



Open Neighborhood Coloring of Star and Double Star Graph Families

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Abstract: In this paper, we determine the open neighborhood chromatic number for star and double star graph families.

Keywords: Open neighborhood coloring, central graph, middle graph, total graph.

I. INTRODUCTION

All the graphs considered here are simple, connected and undirected. A proper coloring of a graph is an assignment of colors to the vertices such that adjacent vertices receive different colors. The minimum number of colors required to color the vertices is called the chromatic number of the graph $\chi(G)$. An open neighborhood coloring of a graph $G(V, E)$ as a coloring $f: V(G) \rightarrow Z^+$, such that for each $w \in V(G)$ and $\forall u, v \in N(w)$, $f(u) \neq f(v)$. The maximum value of $f(w)$, $\forall w \in V(G)$ is called the span of the open neighborhood coloring f . The minimum value of span of f overall open neighborhood colorings f is called open neighborhood chromatic number of G denoted by $\chi_{onc}(G)$. Geetha K. N., et al. [4] introduced the notion of open neighborhood coloring and they have discussed the open neighborhood chromatic number of some standard graphs, trees and the infinite triangular lattice. Geetha K. N., et al. have studied the open neighborhood chromatic number of prism graph which is generalized Peterson graph $GP(n, k)$ for $n \geq 3$ and $k = 1$ in [5]. An open neighborhood coloring of a graph G need not be a proper coloring of G .

Double star $K_{1,n,n}$ [7] is a tree obtained from the star $K_{1,n}$ by adding a new pendent edge of the existing n pendant vertices. It has $2n+1$ vertices and $2n$ edges.

For a given graph $G=(V, E)$, we do an operation on G , by subdividing each edge exactly once and joining all the non-adjacent vertices of G . The graph obtained by this process is called central graph [8] of G and it is denoted by $C(G)$.

The middle graph [2,8] of G , is defined with the vertex set $V(G) \cup E(G)$ where two vertices are adjacent if they are either adjacent edges of G or one is the vertex and the other is an edge incident with it and it is denoted by $M(G)$.

The total graph [2,8] $T(G)$ of G has vertex set $V(G) \cup E(G)$, and edges joining all elements of this vertex set which are adjacent or incident in G .

Additional graph theory terminology used in this paper can found in [1,3,6].

II. OPEN NEIGHBORHOOD COLORING ON CENTRAL GRAPH OF STAR GRAPH

Theorem 2.1. For any star graph $K_{1,n}$, $\chi_{onc}[C(K_{1,n})] = n + 1$.

Proof.

Let $v_1, v_2, \dots, v_n$ be the pendant vertices of $K_{1,n}$ and v be the vertex of $K_{1,n}$ adjacent to $v_i, 1 \leq i \leq n$. Obviously $\deg(v) = n$. Now the edge vv_i be subdivided by the vertex $u_i, 1 \leq i \leq n$ in $C(K_{1,n})$.

Clearly, $V(C(K_{1,n})) = \{v_1, v_2, \dots, v_n\} \cup \{v\} \cup \{u_1, u_2, \dots, u_n\}$ and each $u_i, 1 \leq i \leq n$ adjacent with $v_i, 1 \leq i \leq n$ respectively. Note that in $C(K_{1,n})$ the induced subgraph $\{v_1, v_2, \dots, v_n\}$ is complete.

For $1 \leq i \leq n$, assign the color c_i to v_i and u_i respectively and assign the color c_{n+1} to v . Therefore, $\chi_{onc}[C(K_{1,n})] \leq n + 1$.

An easy check shows that, it is not possible for assign one of the allowed color from C_i to v . Therefore, $\chi_{onc} C[K_{1,n}] \geq n+1$.

Hence, $\chi_{onc} [C(K_{1,n})] = n+1$.

III. OPEN NEIGHBORHOOD COLORING ON MIDDLE GRAPH OF STAR GRAPH

Theorem 3.1. For any star graph $K_{1,n}$, $\chi_{onc} [M(K_{1,n})] = n+1$.

Proof.

Let $V(K_{1,n}) = \{v, v_1, v_2, \dots, v_n\}$. By the definition of middle graph, each edge $vv_i, (1 \leq i \leq n)$ of $K_{1,n}$ is subdivided by the new vertex e_i in $M(K_{1,n})$.

$V[M(K_{1,n})] = \{v\} \cup \{v_i / 1 \leq i \leq n\} \cup \{e_i / 1 \leq i \leq n\}$ and the vertices $\{v, e_1, e_2, \dots, e_n\}$ induce a clique of order $(n+1)$ in $M(K_{1,n})$. Therefore, $\chi_{onc} M[K_{1,n}] \geq n+1$.

Now, assign the open neighborhood coloring to $V[M(K_{1,n})]$ as follows.

For $1 \leq i \leq n$, assign the colors c_i to e_i and v_i respectively and assign the color c_{n+1} to v . Therefore, $\chi_{onc} M[K_{1,n}] \leq n+1$.

An easy check shows that, it is not possible to assign one of the allowed colors from C_i to v .

Hence, $\chi_{onc} [M(K_{1,n})] = n+1$.

IV. OPEN NEIGHBORHOOD COLORING ON TOTAL GRAPH OF STAR GRAPH

Theorem 4.1. For any star graph $K_{1,n}$, $\chi_{onc} [T(K_{1,n})] = 2n+1$.

Proof.

Let $V(K_{1,n}) = \{v, v_1, v_2, \dots, v_n\}$. By the definition of total graph, each edge $vv_i, (1 \leq i \leq n)$ of $K_{1,n}$ is subdivided by the vertex e_i in $T(K_{1,n})$. $V[M(K_{1,n})] = \{v\} \cup \{v_i / 1 \leq i \leq n\} \cup \{e_i / 1 \leq i \leq n\}$ and the vertices $\{v, e_1, e_2, \dots, e_n\}$, induce a clique of order $(n+1)$ in $T(K_{1,n})$.

Now, assign the open neighborhood coloring to $V[M(K_{1,n})]$ as follows.

For $1 \leq i \leq n$, assign the color c_i to e_i . For $1 \leq i \leq n$, assign the color c_{n+i} to v_i and assign the color c_{2n+1} to v . Therefore, $\chi_{onc} C[K_{1,n}] \leq 2n+1$.

An easy check shows that, it is not possible to assign one of the allowed colors from $\{c_i / 1 \leq i \leq 2n\}$ to v . Therefore, $\chi_{onc} C[K_{1,n}] \geq 2n+1$.

Hence, $\chi_{onc} [C(K_{1,n})] = 2n+1$.

V. OPEN NEIGHBORHOOD COLORING ON CENTRAL GRAPH OF DOUBLE STAR GRAPH

Theorem 5.1

For any double star graph $K_{1,n,n}$, $\chi_{onc} [C(K_{1,n,n})] = 2n+1$.

Proof

Let $v, v_1, v_2, \dots, v_n$ and $u_1, u_2, \dots, u_n$ be the vertices in $K_{1,n,n}$, the vertex v be adjacent to $v_i, 1 \leq i \leq n$. The vertices $v_i, 1 \leq i \leq n$ be adjacent to $u_i, 1 \leq i \leq n$ respectively. Let the edge $vv_i, 1 \leq i \leq n$ and $v_i u_i, 1 \leq i \leq n$ be subdivided by the vertices $e_i, 1 \leq i \leq n$ and $s_i, 1 \leq i \leq n$ respectively in $C(K_{1,n,n})$.

Clearly,

$$V[C(K_{1,n,n})] = \{v\} \cup \{v_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n\} \cup \{e_i : 1 \leq i \leq n\} \cup \{s_i : 1 \leq i \leq n\}$$

The vertices $\{v_i, 1 \leq i \leq n\}$ induce a clique of order n (say K_n) and the vertices $v, u_i, 1 \leq i \leq n$ induce a clique of order $n+1$ (say K_{n+1}) in $C(K_{1,n,n})$ also each v_i adjacent to $u_j, \forall i \neq j$.

For $1 \leq i \leq n$, assign the color c_i to v_i and e_i respectively and assign the color c_{n+i} to s_i and u_i also assign the color c_{2n+1} to v . Therefore, $\chi_{onc} C[K_{1,n,n}] \leq 2n + 1$.

An easy check shows that, it is not possible for assign any one of the allowed colors from $c_i : 1 \leq i \leq 2n$ to v . Therefore, $\chi_{onc} C[K_{1,n,n}] \geq 2n + 1$.

Hence, $\chi_{onc} [C(K_{1,n,n})] = 2n + 1$.

VI. OPEN NEIGHBORHOOD COLORING ON MIDDLE GRAPH OF DOUBLE STARGRAPH

Theorem 6.1

For any double star graph $K_{1,n,n}$, $\chi_{onc} [M(K_{1,n,n})] = 2n + 1$.

Proof

Let $V(K_{1,n,n}) = \{v\} \cup \{v_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n\}$. By the definition of middle graph, each edge vv_i and $v_i u_i$ ($1 \leq i \leq n$) subdivided by the vertices $e_i, 1 \leq i \leq n$ and $s_i, 1 \leq i \leq n$ respectively.

Let $V[M(K_{1,n,n})] = \{v\} \cup \{v_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n\} \cup \{e_i : 1 \leq i \leq n\} \cup \{s_i : 1 \leq i \leq n\}$, the vertices $v, e_1, e_2, \dots, e_n$ induce a clique of order $n + 1$ (say K_{n+1}) in $M(K_{1,n,n})$.

For $1 \leq i \leq n$, assign the color c_i to e_i and assign the color c_{n+i} to v_i . Assign the colors $c_{n+2}, c_{n+3}, \dots, c_{2n}, c_{n+1}$ to u_i and s_i respectively and assign the color c_{2n+1} to v . Therefore, $\chi_{onc} \{M[K_{1,n,n}]\} \leq 2n + 1$.

An easy check shows that, it is not possible for assign one of the allowed colors from $c_i : 1 \leq i \leq 2n$ to v . Therefore, $\chi_{onc} M[K_{1,n,n}] \geq 2n + 1$.

Hence, $\chi_{onc} [M(K_{1,n,n})] = 2n + 1$.

VII. OPEN NEIGHBORHOOD COLORING ON TOTAL GRAPH OF DOUBLE STAR GRAPH

Theorem 7.1. For any double star graph $K_{1,n,n}$, $\chi_{onc} [T(K_{1,n,n})] = 2n + 1$.

Proof.

Let $V(K_{1,n,n}) = \{v\} \cup \{v_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n\}$ and $E(K_{1,n,n}) = \{e_i : 1 \leq i \leq n\} \cup \{s_i : 1 \leq i \leq n\}$. By the definition of total graph, we have $V[T(K_{1,n,n})] = \{v\} \cup \{v_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n\} \cup \{e_i : 1 \leq i \leq n\} \cup \{s_i : 1 \leq i \leq n\}$ in which the vertices $v, e_1, e_2, \dots, e_n$ induce a clique of order $n + 1$ (say K_{n+1}).

For $1 \leq i \leq n$, assign the color c_i to e_i and assign the colors $c_{n+1}, c_{n+2}, \dots, c_{2n}$ to v_i . Assign the colors $c_{n+2}, c_{n+3}, \dots, c_{2n}, c_{n+1}$ to s_i and assign the colors $c_2, c_3, \dots, c_n, c_1$ to u_i and assign the color c_{2n+1} to v . Therefore, $\chi_{onc} T[K_{1,n,n}] \leq 2n + 1$.

An easy check shows that, it is not possible for assign one of the allowed colors from $c_i : 1 \leq i \leq 2n$ to v . Therefore, $\chi_{onc} T[K_{1,n,n}] \geq 2n + 1$.

Hence, $\chi_{onc} [T(K_{1,n,n})] = 2n + 1$.

VIII. CONCLUSION

In this note, we have proved that the open neighborhood chromatic number for the middle graph, central graph and total graph of star and double star graphs. As a motivation this work can be extended by finding the exact values for generalization of these numbers, for graphs according to maximum degree.

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