7

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