



Design of a Model of a System to Control the Pitch angle of Helicopter using State Estimator

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Abstract: This paper describes the design of a model of state feedback System to control the pitch point of one helicopter by altering the rotor point, δ . The autopilot completely controls the overall framework by the pitch point. It begins with a determination of a reasonable scientific show to depict the flow of an airplane. The design of controllers utilizing state feedback model is the subject of the proposed work. Here we first present a system test for controllability and observability and proceed to describe one procedure for determining an optimal control system. Ackermann's equation can be utilized to decide the state variable input pick up the framework to put the framework posts in the specified areas. The closed – circle framework post areas can be self-assertively set in the event that and as it were in the event that the framework is controllable. We moreover portray the use of inside demonstrate plan to attain the endorsed steady-state reaction to choose input commands.

Key Words: Aircraft pitch angle, rotor angle, state space model, state feedback model, state variable feedback

I. Introduction

Over the past few decades, a vast amount of research has been expended on linear feedback control theory and its application to the design of control systems. If we look back over the development falls into the following two clearly-defined phases: classical approach and state variable approach. The transfer function method is very convenient in the frequency domain analysis of the system. However, this method suffers a major disadvantage, in that all initial conditions of the system are neglected. Therefore when one is interested in time domain solution, the transfer function method is inadequate. Further, we cannot apply the transfer models to nonlinear time-varying systems. The transfer module cannot be effective for the multivariable system (i.e. system with many inputs and outputs). The transfer function approach confines the inputs and outputs behavior of linear system only. On the contrary, the state space representation gives information about the internal behavior of the system as well as information about its the input – output behavior. In state- variable approach, perfect control of the system is possible. The beauty of the state variable approach, perfect control of the system is possible. The beauty of the state –variable approach is that it can convert a high order differential equation to a first order vector matrix differential equation. We develop a state variable controller that operates on the information available in measured form. The type of system compensation is quite useful for system optimization. We first determine an optimal controller for a system described in terms of state variables. We then describe the design of state variable feedback systems with specified characteristic root locations. We also briefly consider the internal model design method.

II. Transient Response

The pole and zero locations affect the transient response of a system is critical when designing a state feedback controller based on pole placement.

The equations for 2nd order systems that relate percent overshoot and settling time for the step response to the damping coefficient and natural frequency of the closed-loop system poles are given by:

$$\text{Percent Overshoot} = \exp(-\pi\xi / \sqrt{1-\xi^2})$$

$$\text{Settling Time} = 4 / (\xi\omega_n)$$

Where: ω_n = natural frequency, ξ = damping ratio.



System A is a 2nd order system

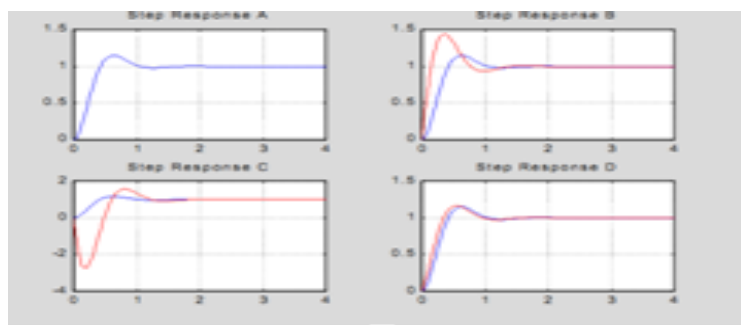


Figure 1 Effect of Zeros on System Transient Response

with complex poles.



System B has the same poles as System A with an additional zero.



System C has the same poles as System A with an additional zero at +1 (non-minimum phase system).



System D has the same poles as System A with an additional zero. Since this zero is not close to the complex pole, it has virtually no effect on the step response

III. Advantages of State Feedback over Output Feedback

Output feedback is sufficient for many systems; state feedback is very useful for multi-input multi-output systems and for control systems with optimal constraints such as those requiring minimal control effort or minimum time to final value. The response for one particular set of gains is shown in the scope plot in Figure 2

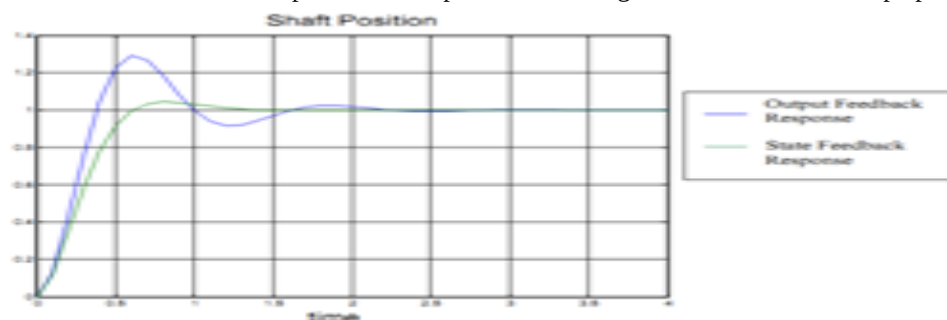


Figure 2: Shaft Position of Motor

IV. Pole Placement Using State Feedback

The state equation for state x with input u is given by a relation as:

$$\begin{aligned} \frac{dx(t)}{dt} &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned}$$

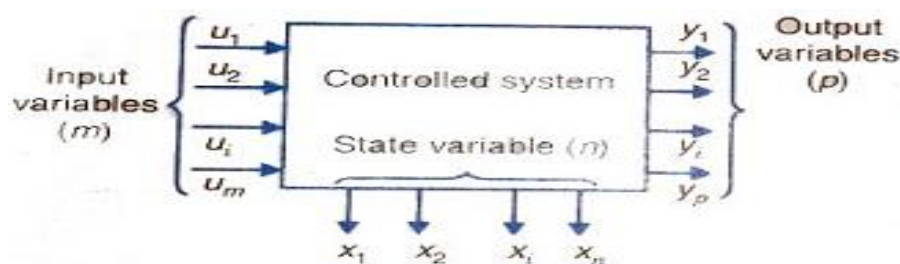


Fig: A

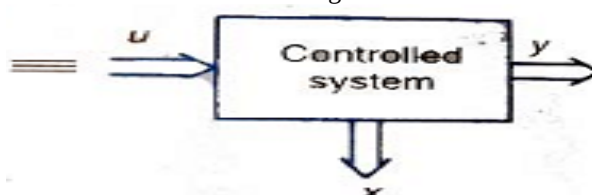


Fig. B

Fig. A & Fig. B Structure of a general control system

Where A is called system matrix, B the input coupling, C the output coupling matrix and D input- output coupling or direct transmission matrix

The controllability matrix is then defined as

$$[B \ AB \ A^2B \ \dots \ A^{n-1}B]$$

Where n is the system order or equivalently the number of states. The theorem concerning pole placement is then introduced.

Theorem: The system is controllable (i.e., the closed-loop poles can be placed in any desired position) if the determinant of the controllability matrix $[B \ AB \ A^2B \ \dots \ A^{n-1}B]$ is non-zero.

The controllability model for a system is then introduced. It is shown that for any given set of desired poles, a set of feedback gains can be derived to place the system closed-loop poles at the desired positions. Students are reminded of the exercise with the dc motor (discussed in the previous section) where they derived a closed-loop transfer function and showed they had complete freedom in specifying the location of the poles by properly choosing the feedback gains. Finally, Ackermann's formula is introduced to compute the state-feedback gains to place the poles in the desirable positions:

$$K = [0 \ 0 \ \dots \ 0 \ 1] [B \ AB \ A^2B \ \dots \ A^{n-1}B]^{-1} \alpha_c(A)$$

$(1 \times n) \qquad \qquad \qquad (n \times n) \qquad \qquad \qquad (n \times n)$

Where $\alpha_c(s)$ is the desired characteristic polynomial for the closed-loop system.

We want to design a model of state feedback System to control the pitch angle of a helicopter. A high performance helicopter has a model shown in fig.3

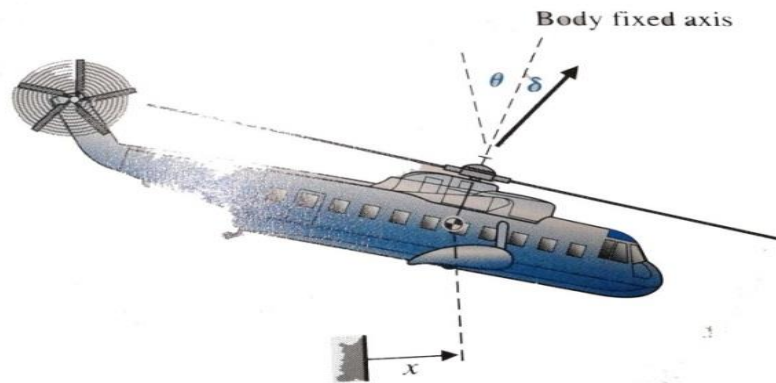


Fig 3: Helicopter pitch angle, θ , control

The equations of motion of the Helicopter are given by:

$$\frac{d^2\theta}{dt^2} = -\sigma_1 \frac{d\theta}{dt} - \alpha_1 \frac{dx}{dt} + n\delta$$

$$\frac{d^2x}{dt^2} = g\theta - \alpha_2 \frac{d\theta}{dt} - \sigma_2 \frac{dx}{dt} + g\delta$$

Where x is the translation in the horizontal direction. For a military, high-performance helicopter we find that [3]

$$\begin{array}{ll} \sigma_1 = 0.415 & \alpha_2 = 1.43 \\ \sigma_2 = 0.0198 & \alpha_1 = 6.27 \\ \alpha_1 = 0.0111 & g = 9.8 \end{array}$$

all in appropriate S.I, units.

So the equations of motion of the Helicopter become

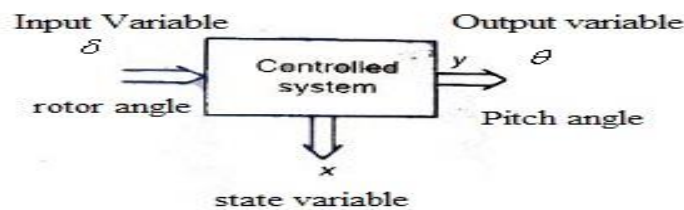
$$\frac{d^2\theta}{dt^2} = -0.415 \frac{d\theta}{dt} - 0.0111 \frac{dx}{dt} + 6.27\delta$$

$$\frac{d^2x}{dt^2} = 9.8\theta - 1.43 \frac{d\theta}{dt} - 0.0198 \frac{dx}{dt} + 9.8\delta$$

where θ is the pitch angle, x is the translation in the horizontal direction, and δ is the rotor angle. Here described a step-by-step procedure for designing a state-feedback controller. The steps with application to the pitch control system for the helicopter are included here.

The performance specifications for this controller are a maximum 20% overshoot to a step change in the rotor angle and a maximum settling time of 10 seconds.

Step 1: Derive the state model and enter it into MATLAB.



The states are defined as:

- $x_1 = \theta$ (pitch angle)
- $x_2 = d\theta/dt$ (pitch velocity)
- $x_3 = dx/dt$ (horizontal velocity)

The input is the rotor angle, δ , and the output is the resulting pitch angle, θ .

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \frac{dx_3}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.415 & -0.0111 \\ 9.8 & -1.43 & -0.0198 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 6.27 \\ 9.8 \end{bmatrix} \delta$$

$$a = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.415 & -0.0111 \\ 9.8 & -1.43 & -0.0198 \end{bmatrix}; b = \begin{bmatrix} 0 \\ 6.27 \\ 9.8 \end{bmatrix}$$

So,

If the Helicopter pitch angle is regarded as the output, it can be written as

$$y = \theta = x_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

So, $c = [1 \ 0 \ 0]$; $d=0$;

The MATLAB statements for entering the state model are:

```

a=[1 1 0;0 -0.415 -0.011;9.8 -1.43 -0.0198];
b=[0; 6.27; 9.8 ];
c=[1 0 0]; d=0;
  
```

Step 2: Check for controllability of the system.

```

Pc=ctrb(a,b); ⇐ Compute controllability matrix
n=det(Pc) ⇐ [n= determinant of controllability matrix]
  
```

ans n = -2.4161e+003 ⇐ The system is controllable.

Step 3: Find the zeros of the system.

```
[num,den]=ss2tf(a,b,c,d);
zeros=roots(num)
```

zeros = -2.4507e-003

Step 4: Determine the desired pole locations based on the performance specifications.

The damping coefficient corresponding to a percent overshoot of 20% is $\xi = 0.45$. The natural frequency, w_n , needed to satisfy the settling time specification and the resulting closed-loop poles are calculated using the MATLAB statements:

```
zeta=0.45; ts=10;
wn=4/(zeta*ts)
realpart=-zeta*wn
imagpart=wn*sqrt(1-zeta^2)
```

$w_n = 8.8889\text{e-}001$

realpart = -4.0000e-001

imagpart = 7.9380e-001

The desired dominant complex poles are at $-0.4 + j 0.8$. What about the other pole? Place the other pole at -0.0025 to cancel the effects of the zero. It is a very interesting if this zero is neglected and the third pole is chosen far to the left of the complex poles, say at -2.0.

Step 5: Consider to find the feedback gains to place the poles at the desired positions.

(a) Find the desired denominator for the closed-loop transfer function:

```
>> desired = conv([1 0.4 + 0.8j],[1 0.4-0.8j]);
desired = conv(desired, [1 0.0025])
```

desired = 1.0000 0.8025 0.8020 0.0020

(b) Calculating the feedback gains using Ackermann's formula

```
k=[0 0 1]*inv(ctrb(a,b))*polyvalm(desired,a)
```

Ans: k = 0.1276 0.0614 -0.0018

(c) Check to make sure the closed-loop poles are in the desired locations:

```
anew=a-b*k;[clnum,clden]=ss2tf(anew,b,c,d); poles=roots(clden)
```

poles =

-0.4000 + 0.8000i

-0.4000 - 0.8000i

-0.0025

```
anew=a-b*k;
[clnum,clden]=ss2tf(anew,b,c,d);
poles=roots(clden)
poles =
-4.0000e-001 +8.0000e-001i
-4.0000e-001 -8.0000e-001i
-2.5000e-003
```

Step 6: Simulate the system response to check specifications:

The system response can be generated using the MATLAB commands step or lsim but students get a better understanding of the feedback system by building it in SIMULINK using the State Variable block. The block diagram for the pitch angle control system and the system response to a step change in the rotor angle are shown in Figure 6.

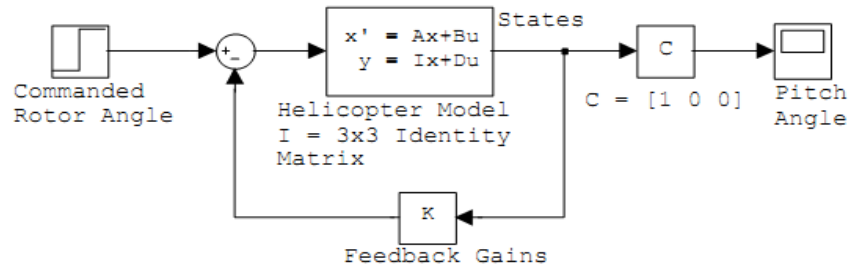


Figure 6 (a): State Feedback Controller for Pitch Angle of Helicopter

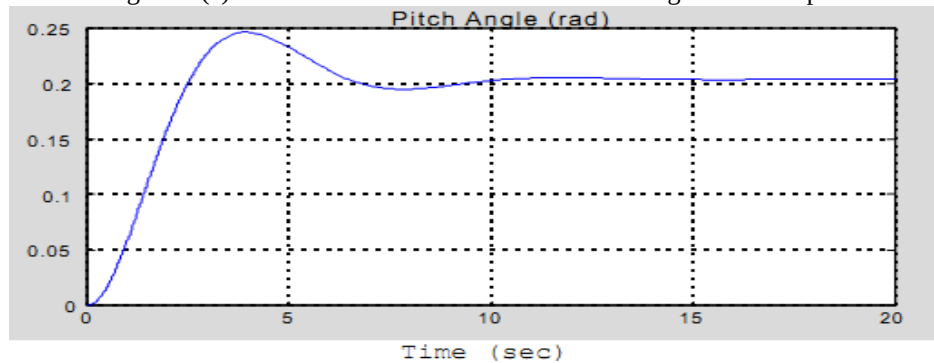


Figure 6 (b): Pitch Angle of Helicopter

V. Designing Observers for State Estimation

In many systems, some of the states of the system may not be measurable and available for state feedback. In this case, the actual state is replaced by an estimate of that state derived from a state estimator or a state observer. As in the case of state feedback, the theory behind state estimation is discussed in lecture without any formal proofs of theorems. The performance specifications for the controller are: a maximum 20% overshoot to a step change in the rotor angle and a maximum settling time of 10 seconds. Therefore, the feedback controller gains are identical to those computed in the previous section. The desired pitch angle of 0.2 radians is assumed. Also, the initial conditions of the system are assumed to be $q = 0$; $\dot{q}/dt=0$; and $dx/dt=15$. The system block diagram and the system response are shown in Figure 7(a) and Figure 7(b).

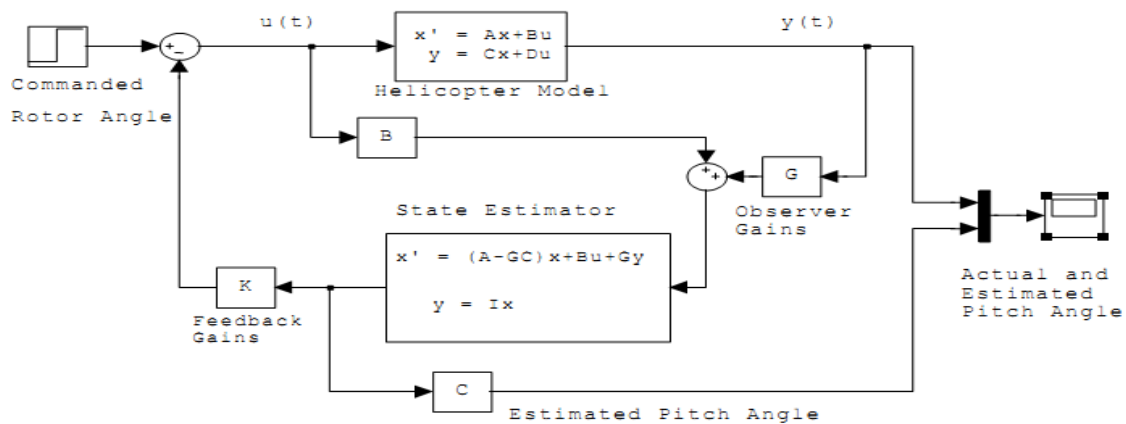


Figure 7 (a): Pitch Control for Helicopter using State Estimation

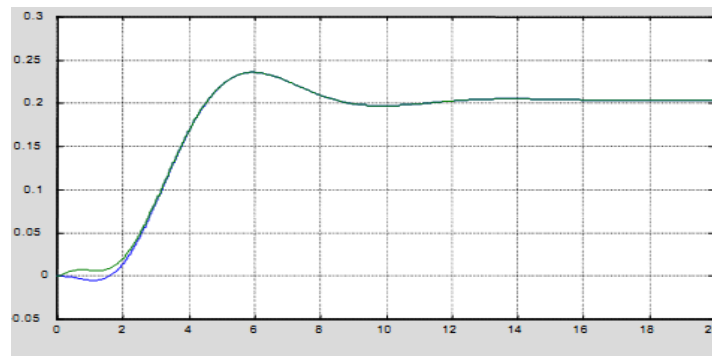


Figure 7 (b): Actual and Estimated Pitch Angles

VI. CONCLUSION

In this paper, it is seen that for control the pitch point, θ of the helicopter by altering the rotor point, δ , we to begin with show a framework test for controllability and perceptibility and continue to depict one method for deciding an ideal control framework. Utilizing the capable idea of state variable input, we present the shaft arrangement plan strategy. Ackermann's equation can be utilized to decide the state variable criticism pick up framework to put the framework posts at the specified areas. The closed-loop framework shaft areas can be self-assertively put on the off chance that and as it were on the off chance that the framework is controllable.

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REFERENCES

- [1] Heffley, R.K., "A Compilation and Analysis of Helicopter Handling Qualities Data" 1979.
- [2] Tischler, M.B. and M.G. Cauffman, "Frequency-Response Method for Rotorcraft System Identification" Journal of the American Helicopter Society, pp.3-17 1992.
- [3] C.Kuo., Digital Control Systems. Oxford Univ. Press, 1992.
- [4] Dorf and Bishop, Modern Control Systems, 7th Edition, Addison-Wesley, 1995.
- [5] Prouty, R.W., "Helicopter Performance, Stability and Control", 1995.
- [6] Mettler, B.F., M.B. Tischler, and T. Kanade, "System Identification of Small-Size Unmanned Helicopter Dynamics." 1999.
- [7] Phillips and Harbor, Feedback Control Systems, 4th Edition, Prentice Hall, 2000.
- [8] W. Staruk, B.Woods, B. Berry, and G. Bowen-Davies, "DESIGN AND FAB- RICATION OF ULTRA- LIGHTWEIGHT COMPOSITE STRUCTURES FOR THE GAMERA HUMAN- POWERED HELICOPTERS", 2011.