



## Selection of Bayesian Single Sampling Plan with Gamma Zero – Inflated Poisson Model based on OAOQL

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**Abstract:** Sampling plans, systems and schemes are used to accept/reject the lots when inspecting a series of lots. A sampling system is a grouping of two or more sampling plans, with specified rules for switching between the plans and selecting the lots of finished products. Count a defecting unit in a production is contracted process in some situation that may have zeros. This situation very much suited distribution is Zero – inflated Poisson distribution than other process. Single sampling plan by attribute in using Bayesian approach assuming that the defective units are distributed to a gamma zero – inflated Poisson distribution. In this paper, a procedure is developed to draw an Overall Average Outgoing Quality Level (OAOQL), curve by using Zero - Poisson distribution and compare the different procedures in order to show the ambiguity of the procedure and their results.

**Keywords:** Single sampling plan, Zero – Inflated Poisson distribution with gamma prior, Overall Average Outgoing Quality Level (OAOQL).

### I. Introduction

The zero-inflated Poisson (ZIP) is an alternative process is considered here. This model allows for over dispersion assuming that there are two different types of individuals in the data: (1) Those who have a zero count with a probability of 1 (*Always-0 group*), and (2) those who have counts predicted by the standard Poisson (*Not always-0 group*). Observed zero could be from either group, and if the zero is from the Always-0 group, it indicates that the observation is free from the probability of having a positive outcome (Long 1997). The overall model is a mixture of the probabilities from the two groups, which allows for both the over dispersion and excess zeros that cannot be predicted by the standard Poisson model. Acceptance sampling is a major field/tool in Statistical Quality Control to inspect the quality of the product or raw material at various stages against the specified quality standards. Various sampling plans, systems and schemes were developed and applied in the industries based on the need of the shop floor situations. When 100% inspection is not possible there may be use acceptance sampling technique, because when using sampling methods there arise more risks, but in acceptance sampling will reduce the risks of producer's as well as consumer's sides. Now a days 100% inspection is not possible for all the industries. Buyer's thought will change day by day. So an industry wanted to maintain the quality standard becomes important and availability of that product in market is essential and price of the product must be considerable.

ZIP distribution has been used in a wide range of disciplines such as agriculture, epidemiology, econometrics, public health, process control, medicine, manufacturing, etc. Some of the applications of ZIP distribution can be found in Bohning et al. (1999), Lambert (1992), and Yang et al. (2011). Construction of control charts using ZIP distribution are discussed in Sim and Lim (2008). Some theoretical aspects of ZIP distributions are mentioned in McLachlan and Peel (2000). Single sampling plans by attributes under the conditions of Zero – inflated Poisson distribution are determined by Loganathan and Shalini (2013), Suresh and Latha (2001), (2002) have given a procedure and tables for the selection of Bayesian SSP with gamma Poisson model. This paper attempts gamma zero - inflated Poisson distribution by attributing single sampling plan.

In most sampling plans, when a lot is rejected, the entire lot is inspected and all of the defective items are replaced. Use of this replacement technique improves the average outgoing quality in terms of percent defective. In fact, given any sampling plan that replaces all defective items encountered the true incoming percent defective for the lot, it is possible to determine the average outgoing quality (AOQ) in percent defective. Sampling risks are of two kinds (i) Producer's risk (ii) Consumer's risk. This study examines single sampling plans for variables indexed by AQL and AOQL under measurement error. Procedures and tables are provided for selecting single sampling plans for attributes for given AQL and AOQL when rejected lots are 100% inspected for replacement of nonconforming units. For a particular sampling plan in operation for observed measurement, a method of determining the AOQ curves is described for various errors sizes.

## Operating procedure of SSP

The operating procedure of a Bayesian SSP described as follows:

- 1) Draw a random sample of  $n$  units from a lot of  $N$  units.
- 2) Count the number of defective units,  $x$  in the sample
- 3) If  $x \leq c$ , accept the lot, otherwise, reject the lot.

Where,  $N$ ,  $n$ ,  $c$  are parameters of SSP,  $N$  is lots,  $n$  is sample, and  $c$  is acceptance constant.

## II. Operating Characteristic function of SSP with Gamma- ZIP model

The OC function of SSP is defined as

$$P_a(p) = P[X \leq c] \quad (1)$$

Where  $p$  is the probability of fraction defective

The numbers of defects are zero for many samples there may consider Zero – inflated Poisson probability distribution. The probability mass function of the ZIP ( $\phi, \lambda$ ) distribution is given by Lambert (1992) and McLachlan and Peel (2000)

$$P(X = x | \phi, \lambda) = \phi f(x) + (1 - \phi) P(X = x | \lambda) \quad (2)$$

where

$$f(x) = \begin{cases} 1, & \text{if } x = 0 \\ 0, & \text{if } x \neq 0 \end{cases}$$

and

$$P(X = x | \lambda) = \frac{e^{-\lambda} \lambda^x}{x!}, \quad \text{when } x = 0, 1, 2, \dots$$

The above probability mass function can also be expresses as

$$P(X = x | \phi, \lambda) = \begin{cases} \phi + (1 - \phi)e^{-\lambda} & \text{when } x = 0 \\ (1 - \phi) \frac{e^{-\lambda} \lambda^x}{x!}, & \text{when } x = 1, 2, \dots, 0 < \phi < 1, \lambda > 0 \end{cases}$$

In this distribution,  $\phi$  may be termed as the mixing proportion.  $\phi$  and  $\lambda$  are the parameters of the ZIP distribution. According to McLachlan and Peel (2000), a Zip distribution is a special kind of mixture distribution.

The OC function of the SSP under the conditions of ZIP ( $\phi, \lambda$ ) distribution can be defined as

$$P_a(p) = \sum_{x=0}^c P(X = x | \phi, \lambda)$$

$$P_a(p) = \phi + (1 - \phi)e^{-\lambda} + \sum_{x=1}^c (1 - \phi) \frac{e^{-\lambda} \lambda^x}{x!}, \quad x > 0, \lambda > 0, 0 < \phi < 1. \quad (3)$$

From the history of inspection it is known that  $p$  follows a Beta distribution which is for convenience approximated by a Gamma distribution (see Hald, 1981, p. 133) with density function  $f(p)$ .

$$f(p) = \frac{t^s}{\Gamma(s)} p^{(s-1)} e^{-tp} \quad s, t > 0, 0 \leq p \leq 1 \quad (4)$$

Thus, the average probability of acceptance  $\bar{P}$  is approximately obtained by latha and Palanisamy (2018) as

$$\bar{P} = \int_1^c P_{(a)}(p) f(p) dp$$

## II. Average Outgoing Quality Limit (AOQL)

The second fundamental curve in the description of a sampling plan is the average outgoing quality (AOQ) curve. This is another indication of the protection afforded by the sampling plan. Naturally the average quality, after the sampling inspection and sorting of rejected lots, depends upon the lot quality being offered. Hence, we shall for the present assume that lots of but one quality  $p'$  are offered. Then the outgoing quality will depend upon  $p'$ . There are several interpretations possible in the meaning of AOQ (refer Irving W. Burr – [6]). In practice, they give about the same results, and hence the easiest method is commonly used. The differences depend upon how the defectives are treated. We have the following:

1. The defectives found in the samples may be retained with the accepted lots and not replaced by good pieces. This is the simplest assumptions.
2. The defectives found in the samples and sorting may all be removed, but not replaced by good pieces. This is the usual assumption where sampling inspection is done at same considerable distance from the places when the pieces were produced. Hence it is not feasible to replace the defectives.
3. The defectives found in the samples and sorting may all be removed and replaced by good pieces. This is the usual assumption when sampling inspection is done close to the place where the pieces are produced, so that the defectives may be replaced. This is typical of final inspection.

In all three cases, as we do throughout this paper, we assume that the inspectors find all the defectives, and only the defectives in the samples and lots. Knowing the value of the AOQL for a sampling plan is very helpful,

because it tells, what is the worst long –run average quality which will leave the inspection department. The AOQ curve pictures this relationship between incoming quality and outgoing quality .It will be noticed that the curves show the fraction defective of the outgoing material and therefore fluctuations inversely with quality. The maximum ordinates of the curve measures the worst possible quality of material turned out by the plan and is known as the Average Outgoing Quality Limit or briefly the AOQL. It is to be noted once again that this is an average value over many lots. The AOQL is the maximum of the AOQs for all possible incoming qualities for a given acceptance sampling plan (refer Duncan - [7]). The use of the AOQ Curve and the concept of the AOQL assume that it is feasible to sort a lot 100 percent for defectives.

Thus, for a Bayesian Single Sampling Plan the Overall Average Outgoing Quality (OAOQ) is approximately obtained by:

$$\text{OAOQ} = \int_0^1 p P_{(a)}(p) f(p) dp$$

Gives

nOAOQ=

$$\phi n\mu + (1-\phi) \left( \frac{s n\mu}{n\mu + s} \right) \left( \frac{s}{n\mu + s} \right)^s + \sum_{x=1}^c (1-\phi) (x+s) \binom{x+s-1}{s-1} \left( \frac{n\mu}{n\mu + s} \right)^x \left( \frac{s}{n\mu + s} \right)^s \left( \frac{n\mu}{n\mu + s} \right) \quad (5)$$

Differentiating OAOQ with respect to  $n\mu$  and equating to 0, the value of Overall Average Outgoing limit( OAOQL =  $n\mu_m$ ) can be obtained by solving the equation.

nOAOQL=

$$\phi + (1-\phi) \left( \frac{s}{s+n\mu} \right)^{s+1} \left( \frac{s(1-n\mu)}{s+n\mu} \right)^s + \sum_{x=1}^c (1-\phi) (x+s) \binom{x+s-1}{s-1} S^s \left( \frac{n\mu}{s+n\mu} \right)^x \left( \frac{1}{s+n\mu} \right)^{s+x} \left( \frac{s}{s+n\mu} \right) (x+1-n\mu) = 0 \quad (6)$$

AOQL for conversion single sampling with ZIP model

The OC function of the SSP under the conditions of ZIP ( $\phi, \lambda$ ) distribution can be defined as

$$P_a(\phi) = \sum_{x=0}^c P(X=x|\phi, \lambda)$$

$$P_a(p) = \phi + (1-\phi) e^{-\lambda} + \sum_{x=1}^c (1-\phi) \frac{e^{-\lambda} \lambda^x}{x!}$$

Thus, the average probability of acceptance Overall Average Outgoing Quality (OAOQ) is approximately obtained by

$$\text{nAOQ} = (np) P_{(a)}(P)$$

$$\text{nAOQ} = \phi np + (1-\phi) e^{-np} np + \sum_{x=1}^c (1-\phi) \left( \frac{e^{-np} (np)^x}{x!} \right) np. \quad (7)$$

Differentiating nAOQ with respect to  $np$  and equations to 0, the value of Overall Average Outgoing limit( OAOQL =  $np_m$ ) can be obtained by solving the equation.

$$\text{nAOQ} = \phi + (1-\phi) e^{-np} (1-np) + \sum_{x=1}^c (1-\phi) \left( \frac{e^{-np} (np)^x}{x!} \right) (x+1-np). \quad (8)$$

### III. Designing Bayesian single sampling plans

Sampling plans are constructed in such a way that protection to the producer as well as the consumer is ensured. The optimum (n, c) could be determined satisfying the conditions

$$P_a(p_1) = 1-\alpha \quad (9)$$

$$P_a(p_2) = \beta \quad (10)$$

where  $p_1$ ,  $\alpha$ ,  $p_2$ , and  $\beta$  represent respectively, acceptable quality level, producer's risk, reject able quality level and consumer's risk.

The fixed values of  $\phi$  are 0.0001, 0.001, 0.01, 0.05, and 0.09 and  $s$  is specified on substituting the values of ( $n\mu_m$ ) in equation (6) the values of nOAOQL can be obtained. The values of  $n\mu_m$  and nOAOQL for the given the values  $s$  and  $c$  are given in the Table I (i) to I (ii).

The fixed values of  $\phi$  are 0.0001, 0.001, 0.01, 0.05, and 0.09 and  $c$  is specified on substituting the values of ( $np_m$ ) in equation (8) the values of nOAOQL can be obtained. The values of  $np_m$  and nOAOQL for the given the values  $c$  are given in the Table 2 .

#### IV. Analysis of AOQL values

From Table I (i) to I (ii) it is observed that for fixed value of  $\phi$ ,  $c$ , and as  $s$  value increases OAOQL values increase at all levels of  $np_m$  value. Further comparing with conventional sampling plan, the AOQL values increasing at all levels of  $np_m$ . This ensure protection to the consumer against satisfactory lot quality.

**$np_m$  and nOAOQL values for Bayesian single sampling plan with ZIP model given values  $\phi, c$ , and  $s$ .**

| $\phi$ | $c$ | $s$ | $np_m$ | nOAOQL |
|--------|-----|-----|--------|--------|
| 0.0001 | 1   | 2   | 1.5692 | 0.6403 |
|        |     | 3   | 1.5801 | 0.6923 |
|        |     | 4   | 1.5871 | 0.7226 |
|        |     | 5   | 1.5919 | 0.7425 |
|        |     | 6   | 1.5954 | 0.7566 |
|        |     | 7   | 1.5981 | 0.7671 |
|        |     | 8   | 1.6002 | 0.7753 |
|        |     | 9   | 1.6019 | 0.7818 |
|        |     |     |        |        |
| 0.001  |     | 2   | 1.5735 | 0.6412 |
|        |     | 3   | 1.5836 | 0.6931 |
|        |     | 4   | 1.5902 | 0.7234 |
|        |     | 5   | 1.5948 | 0.7433 |
|        |     | 6   | 1.5982 | 0.7574 |
|        |     | 7   | 1.6007 | 0.7679 |
|        |     | 8   | 1.6028 | 0.7761 |
|        |     | 9   | 1.6044 | 0.7826 |
|        |     |     |        |        |
| 0.01   |     | 2   | 1.6193 | 0.6498 |
|        |     | 3   | 1.6202 | 0.7013 |
|        |     | 4   | 1.6225 | 0.7313 |
|        |     | 5   | 1.6246 | 0.7511 |
|        |     | 6   | 1.6264 | 0.7651 |
|        |     | 7   | 1.6279 | 0.7755 |
|        |     | 8   | 1.6291 | 0.7836 |
|        |     | 9   | 1.6301 | 0.7901 |
|        |     |     |        |        |
| 0.05   |     | 2   | 1.9043 | 0.6942 |
|        |     | 3   | 1.8283 | 0.7424 |
|        |     | 4   | 1.7997 | 0.7708 |
|        |     | 5   | 1.7851 | 0.7895 |
|        |     | 6   | 1.7764 | 0.8028 |
|        |     | 7   | 1.7708 | 0.8128 |
|        |     | 8   | 1.7668 | 0.8205 |
|        |     | 9   | 1.7638 | 0.8266 |
|        |     |     |        |        |
| 0.09   |     | 2   | 1.5692 | 0.6403 |
|        |     | 3   | 1.5801 | 0.6923 |
|        |     | 4   | 1.5871 | 0.7226 |
|        |     | 5   | 1.5919 | 0.7425 |
|        |     | 6   | 1.5954 | 0.7566 |
|        |     | 7   | 1.5981 | 0.7671 |
|        |     | 8   | 1.6002 | 0.7753 |
|        |     | 9   | 1.6019 | 0.7818 |
|        |     |     |        |        |

| $\phi$ | $c$ | $s$ | $np_m$ | nOAOQL |
|--------|-----|-----|--------|--------|
| 0.0001 | 2   | 2   | 2.1465 | 1.0023 |
|        |     | 3   | 2.1722 | 1.0926 |
|        |     | 4   | 2.1889 | 1.1467 |
|        |     | 5   | 2.2008 | 1.1830 |
|        |     | 6   | 2.2095 | 1.2091 |
|        |     | 7   | 2.2163 | 1.2287 |
|        |     | 8   | 2.2217 | 1.2441 |
|        |     | 9   | 2.2261 | 1.2565 |
|        |     |     |        |        |
| 0.001  |     | 2   | 2.1511 | 1.0034 |
|        |     | 3   | 2.1759 | 1.0935 |
|        |     | 4   | 2.1922 | 1.1474 |
|        |     | 5   | 2.2037 | 1.1835 |
|        |     | 6   | 2.2123 | 1.2100 |
|        |     | 7   | 2.2190 | 1.2296 |
|        |     | 8   | 2.2243 | 1.2450 |
|        |     | 9   | 2.2286 | 1.2574 |
|        |     |     |        |        |
| 0.01   |     | 2   | 2.1997 | 1.0139 |
|        |     | 3   | 2.2139 | 1.1035 |
|        |     | 4   | 2.2254 | 1.1572 |
|        |     | 5   | 2.2341 | 1.1932 |
|        |     | 6   | 2.2409 | 1.2191 |
|        |     | 7   | 2.2463 | 1.2387 |
|        |     | 8   | 2.2506 | 1.2539 |
|        |     | 9   | 2.2542 | 1.2662 |
|        |     |     |        |        |
| 0.05   |     | 2   | 2.4812 | 1.0672 |
|        |     | 3   | 2.4197 | 1.1524 |
|        |     | 4   | 2.3996 | 1.2038 |
|        |     | 5   | 2.3910 | 1.2384 |
|        |     | 6   | 2.3868 | 1.2633 |
|        |     | 7   | 2.3846 | 1.2821 |
|        |     | 8   | 2.3835 | 1.2968 |
|        |     | 9   | 2.3829 | 1.3087 |
|        |     |     |        |        |
| 0.09   |     | 2   | 3.0300 | 1.1366 |
|        |     | 3   | 2.7318 | 1.2118 |
|        |     | 4   | 2.6431 | 1.2590 |
|        |     | 5   | 2.6015 | 1.2911 |
|        |     | 6   | 2.5778 | 1.3145 |
|        |     | 7   | 2.5628 | 1.3321 |
|        |     | 8   | 2.5525 | 1.3460 |
|        |     | 9   | 2.5450 | 1.3572 |
|        |     |     |        |        |

**Table (2):  $np_m$  and AOQ values for Single Sampling Plan with ZIP model given values  $\phi, c$**

| $\phi$ | $c$ | $np_m$ | nAOQ   |
|--------|-----|--------|--------|
| 0.0001 | 1   | 1.0001 | 1.0000 |
|        | 2   | 1.0000 | 2.0000 |
|        | 3   | 1.0000 | 3.0000 |
|        | 4   | 1.0000 | 4.0000 |
|        | 5   | 1.0000 | 5.0000 |
|        | 6   | 1.0000 | 6.0000 |
|        | 7   | 1.0000 | 7.0000 |
|        | 8   | 1.0000 | 8.0000 |

|       |   |        |        |
|-------|---|--------|--------|
|       | 9 | 1.0000 | 9.0000 |
| 0.001 | 1 | 1.0017 | 1.0000 |
|       | 2 | 1.0007 | 2.0000 |
|       | 3 | 1.0002 | 3.0000 |
|       | 4 | 1.0000 | 4.0000 |
|       | 5 | 1.0000 | 5.0000 |
|       | 6 | 1.0000 | 6.0000 |
|       | 7 | 1.0000 | 7.0000 |
|       | 8 | 1.0000 | 8.0000 |
|       | 9 | 1.0000 | 9.0000 |
| 0.01  | 1 | 1.0000 | 1.0063 |
|       | 2 | 1.0073 | 2.0000 |
|       | 3 | 1.0022 | 3.0000 |
|       | 4 | 1.0005 | 4.0000 |
|       | 5 | 1.0000 | 5.0000 |
|       | 6 | 1.0000 | 6.0000 |
|       | 7 | 1.0000 | 7.0000 |
|       | 8 | 1.0000 | 8.0000 |
|       | 9 | 1.0000 | 9.0000 |
| 0.05  | 1 | 1.1072 | 1.0000 |
|       | 2 | 1.0406 | 2.0000 |
|       | 3 | 1.0114 | 3.0000 |
|       | 4 | 1.0026 | 4.0000 |
|       | 5 | 1.0004 | 5.0000 |
|       | 6 | 1.0000 | 6.0000 |
|       | 7 | 1.0000 | 7.0000 |
|       | 8 | 1.0000 | 8.0000 |
|       | 9 | 1.0000 | 9.0000 |
| 0.09  | 1 | 1.2541 | 1.0568 |
|       | 2 | 1.0826 | 2.0237 |
|       | 3 | 1.0214 | 3.0000 |
|       | 4 | 1.0047 | 4.0000 |
|       | 5 | 1.0009 | 5.0000 |
|       | 6 | 1.0001 | 6.0000 |
|       | 7 | 1.0000 | 7.0000 |
|       | 8 | 1.0000 | 8.0000 |
|       | 9 | 1.0000 | 9.0000 |

## V. Comparison with conversional plan

From Table 1 and 2 it is observed that for fixed value of  $\phi$ ,  $c$ , and  $s$  as  $np_m$  value increases the  $nAOQL$  value increasing approaches to the  $np_m$  value of the conversional plan (Bayesian Single sampling Plan with ZIP Model).  $np_m$  values decrease  $nAOQL$  value increasing. For large value of  $nAOQ$  (Single sampling Plan with ZIP Model), Where for  $np_m$  values decrease with  $nAOQ$  value increase of  $c$  value and tends to the  $np_m$  values of conversional plan for large value of  $c$ .

## VII. Conclusion

In this article Bayesian sampling is the best technique which deals with the procedure in which decision to accept or reject lots or process based on their examination of past history or knowledge. Bayesian single sampling plan Zero –inflated Poisson distribution with average outgoing quality limit as an index and developed. When the process's and the acceptance number increases which is favorable for producer increase of the same lot quality. The tables are made elegant, handy and also made easy to use in industrial shop floor situations.

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