



Introducing a New Integral Transform: Sadik Transform

Sadikali Latif Shaikh
Department of Mathematics,
Maulana Azad College of arts, Science and Commerce,
RozaBagh, Aurangabad. Maharashtra. 431001, INDIA.

Abstract: In this paper we introduced a new integral transform named Sadik transform. Proved operational properties of Sadik transform. Proved duality theorem of Laplace transform and Sadik transform. Proved that Laplace transform, Sumudu transform, Elzaki transform, Kamal transform, Tarig transform, Laplace- Carson transform, Aboodh transform are particular cases of Sadik transform. Also proved convolution theorem of Sadik transform.

Keywords: Integral transforms, Operational properties, Convolution theorem .

I. Introduction

At the oversight Integral transform method is useful and effective tool for solving differential equations. But it is also true that all types of differential equations are not solvable by integral transform technique. There are so many integral transforms has been developed for eliminating different differential operators see in [1], [2], [3], [4], [5]. The most popular integral transform with exponential type kernel is the Laplace transform. Laplace transform has proved its dominancy in the applications of engineering and applied sciences. Since last few years so many integral transforms with exponential type kernels like Sumudu transform, Elzaki transform, Tarig transform, Natural transform, Kamal Transform, and many more transforms has been introduced and claimed their own superiority as far as their applications are consent. So many authors developed relations between each other transforms. By means of counter examples some authors has been proved their new transform is powerful than the previous ones. But my opinion is that all the integral transforms introduced after the Laplace transform are very much similar to the Laplace transform. Sumudu transform claimed and proved its unit and scale preservation property and also proved that it will not resorting a new frequency domain. The following table explores all new integral transforms of exponential type.

Table 1.1:

Name of Integral Transform	Kernel
1. Laplace Transform	$K(t, v) = e^{-vt}$
2. Sumudu Transform	$K(t, v) = \frac{1}{v} e^{-\frac{t}{v}}$
3. Elzaki Transform	$K(t, v) = v e^{-\frac{t}{v}}$
4. Tarig Transform	$K(t, v) = \frac{1}{v} e^{-\frac{t}{v^2}}$
5. Kamal Transform	$K(t, v) = e^{-\frac{t}{v}}$
6. Natural Transform	$K(t, s, v) = \frac{1}{v} e^{-\frac{st}{v}}$
7. Laplace –Carson transform	$K(t, v) = v e^{-vt}$
8. Aboodh Transform	$K(t, v) = \frac{1}{v} e^{-vt}$
9. ZZ- transform	$K(t, s, v) = \frac{s}{v} e^{-\frac{st}{v}}$

Now I am introducing a new integral transform called Sadik transform, beauty of this transform is that all above transforms in the table will become particular cases of Sadik transform.

II. Preliminaries

Definition 2.2. Sadik transform:

If,

- 1) $f(t)$ is piecewise continuous on the interval $0 \leq t \leq A$ for any $A > 0$.
- 2) $|f(t)| \leq K \cdot e^{at}$ when $t \geq M$, for any real constant a . and some positive constant K and M .

Then Sadik transform of $f(t)$ is defined by

$$F(v^\alpha, \beta) = S[f(t)] = \frac{1}{v^\beta} \int_0^\infty e^{-t v^\alpha} f(t) dt \quad (1)$$

Where,

v is complex variable,

α is any non zero real numbers, and

β is any real number.

Now the following table elaborates that the Sadik transform will be convert into the all transforms which are mentioned in table 1 by changing values of α, β .

Table 2.2:

Values of α, β .	Sadik transform converts into
$\beta = 0$ and $\alpha = 1$	Laplace Transform
$\beta = 1$ and $\alpha = -1$	Sumudu Transform
$\beta = -1$ and $\alpha = -1$	Elzaki Transform
$\beta = 1$ and $\alpha = -2$	Tarig Transform
$\beta = 0$ and $\alpha = -1$	Kamal Transform
$\beta = -1$ and $\alpha = 1$	Laplace –Carson transform
$\beta = 1$ and $\alpha = 1$	Aboodh Transform

By changing the values of alpha and beta, the Sadik transform not only converts into the Laplace, Sumudu, Elzaki, Tarig, Kamal, Laplace-Carson, Aboodh transforms but also will be convert into those integral transforms which are actually not present in the literature and till yet not be proposed by anyone. Sadik transform is now very powerful tool because after applying the Sadik transform you have a choice whether you wish to proceed by Sadik transform or any other existing, non-existed integral transforms just by fixing values of alpha and beta according to a convenience and situation of the problem.

III. Operational Properties of Sadik transform:

1) If $f(t) = t^n$ then, Sadik transform of $f(t) = t^n$ is

$$S[t^n] = \frac{n!}{v^{n\alpha + (\alpha + \beta)}} \quad (2)$$

2) If $f(t) = \sin(at)$ then, Sadik transform of $f(t) = \sin(at)$ is

$$S[\sin(at)] = \frac{av^{-\beta}}{v^{2\alpha + a^2}} \quad (3)$$

3) If $f(t) = \cos(at)$ then, Sadik transform of $f(t) = \cos(at)$ is

$$S[\cos(at)] = \frac{v^{\alpha - \beta}}{v^{2\alpha + a^2}} \quad (4)$$

4) If $f(t) = e^{at}$ then, Sadik transform of $f(t) = e^{at}$ is

$$S[e^{at}] = \frac{v^{-\beta}}{v^{\alpha - a}} \quad (5)$$

5) Sadik transform of hyperbolic functions

$$S[\sinh(at)] = \frac{av^{-\beta}}{v^{2\alpha - a^2}} \quad (6)$$

$$S[\cosh(at)] = \frac{v^{\alpha - \beta}}{v^{2\alpha - a^2}} \quad (7)$$

6) Sadik transform of derivatives:

If $F(v)$ is Sadik transform of $f(t)$ then,

$$S[f'(t)] = v^\alpha F(v) - v^{-\beta} f(0) \quad (8)$$

In general, Sadik transform of nth derivative of $f(t)$ is

$$S[f^{(n)}(t)] = v^{n\alpha} F(v) - \sum_{k=0}^{n-1} v^{k\alpha - \beta} f^{(n-1)-k}(0) \quad (9)$$

In the above all properties if we fix values of α and β as per table 2.1, then these properties will become to properties of Laplace transform, Sumudu transform, Elzaki transform, Aboodh transform, Kamal Transform, Tarig transform, Laplace- Carson transform respectively. It is not compulsory that we have to convert Sadik

transform into any other transform, we can handle it with Sadik transform itself or it can be convert into such a expression in the frequency domain which we feel suitable by fixing values of α and β .

Theorem 3.1: Laplace – Sadik transform duality theorem

If $F(s)$ is Laplace transform of $f(t)$ and $G(v^\alpha, \beta)$ is a Sadik transform of $f(t)$ then

$$G(v^\alpha, \beta) = \frac{1}{v^\beta} F(v^\alpha) \quad (10)$$

Theorem 3.2: Convolution Theorem of Sadik transform

If $F(v^\alpha, \beta)$ and $G(v^\alpha, \beta)$ are Sadik transforms of $f(t)$ and $g(t)$ respectively and $(f * g)(t)$ is a convolution of $f(t)$ and $g(t)$ then, Sadik transform of $(f * g)(t)$ is

$$S[(f * g)(t)] = v^\beta F(v^\alpha, \beta) \cdot G(v^\alpha, \beta) \quad (11)$$

Proof: we can easily prove this theorem by using convolution theorem of Laplace transform and theorem 3.1.

IV. Conclusion

A new Sadik transform is a very powerful transform among all the integral transforms of exponential type kernels, which are described above. Due to Sadik transform we have choice to solve the problems through any transform existed in the literature or transform which are yet not to be exist, just by fixing suitable values of α and β . So many problems in engineering and applied sciences can be consider to solve by Sadik transform, it will be our further research.

V. References

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