



Factorial Labeling For Some Classes of Graphs

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Abstract: In this paper, we discuss the factorial labeling conditions satisfy to some special classes of graphs. A factorial labeling of a connected graph is bijection $f : V(G) \rightarrow \{1, 2, \dots, p\}$ such that the induced function $g_f : E(G) \rightarrow N$ defined as $g_f(uv) = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$ then the edges labels are distinct. Graph which admits a factorial labeling is a factorial graph. A special classes of graphs likely path, cycle, brush, friendship, star, sun let, wheel, helm, ladder, triangular snake, dragon, coconut tree, umbrella, mangolian tent, key graphs. Also, we find the factorial number and the chromatic number for that graphs.

Keywords : Factorial labeling, Factorial graph, Factorial number, Classes of graphs, Chromatic number.

I. Introduction

Graph theory is the very simplicity and interesting area of the combinatorics and also graph labeling is one of the most grown areas of graph theory. Graph labeling is assign the values to set of vertices and set of edges of graphs. This was first introduced Alexander Rosa in the late 1960's. Gallian .J.A[7] updating the dynamic survey of graph labeling upto 2017. In the last four decades more than 1000 research papers are published. Graph labeling applying many places it was likely computer science, engineering, medical line, etc., we discuss the factorial labeling conditions satisfy to some special classes of graphs likely path, cycle, brush, friendship, star, sun let, wheel, helm, ladder, triangular snake, dragon, coconut tree, umbrella, mangolian tent, key graphs. Also, we find the factorial number and the chromatic number for these graphs.

II. Preliminaries

2.1. Definition[5]

A walk in which no vertex is repeated is called a path P_n . It has n vertices and $n - 1$ edges.

2.2. Definition[5]

A closed path is called a cycle $C_n, n \geq 3$. It has both n vertices and n edges.

2.3. Definition[2]

The brush graph $B_n, (n \geq 2)$ can be constructed by path graph $P_n, (n \geq 2)$ by joining the star graph $K_{1,1}$ at each vertex of the path. i.e., $B_n = P_n + nK_{1,1}$. It has $2n$ vertices and $2n - 1$ edges.

2.4. Definition[12]

A cycle C_3 with n copies having a common central vertex is called a friendship graph T_n . It has $2n + 1$ vertices and $3n$ edges.

2.5. Definition[5]

A star graph $K_{1,n}$ is said to be n pendant vertices with common central vertex. It has $n + 1$ vertices and n edges.

2.6. Definition[9]

The cycle graph C_n with attaching n pendant vertices at each vertex is called the sun let graph $S_n, n \geq 3$. It has both $2n$ vertices and $2n$ edges.

2.7. Definition[13]

The cycle graph $C_n, n \geq 3$ joining the complete graph of one vertex K_1 is called the wheel graph $W_n, n \geq 3$. It has $n + 1$ vertices and $2n$ edges.

2.8. Definition[1]

The wheel graph W_n by adding a pendant edge at each vertex on the rim of W_n is called the helm graph $H_n, n \geq 3$. It has $2n + 1$ vertices and $3n$ edges.

2.9. Definition[3]

The cartesian product of path graphs $P_n \times P_2$ is known as ladder graph $L_n, n \geq 2$. It has $2n$ vertices and $3n - 2$ edges.

2.10. Definition[4]

A Path graph P_n by joining the vertices u_i and u_{i+1} to a new vertex $v_j, 1 \leq i, j \leq n - 1$ is said to be a triangular snake graph $TS_n, n \geq 2$. That means every edge of the path is replaced by a triangle C_3 . It has $2n - 1$ vertices and $3n - 3$ edges.

2.11. Definition[8]

A path P_m is attached at one vertex of cycle C_n is called the dragon graph $D_{n,m}, n \geq 3, m \geq 1$. It has both $n + m$ vertices and $n + m$ edges.

2.12. Definition[8]

A path graph P_n by appending m new pendant edges at an end vertex of P_n is called a coconut tree $CT_{m,n}$. It has $m + n$ vertices and $m + n - 1$ edges.

2.13. Definition[10]

A common central vertex of fan graph F_m is joining the end vertex of path graph P_n is said to be an umbrella graph $U_{m,n}, m \geq 3, n \geq 2$. It has $m + n$ vertices and $2m + n - 2$ edges.

2.14. Definition[10]

The grid graph $P_m \times P_n$ by adding one extra vertex above the grid and joining every other of the top row of $P_m \times P_n$ to the new vertex is called the Mongolian tent $M_{m,n}, (m, n) \geq 2$. It has $(mn) + 1$ vertices and $2(mn) - n$ edges.

2.15. Definition[11]

The key graph $KY_{m,n}, m \geq 3, n \geq 2$ is a graph obtained from K_2 by appending one vertex of C_m to one end point and comb graph $P_n \odot K_1$ to the other end of K_2 . It has both $(2n + m)$ vertices and $(2n + m)$ edges.

2.16. Definition[6]

A coloring of a graph is an assigned color to its points so that two adjacent points have different colors and also non-adjacent vertices have either same color or any other colors. The chromatic number $\chi(G)$ is defined as the minimum l for which a graph G has an l -coloring.

III. Factorial Labeling For Some Special Classes Of Graphs

3.1. Definition

A factorial labeling of a connected graph G is a bijection $f : V(G) \rightarrow \{1, 2, \dots, p\}$ such that the induced function $g_f : E(G) \rightarrow N$ defined as $e = uv = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$, then the edges labels are distinct. Any graph which admits a factorial labeling is called a factorial graph.

3.2. Definition

The minimum number of the edge label of the factorial graph is said to be a factorial number. It is denoted by $\omega(n)$.

3.3. Theorem

The path graph P_n admits a factorial graph.

Proof

Let P_n be a path of n vertices and $n - 1$ edges. Let $u_1, u_2, \dots, u_n$ be the successive vertices and $e_j = u_i u_{i+1}$ for $1 \leq i, j \leq n - 1$ be the edges of a path P_n . Here, $V(P_n) = \{u_i / 1 \leq i \leq n\}$ and $E(P_n) = \{u_i u_{i+1} / 1 \leq i \leq n - 1\}$. The maximum degree of path graph is $\Delta = 2$ and the minimum degree of path graph is $\delta = 1$. Note that, $|V(P_n)| = n$ and $|E(P_n)| = n - 1$. The function is defined $f : V(P_n) \rightarrow \{1, 2, \dots, n\}$ as follows, $f(u_i) = i$ for $1 \leq i \leq$

n . Then every edge $e = uv = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$ and for any edge $f(e_i) \neq f(e_j), i \neq j$. Clearly, vertices and edges labels are distinct. The function f is a factorial labeling for a graph P_n . Hence, the path graph P_n admits a factorial graph. That is, the minimum number of the edge label of the factorial graph is called factorial number $\omega(n) = \min\{\text{edge label of } P_n\} = 3$. The chromatic number $\chi(P_n)$ is 2.

3.4.Example

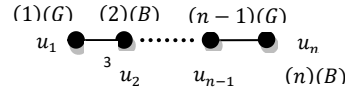


Figure 1 Factorial labeling of path graph P_n .

3.5.Theorem

The cycle graph $C_n, n \geq 3$ is a factorial graph.

Proof

Let the cycle graph $C_n, n \geq 3$ be n vertices and n edges. Let the edges of a cycle graph is $e_j = u_i u_{i+1}$ for $1 \leq i, j \leq n-1, e_n = u_n u_{n+1}$. Here, $V(C_n) = \{u_i / 1 \leq i \leq n\}$ and $E(C_n) = \{u_i u_{i+1} / 1 \leq i \leq n-1\} \cup \{u_n u_1\}$. The maximum and minimum degree of cycle graph C_n are $\Delta = 2$ and $\delta = 2$ respectively. i.e. $|V(C_n)| = n = |E(C_n)|$. The bijection mapping f from $V(C_n)$ to $\{1, 2, \dots, n\}$ is defined by, $f(u_i) = i$ for $1 \leq i \leq n$. Then every edge $e = uv = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$ and for any edge $f(e_i) \neq f(e_j), i \neq j$. Clearly, vertices and edges labels are distinct. Hence, f is a factorial labeling of C_n and also it is a factorial graph. The factorial number of factorial graph of cycle graph C_n , $\omega(n) = \min\{\text{edge label of } C_n\} = 3$. The chromatic number of factorial graph of cycle graph is $\chi(C_n) = \begin{cases} 3 & \text{if } n \text{ is odd} \\ 2 & \text{if } n \text{ is even} \end{cases}$.

3.6.Example

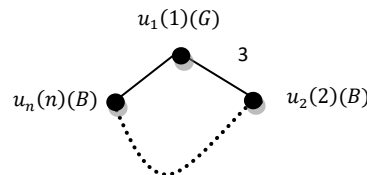


Figure 2 Factorial labeling of cycle graph C_n .

3.7. Theorem

The brush graph B_n produces a factorial graph.

Proof

Let B_n be the brush graph with $2n$ vertices and $2n-1$ edges. Let $V(B_n) = \{u_i / 1 \leq i \leq 2n\}$ and $E(B_n) = \{u_i u_{i+1} / 1 \leq i \leq n\} \cup \{u_i u_{2n+1-i} / 1 \leq i \leq n\}$. Here, $|V(B_n)| = 2n$ and $|E(B_n)| = 2n-1$. The maximum degree $\Delta = 3$ and the minimum degree $\delta = 1$ of brush graph B_n . The mapping $f: V(B_n) \rightarrow \{1, 2, \dots, 2n\}$ as follows, $f(u_i) = i$ for $1 \leq i \leq 2n$. Then for every edge $e = uv = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$ and for any edge $f(e_i) \neq f(e_j), i \neq j$. Clearly, vertices and edges labels are distinct. Hence, f is a factorial labeling. Thus, B_n produces a factorial graph. The factorial number, $\omega(n) = \min\{\text{edge label of } B_n\} = 3$. The chromatic number, $\chi(B_n) = 2$.

3.8.Example

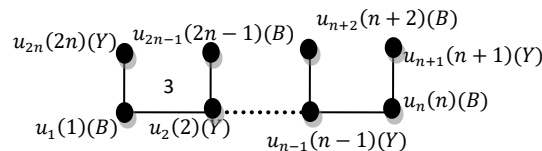


Figure 3 Factorial labeling of brush graph B_n .

3.9. Theorem

The friendship graph T_n admits a factorial graph.

Proof

Let T_n be the friendship graph with n copies of cycle C_3 . Let u_1 be the apex vertex and $u_2, u_3, \dots, u_{2n+1}$ be the remaining vertices and $e_1, e_2, \dots, e_{3n}$ be the edges of T_n . Let T_n be $2n+1$ vertices and $3n$ edges. Here, $V(T_n) = \{u_i / 1 \leq i \leq 2n+1\}$ with u_1 as the centre vertex and $E(T_n) = \{u_1 u_i / 2 \leq i \leq 2n+1\} \cup \{u_{2i} u_{2i+1} / 1 \leq i \leq n\}$. Note that, $|V(T_n)| = 2n+1$ and $|E(T_n)| = 3n$. The maximum degree $\Delta = 2n$ and the minimum degree $\delta =$

2 of friendship graph T_n . The function is defined $f: V(T_n) \rightarrow \{1, 2, \dots, 2n+1\}$ as follows, $f(u_i) = i$ for $1 \leq i \leq 2n+1$. Then for every edge $e = uv = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$ and for any edge $f(e_i) \neq f(e_j), i \neq j$. Clearly, vertices and edges labels are distinct. Hence, T_n is a factorial labeling and T_n also admits a factorial graph. The factorial number, $\omega(n) = \min\{\text{edge label of } T_n\} = 3$. The chromatic number, $\chi(T_n) = 3$.

3.10. Example

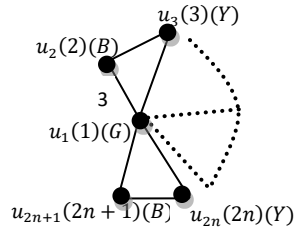


Figure 4 Factorial labeling of friendship graph T_n .

3.11. Theorem

The star graph $K_{1,n}$ acknowledges the factorial graph.

Proof

Let $K_{1,n}$ be the star graph with $n+1$ vertices and n edges. Here, $V(K_{1,n}) = \{u_i / 1 \leq i \leq n+1\}$ and $E(K_{1,n}) = \{u_1 u_i / 1 \leq i \leq n\}$. Note that, $|V(K_{1,n})| = n+1$ and $|E(K_{1,n})| = n$. The maximum and minimum degree of star graph $K_{1,n}$ are $\Delta = n$ and $\delta = 1$ respectively. The labeling function $f: V(K_{1,n}) \rightarrow \{1, 2, \dots, n+1\}$ is defined by, $f(u_i) = i$ for $1 \leq i \leq n+1$. Then, every edge $e = uv = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$ and for any edge $f(e_i) \neq f(e_j), i \neq j$. Clearly, vertices and edges labels are distinct. Hence, the labeling function f is a factorial labeling. Therefore, $K_{1,n}$ acknowledges the factorial graph. The factorial number is $\omega(n) = \min\{\text{edge label of } K_{1,n}\} = 3$. The chromatic number is $\chi(K_{1,n}) = 2$.

3.12. Example

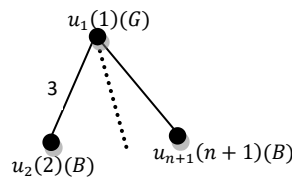


Figure 5 Factorial labeling of star graph $K_{1,n}$.

3.13. Theorem

The sun let graph $S_n, n \geq 3$ produces a factorial graph.

Proof

Let the sun let graph $S_n, n \geq 3$ having $2n$ vertices and also $2n$ edges. Here, $V(S_n) = \{u_i / 1 \leq i \leq 2n\}$ and $E(S_n) = \{u_i u_{i+1} / 1 \leq i \leq n-1\} \cup \{u_n u_1\} \cup \{u_n u_{n+i} / 1 \leq i \leq n\}$. Note, $|V(S_n)| = 2n = |E(S_n)|$. The maximum degree of S_n is $\Delta = 3$ and that the minimum degree of S_n is $\delta = 1$. The mapping is defined $f: V(S_n) \rightarrow \{1, 2, \dots, 2n\}$ as follows, $f(u_i) = i$ for $1 \leq i \leq 2n$. Clearly, vertices and edges labels are distinct. Then, for every edge $e = uv = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$ and for any edge $f(e_i) \neq f(e_j), i \neq j$. Now, all that edges are satisfied these conditions and also f is a factorial labeling. Hence, $S_n, n \geq 3$ produces a factorial graph. The factorial number, $\omega(n) = \min\{\text{edge label of } S_n\} = 3$. The chromatic number, $\chi(S_n) = 2$.

3.14. Example

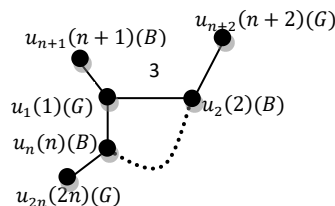


Figure 6 Factorial labeling of sun let graph $S_n, n \geq 3$.

3.15. Theorem

The wheel graph $W_n, n \geq 3$ admits a factorial graph.

Proof

Let $W_n, n \geq 3$ be a wheel graph with $n + 1$ vertices and $2n$ edges. Let $V(W_n) = \{u_i / 1 \leq i \leq n + 1\}$ and $E(W_n) = \{u_1 u_i / 2 \leq i \leq n + 1\} \cup \{u_i u_{i+1} / 2 \leq i \leq n\} \cup \{u_{n+1} u_2\}$. Here, we note that, $|V(W_n)| = n + 1$ and $|E(W_n)| = 2n$. The maximum degree of wheel graph is $\Delta = n$ and the minimum degree of wheel graph is $\delta = 3$. The function is defined $f : V(W_n) \rightarrow \{1, 2, \dots, n + 1\}$ as follows, $f(u_i) = i$ for $1 \leq i \leq n + 1$. Clearly, vertices labels are distinct. All edges are distinct labels with these conditions for every edge $e = uv = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$ and for any edge $f(e_i) \neq f(e_j), i \neq j$. The function f is a factorial labeling and also wheel graph $W_n, n \geq 3$ admits a factorial graph. The minimum number of the edge label of the factorial graph which means factorial number is $\omega(n) = \min\{\text{edge label of } W_n\} = 3$. The chromatic number $\chi(W_n) = \begin{cases} 4 & \text{if } n \text{ is odd} \\ 3 & \text{if } n \text{ is even} \end{cases}$.

3.16. Example

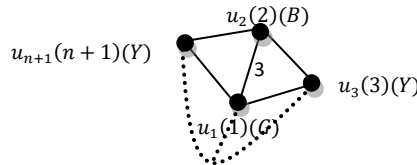


Figure 7 Factorial labeling of wheel graph $W_n, n \geq 3$.

3.17. Theorem

The helm graph $H_n, n \geq 3$ is a factorial graph.

Proof

Let $H_n, n \geq 3$ be a helm graph with $2n + 1$ vertices and $3n$ edges. Let $V(H_n) = \{u_i / 1 \leq i \leq 2n + 1\}$ and $E(H_n) = \{u_1 u_i / 2 \leq i \leq n + 1\} \cup \{u_i u_{i+1} / 2 \leq i \leq n\} \cup \{u_i u_{n+i} / 2 \leq i \leq n + 1\} \cup \{u_{n+1} u_2\}$. Clearly, $|V(H_n)| = 2n + 1$ and $|E(H_n)| = 3n$. The maximum degree of H_n is $\Delta = n$ and the minimum degree of H_n is $\delta = 1$. The function f is defined from vertex set of H_n to $\{1, 2, \dots, 2n + 1\}$ as follows, $f(u_i) = i$ for $1 \leq i \leq 2n + 1$. Now, vertices and edges labels are distinct in these conditions for every edge $e = uv = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$ and for any edge $f(e_i) \neq f(e_j), i \neq j$. Hence, the function f is a factorial labeling and thus the helm graph $H_n, n \geq 3$ is a factorial graph. The factorial number is the minimum number of edge label of factorial graph $H_n, n \geq 3$. i.e., $\omega(n) = \min\{\text{edge label of } H_n\} = 3$. And the chromatic number $\chi(H_n) = \begin{cases} 4 & \text{if } n \text{ is odd} \\ 3 & \text{if } n \text{ is even} \end{cases}$.

3.18. Example

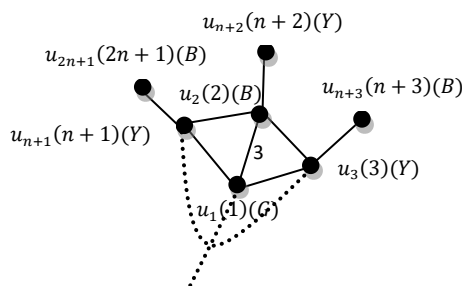


Figure 8 Factorial labeling of helm graph $H_n, n \geq 3$.

3.19. Theorem

The ladder graph $L_n, n \geq 2$ acknowledges a factorial graph.

Proof

Let $L_n, n \geq 2$ be a ladder graph i.e., $L_n = P_2 \times P_n$ with $2n$ vertices and $3n - 2$ edges. Let $V(L_n) = \{u_i / 1 \leq i \leq 2n\}$ and $E(L_n) = \{u_i u_{i+1} / 1 \leq i \leq 2n - 1\} \cup \{u_i u_{2n-i+1} / 1 \leq i \leq n\}$. Clearly, $|V(L_n)| = 2n$ and $|E(L_n)| = 3n - 2$. The maximum degree of ladder graph $L_n, n \geq 2$ are $\Delta = 3$ and $\delta = 2$ respectively. The bijection mapping $f : V(L_n) \rightarrow \{1, 2, \dots, 2n\}$ as follows, $f(u_i) = i$ for $1 \leq i \leq 2n$. Then, the vertices and edges labels are distinct in these conditions for every edge $e = uv = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$ and for any edge $f(e_i) \neq f(e_j), i \neq j$. Hence, the bijection mapping function f is a factorial labeling and thus the ladder graph $L_n, n \geq 2$ acknowledges a factorial graph. The factorial number is the minimum number of edge label of factorial graph $L_n, n \geq 2$. i.e., $\omega(n) = \min\{\text{edge label of } L_n\} = 3$. And the chromatic number $\chi(L_n) = 2$.

3.20. Example

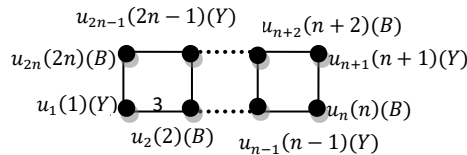


Figure 9 Factorial labeling of ladder graph $L_n, n \geq 2$.

3.21. Theorem

The triangular snake graph $TS_n, n \geq 2$ is a factorial graph.

Proof

Let $TS_n, n \geq 2$ be a triangular snake graph with $2n - 1$ vertices and $3n - 3$ edges. Let $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_{n-1}$ are the successive vertices of a triangular snake graph $TS_n, n \geq 2$. Here, $V(TS_n) = \{u_i / 1 \leq i \leq n\} \cup \{v_j / 1 \leq j \leq n - 1\}$ and $E(TS_n) = \{u_i u_{i+1}, u_i v_j, u_{i+1} v_j / 1 \leq i, j \leq n - 1\}$. The maximum degree of triangular snake graph $TS_n, n \geq 2$ is $\Delta = 4$ and the minimum degree of triangular snake graph $TS_n, n \geq 2$ is $\delta = 2$. Clearly, $|V(TS_n)| = 2n - 1$ and $|E(TS_n)| = 3n - 3$. Define the bijection mapping $f : V(TS_n) \rightarrow \{1, 2, \dots, 2n - 1\}$ as follows, $f(u_i) = i$ for $1 \leq i \leq n$, $f(v_j) = n + j$ for $1 \leq j \leq n - 1$. All edges labels are distinct with respect to these conditions $e = uv = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$ and for any edge $f(e_i) \neq f(e_j), i \neq j$. Clearly, vertices and edges labels are distinct. Hence, f is a factorial labeling for a triangular snake graph $TS_n, n \geq 2$. Thus, the triangular snake graph $TS_n, n \geq 2$ is a factorial graph. The factorial number $\omega(n) = \min\{\text{edge label of } TS_n\} = 3$. The chromatic number for the corresponding graph, $\chi(TS_n)$ is 3.

3.22. Example

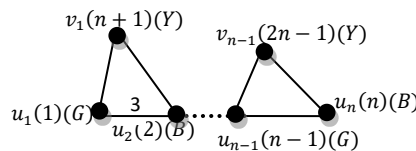


Figure 10. Factorial labeling of triangular snake graph $TS_n, n \geq 2$.

3.23. Theorem

The dragon graph $D_{n,m}, n \geq 3, m \geq 1$ produces a factorial graph.

Proof

Let G be a dragon graph $D_{n,m}, n \geq 3, m \geq 1$ in which any vertex of cycle C_n is attached to path P_m . Then, $D_{n,m}$ has $n + m$ vertices and also $n + m$ edges. Let $u_1, u_2, \dots, u_n$ and $u_{n+1}, u_{n+2}, \dots, u_{n+m}$ be the successive vertices of a dragon graph $D_{n,m}, n \geq 3, m \geq 1$. Here we note that, $V(D_{n,m}) = \{u_i / 1 \leq i \leq n + m\}$ and $E(D_{n,m}) = \{u_i u_{i+1} / 1 \leq i \leq n - 1\} \cup \{u_n u_1\} \cup \{u_1 u_{n+1}\} \cup \{u_{n+i} u_{n+i+1} / 1 \leq i \leq m - 1\}$. Also, $|V(D_{n,m})| = n + m = |E(D_{n,m})|$. The maximum degree of $D_{n,m}$ is $\Delta = 3$. The minimum degree of $D_{n,m}$ is $\delta = 1$. The mapping $f : V(D_{n,m}) \rightarrow \{1, 2, \dots, n + m\}$ is defined by, $f(u_i) = i$ for $1 \leq i \leq n + m$. Clearly, vertices labels are distinct. All edges labels are distinct with respect to these conditions for every edge $e = uv = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$ and for any edge $f(e_i) \neq f(e_j), i \neq j$. Then, f is a factorial labeling. Hence, $D_{n,m}, n \geq 3, m \geq 1$ produces a factorial graph. The factorial number, $\omega(n) = \min\{\text{edge label of } D_{n,m}\} = 3$. The chromatic number, $\chi(D_{n,m}) = \begin{cases} 3 & \text{if } n \text{ is odd} \\ 2 & \text{if } n \text{ is even} \end{cases}$.

3.24. Example

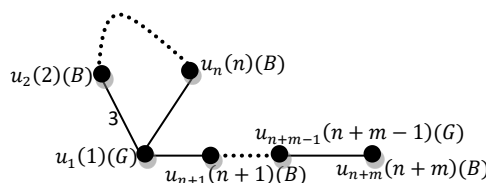


Figure 11 Factorial labeling of dragon graph $D_{n,m}, n \geq 3, m \geq 1$.

3.25. Theorem

The coconut tree graph $CT_{m,n}$ acknowledges a factorial graph.

Proof

Let G be a coconut tree graph $CT_{m,n}$ is the graph obtained from the path P_n by appending m new pendant edges at an end vertex of P_n . It has $m + n$ vertices and $m + n - 1$ edges. Let $V(CT_{m,n}) = \{u_i / 1 \leq i \leq m + n\}$ and

$E(CT_{m,n}) = \{u_i u_{i+1} / 1 \leq i \leq n-1\} \cup \{u_n u_i / n+1 \leq i \leq m+n\}$. We note that, $|V(CT_{m,n})| = m+n$ and $|E(CT_{m,n})| = m+n-1$. The maximum and minimum degree of coconut tree graph $CT_{m,n}$ are $\Delta = m+1$ and $\delta = 1$ respectively. The function is defined $f: V(CT_{m,n}) \rightarrow \{1, 2, \dots, m+n\}$ as follows, $f(u_i) = i$ for $1 \leq i \leq m+n$. The factorial labeling conditions are $e = uv = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$ and for any edge $f(e_i) \neq f(e_j), i \neq j$ satisfied to the coconut tree graph $CT_{m,n}$. Clearly, vertices and edges labels are distinct. Hence, the coconut tree graph $CT_{m,n}$ acknowledges a factorial graph. The factorial number $\omega(n) = \min\{\text{edge label of } CT_{m,n}\} = 3$. The chromatic number of the corresponding graph, $\chi(CT_{m,n})$ is 2.

3.26. Example

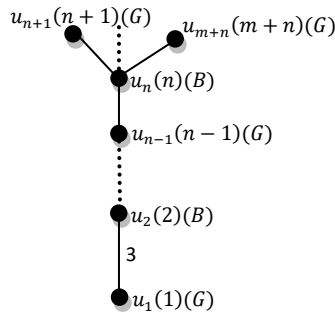


Figure 12 Factorial labeling of a coconut tree graph $CT_{m,n}$.

3.27. Theorem

An umbrella graph $U_{m,n}, m \geq 3, n \geq 2$ produces a factorial graph.

Proof

Let $U_{m,n}, m \geq 3, n \geq 2$ be an umbrella graph with $m+n$ vertices and $2m+n-2$ edges. Let $V(U_{m,n}) = \{u_i / 1 \leq i \leq m+n\}$ and $E(U_{m,n}) = \{u_i u_{i+1} / 1 \leq i \leq n-1\} \cup \{u_n u_1\} \cup \{u_n u_i / n+1 \leq i \leq m+n\} \cup \{u_{n+i} u_{n+i+1} / 1 \leq i \leq m-1\}$. Also, $|V(U_{m,n})| = m+n$ and $|E(U_{m,n})| = 2m+n-2$. The maximum and minimum degree of $U_{m,n}$ are $\Delta = m+1$ and $\delta = 1$ respectively. The mapping $f: V(U_{m,n}) \rightarrow \{1, 2, \dots, m+n\}$ is defined by, $f(u_i) = i$ for $1 \leq i \leq m+n$. Clearly, vertices labels are distinct. All edges labels are distinct with respect to these conditions for every edge $e = uv = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$ and for any edge $f(e_i) \neq f(e_j), i \neq j$. Then, f is a factorial labeling and also $U_{m,n}, m \geq 3, n \geq 2$ produces a factorial graph. The factorial number, $\omega(n) = \min\{\text{edge label of } U_{m,n}\} = 3$. The chromatic number is allocate the minimum number of colors to vertices i.e., $\chi(U_{m,n}) = 3$.

3.28. Example

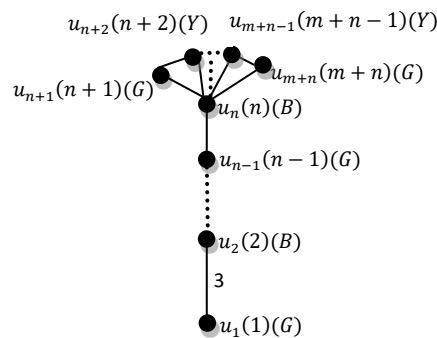


Figure 13 Factorial labeling of an umbrella graph $U_{m,n}, m \geq 3, n \geq 2$.

3.29. Theorem

The mangolian tent graph $M_{m,n}, (m, n) \geq 2$ acknowledges a factorial graph.

Proof

Let $M_{m,n}, (m, n) \geq 2$ be a mangolian tent graph obtained from grid graph $P_m \times P_n$ by adding one extra vertex above the grid graph and joining every other of the top row of $P_m \times P_n$ to the new vertex. It has $(mn) + 1$ vertices and $2(mn) - n$ edges. Here, $V(M_{m,n}) = \{u_i / 1 \leq i \leq (mn) + 1\}$. We note that, $|V(M_{m,n})| = (mn) + 1$ and $|E(M_{m,n})| = 2(mn) - n$. The maximum and minimum degree of mangolian tent graph $M_{m,n}, (m, n) \geq 2$ are $\Delta = m$ and $\delta = 2$ respectively. The bijection mapping function is defined $f: V(M_{m,n}) \rightarrow \{1, 2, \dots, (mn) + 1\}$.

1} as follows, $f(u_i) = i$ for $1 \leq i \leq (mn) + 1$. The factorial labeling conditions are $e = uv = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$ and for any edge $f(e_i) \neq f(e_j), i \neq j$ satisfied to the mangolian tent graph $M_{m,n}, (m, n) \geq 2$. Now, vertices and edges labels are distinct. Hence, the bijection mapping function acknowledges a factorial labeling and also a mangolian tent graph $M_{m,n}, (m, n) \geq 2$ is a factorial graph. The factorial number $\omega(n) = \min\{\text{edge label of } M_{m,n}\} = 3$. The chromatic number for this corresponding graph, $\chi(M_{m,n})$ is 3.

3.30. Example

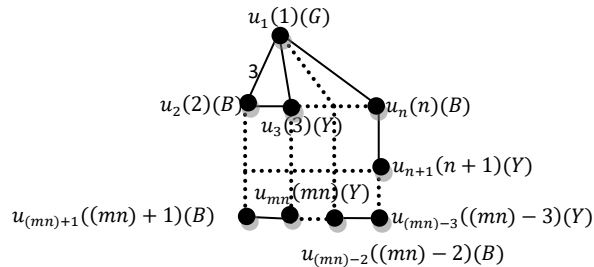


Figure 14 Factorial labeling of a mangolian tent graph $M_{m,n}, (m, n) \geq 2$.

3.31. Theorem

The key graph $KY_{m,n}, m \geq 3, n \geq 2$ admits a factorial graph.

Proof

Let $KY_{m,n}, m \geq 3, n \geq 2$ be a key graph with $2n + m$ vertices and also $2n + m$ edges. Let $u_1, u_2, \dots, u_{2n+m}$ be the successive vertices of a key graph $KY_{m,n}$. Then, $V(KY_{m,n}) = \{u_i / 1 \leq i \leq 2n + m\}$ and $E(KY_{m,n}) = \{u_i u_{i+1} / 1 \leq i \leq m + n\} \cup \{u_m u_1\} \cup \{u_{m+n-i} u_{m+n+j} / 1 \leq i \leq m, 2 \leq j \leq n\}$. Also, $|V(KY_{m,n})| = 2n + m = |E(KY_{m,n})|$. The maximum and minimum degree of $KY_{m,n}$ are $\Delta = 3$ and $\delta = 1$ respectively. The mapping function $f: V(KY_{m,n}) \rightarrow \{1, 2, \dots, 2n + m\}$ is defined by, $f(u_i) = i$ for $1 \leq i \leq 2n + m$. Clearly, vertices and edges labels are distinct with these factorial labeling conditions for every edge $e = uv = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$ and for any edge $f(e_i) \neq f(e_j), i \neq j$ are satisfied. Then, f is a factorial labeling and also $KY_{m,n}, m \geq 3, n \geq 2$ admits a factorial graph. The factorial number, $\omega(n) = \min\{\text{edge label of } KY_{m,n}\} = 3$. The chromatic number, $\chi(KY_{m,n}) = \begin{cases} 3 & \text{if } m \text{ is odd} \\ 2 & \text{if } m \text{ is even} \end{cases}$.

3.32. Example

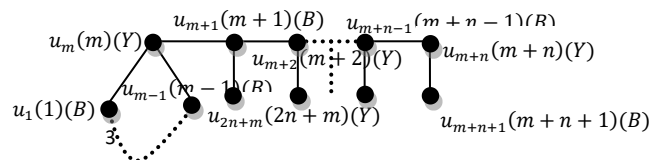


Figure 15 Factorial labeling of a key graph $KY_{m,n}, m \geq 3, n \geq 2$.

IV. Conclusion

In this paper, we discussed the factorial labeling and that factorial graph. The factorial labeling conditions are satisfied the some special classes of graphs likely path, cycle, brush, friendship, star, sun let, wheel, helm, ladder, triangular snake, dragon, coconut tree, umbrella, mangolian tent, key graphs. Also, we found the factorial number and the chromatic number.

V. References

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