



Seepage Analysis of an Embankment

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Abstract: The seepage pressure is one of the most important causes that affects the stability of an embankment. The graphical properties of a flow net are used to obtain the solutions to many seepage problems such as estimation of seepage through an embankment, checking the possibility of piping beneath dams and determination of uplift pressures below dams etc. In order to construct a flow net, the foremost step is to find the location and shape of the phreatic line or the top flow line. In the present study, an attempt is further made to develop the new methods for the construction of a top flow line. For this purpose, the equation of cubic parabola and Laplace's equation are used. Further, using the Laplace's equation, a closed-form solution is derived to compute the potential at various points in the seepage zone of an embankment.

Keywords: seepage pressure, phreatic line.

I. INTRODUCTION

A flow net is a graphical solution to the equations of steady ground water flow. The graphical properties of a flow net are used to obtain the solutions to many seepage problems such as estimation of seepage through an embankment, checking the possibility of piping beneath dams and determination of uplift pressures below dams etc. In order to construct a flow net, the foremost step is to find the location and shape of the phreatic line or the top flow line. A phreatic line is defined as the line within a dam section below which there are positive hydrostatic pressures in the dam. The hydrostatic pressure on the phreatic line itself is atmospheric. Several researchers have proposed different methods, viz., analytical method, graphical method and experimental method to establish the top flow line. In the present study, an attempt is made to develop new methods for establishing the top flow line. Subsequently, the closed-form solutions are proposed to obtain the coordinates of several points on the top flow line. The proposed results are also compared with those given by Casagrande (1937).

I. METHODOLOGY: – Using the Equation of Cubic Parabola

In Fig. 1.0 is shown a schematic diagram of an embankment; the other details for which are as given below. 1) TW – top width, 2) BW – bottom width, 3) α – angle made by the u/s surface with the horizontal, 4) β – angle made by the d/s surface with the horizontal, 5) H = water level and 5) $H_1 = H + \text{freeboard}$.

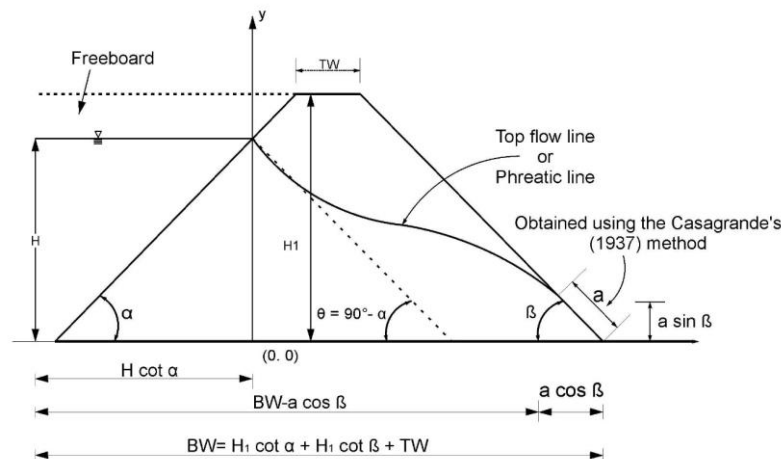


Figure 1.0 Determination of top flow line for seepage through an embankment

As discussed earlier, shape of the top flow line is approximated by parabola. Therefore, referring to Fig. 1.0, the equation of the top flow line is given as

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 \quad (\text{cubic parabola}) \quad (1.0)$$

The constants, $a_i (i = 0-3)$ in Eq. (1.0) are determined using the boundary conditions as given below.

1. At $x = 0$, $y = H$

$$H = a_0 + a_1 \cdot 0 + a_2 \cdot 0 + a_3 \cdot 0 = a_0$$

$$\therefore a_0 = H$$

$$\therefore y = H + a_1x + a_2x^2 + a_3x^3$$

2. At $x = 0$, $dy/dx = -\tan \theta$

$$dy/dx = a_1 + 2a_2x + 3a_3x^2$$

$$\therefore \text{at } x = 0, dy/dx = a_1 = -\tan \theta$$

$$\therefore y = H - \tan \theta \cdot x + a_2x^2 + a_3x^3$$

3. At $x = BW - a \cdot \cos \beta - H \cdot \cot \alpha = x_1$, $y = a \cdot \sin \beta$

$$\therefore a \cdot \sin \beta = H - \tan \theta \cdot x_1 + a_2x_1^2 + a_3x_1^3 \quad (2.0)$$

4. At $x = x_1$, $dy/dx = -\tan \beta$

$$y = H - \tan \theta \cdot x + a_2x^2 + a_3x^3$$

$$dy/dx = -\tan \theta + 2a_2x + 3a_3x^2$$

$$-\tan \beta = -\tan \theta + 2a_2x_1 + 3a_3x_1^2 \quad (3.0)$$

Multiplying Eq. (3.0) by $0.5x_1$

$$-0.5x_1 \cdot \tan \beta = -0.5x_1 \cdot \tan \theta + a_2x_1^2 + 1.5a_3x_1^3 \quad (4.0)$$

Subtracting Eq. (2.0) from Eq. (4.0), the constant, a_3 is obtained as

$$0.5a_3x_1^3 = H - 0.5x_1 \cdot \tan \theta - 0.5x_1 \cdot \tan \beta - a \cdot \sin \beta$$

$$\therefore a_3 = \frac{\{2H - x_1(\tan \theta + \tan \beta) - 2 \cdot a \cdot \sin \beta\}}{x_1^3}$$

Now, rewriting Eqs. (2.0) and (3.0),

$$-\tan \beta = -\tan \theta + 2a_2x_1 + 3a_3x_1^2 \quad (3.0)$$

$$a \cdot \sin \beta = H - \tan \theta \cdot x_1 + a_2x_1^2 + a_3x_1^3 \quad (4.0)$$

Multiplying Eq. (2.0) by Eq. (3.0),

$$3 \cdot a \cdot \sin \beta = 3H - 3 \cdot \tan \theta \cdot x_1 + 3a_2x_1^2 + 3a_3x_1^3 \quad (5.0)$$

Multiplying Eq. (3.0) by x_1

$$-\tan \beta \cdot x_1 = -\tan \theta \cdot x_1 + 2a_2x_1^2 + 3a_3x_1^3 \quad (6.0)$$

Subtracting Eq. (6.0) from Eq. (5.0), the constant, a_2 is obtained as

$$3 \cdot a \cdot \sin \beta + \tan \beta \cdot x_1 = 3H + \tan \theta \cdot x_1 - 3 \cdot \tan \theta \cdot x_1 + a_2x_1^2$$

$$a_2x_1^2 = 3 \cdot a \cdot \sin \beta + \tan \beta \cdot x_1 - 3H - \tan \theta \cdot x_1 + 3 \cdot \tan \theta \cdot x_1$$

$$\therefore a_2 = \frac{\{3(a \cdot \sin \beta - H) + x_1(\tan \beta + 2 \cdot \tan \theta)\}}{x_1^2}$$

Thus, the equation for the top flow line is obtained as

$$y = H - \tan \theta \cdot x + \frac{\{3(a \cdot \sin \beta - H) + x_1(\tan \beta + 2 \cdot \tan \theta)\}}{x_1^2} \cdot x^2 + \frac{\{2H - x_1(\tan \theta + \tan \beta) - 2 \cdot a \cdot \sin \beta\}}{x_1^3} \cdot x^3 \quad (7.0)$$

II. RESULTS AND DISCUSSIONS:

The following is an example that demonstrates the procedure for establishing the top flow line for an embankment using the methods developed in the present study.

Given Data:

Water head (u/s) $H = 10$ m

Freeboard = 2 m

$H_1 = 10 + 2 = 12$ m

$\alpha = 45^\circ$

$\beta = 45^\circ$

Top width (TW) = 3 m

Bottom width (BW) = $H_1 \cot \alpha + H_1 \cot \beta + TW = 27$ m

Solution: Based on the computational steps as shown earlier, the following results are obtained

Table 1.0 Coordinates of several points of the phreatic

x (m)	y (m) [Proposed Method, Eq.(7.0)]	y (m) Casagrande's (1937) Method
0	10.0000	9.2648
1	9.1128	9.0064
2	8.4275	8.7403
3	7.9083	8.4659
4	7.5195	8.1823
5	7.2253	7.8885
6	6.9900	7.5834
7	6.7779	7.2654
8	6.5531	6.9329
9	6.2801	6.5836
10	5.9229	6.2147
11	5.4460	5.8225
12	4.8135	5.4018
13	3.9898	4.9455

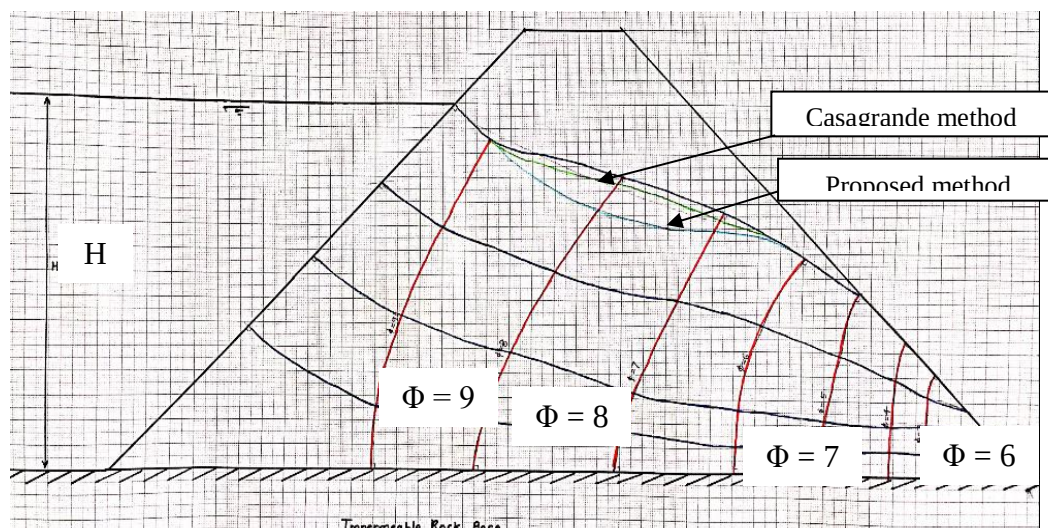


Figure 2.0 Generation of top flow lines using the proposed methods and construction of flow nets

In Fig. 2.0 is shown a plot of the top flow line established using proposed method and Casagrande's (1937) method. As seen from this figure, the top flow line established compares well with the one plotted using the Casagrande's (1937) method. Also, the proposed closed-form solution facilitates the construction of a very accurate flow net, which otherwise would not be possible using the available graphical methods

III. SUMMARY:

In the present study, closed-form solutions are obtained for establishing the top flow line in an embankment. For this purpose, new method is developed using the equation of cubic parabola and Laplace's equation.. The proposed results is observed to be in good agreement reported by casagrande(1987).

IV. CONCLUSION

The conclusions that are drawn from the present study are as follows.

1. New method is proposed to establish the top flow line in the seepage zone of an embankment. For this purpose, the equation of cubic parabola and Laplace's equation are used.
2. The closed-form solution derived using the Laplace's equation facilitates the construction of a very accurate flow net, which otherwise would not be possible using the available graphical methods.

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