



Optimum Allocation in Multi-Objective Geometric Programming in Two-Stage Stratified Sample Surveys

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Abstract: In the real world, many applications of geometric programming are engineering design problems, management problems and transportation problems. Geometric programming provides a powerful tool for solving such types of nonlinear problems where nonlinear relations can be well presented by an exponential or power function. In this paper, the problem of two-stage multivariate stratified sample surveys has been considered and formulated as a multi-objective geometric programming problem (MOGPP) with non-linear objective function and linear constraints. A brief solution procedure of fuzzy geometric programming is given to find the optimum allocations in two-stage multivariate stratified sample surveys. The fuzzy programming approach is used for converting the MOGPP into Single Objective Geometric Programming Problem (SOGPP) with the help of membership function. The formulated SOGPP has been solved with the help of LINGO [LINGO User's Guide. Published by Lindo Systems Inc., 1415 North Dayton Street, Chicago, Illinois-60622; 2013.] Software and the dual solutions are obtained. Consequently with the help of primal-dual relationship theorem and the dual solution, the optimum allocations of two-stage multivariate stratified sample surveys are obtained. Finally a numerical example is solved to illustrate the procedure.

Keywords: Two- Stage, Stratified, Sample Surveys, Multi-Objective, Geometric Programming, Fuzzy programming, Primal-dual relationship.

I. Introduction

In multivariate stratified sampling more than one characteristic is defined on each unit of a stratified population. The problem of optimum allocation of multivariate stratified sampling has been started apparently with Neyman (1934). In multivariate stratified sample surveys, the two-stage sampling designs generally specify two stages of selection: primary sampling units (PSUs) at the first stage, and subsamples from PSUs at second stage as a secondary sampling unit (SSUs). For the large-scale surveys, stratification may precede selection of the sample at any stage. Cochran (1977) described two-stage sampling design in a very simple way in his standard book of the sampling technique. Many authors have worked on two-stage sampling and obtained optimum allocations with the help of different techniques. Some of them are: Kokan and Khan (1967), Arnold, B. F. (1986). Sadooghi-Alvandi, M. (1986), Waters and Chester (1987), Valliant *et al.* (1997), Khan *et al.* (undated). Salehi and Smith (2005), Clark, R. G. (2008), Mwangi and Salim (2012), Moradi *et al.* (2014), Fenta, H. M. (2014), Waters and Chesters (2014), Yang and Thomas (2014) and many others. Geometric Programming (GP) is a mathematical programming technique for optimizing posynomials. If the degree of difficulty of the primal problem is zero, then a unique dual feasible solution exists. If the problem has a positive degree of difficulty, then the objective function can be maximized by finding the dual feasible region, and if there is a negative degree of difficulty then inconsistency of the dual constraints may occur. Duffin and Zener have solved the engineering design problems in the early 1960s with the help of GP techniques and then Duffin *et al.* (1967) have extended the work of Duffin and Zener. GP method was applied by many authors in different fields. Some of them are as: Jitka Dupačová (2010), Maqbool *et al.* (2011), Ghosh and Roy (2013), Shafiullah *et al.* (2013). Multi-objective geometric programming problem was discussed by Ojha and Das (2010), Islam, S. (2010), Kotb and Fergany (2014), Bazikar and Saraj (2014), Shen and Bai (2014), Hou *et al.* (2014), Dey and Roy (2014), Shafiullah *et al.* (2014), Shafiullah and Agarwal (2015) and many others in different fields. A system with indistinguishable information can neither be formulated nor be solved effectively by traditional mathematics-based optimization techniques nor probability-based stochastic optimization approaches. However, fuzzy set theory and fuzzy programming techniques provide a useful and efficient tool for modeling and optimizing such systems. Zadeh (1965), Gupta *et al.* (2013), Li *et al.* (2014), Thuan *et al.* (2014), Zaher *et al.* (2016), Cebi *et al.* (2016) and Zimmermann (1978) have described fuzzy

set theory, fuzzy programming and linear programming respectively. Fuzzy multi-objective programming is discussed by many authors such as: Sakawa and Yano (1994), Kanaya (2010) and many others. In this paper, the problem of two-stage multivariate stratified sample surveys has been formulated as a convex multi-objective geometric programming problem (MOGPP) with non-linear objective function and linear constraints. The fuzzy programming approach is used for converting the (MOGPP) into Single Objective Geometric Programming Problem (SOGPP) with the help of membership function. The formulated SOGPP has been solved with the help of LINGO Software and the dual solution is obtained. Consequently with the help of primal-dual relationship theorem and the dual solution, the optimum allocations of two-stage multivariate stratified sample surveys are obtained. Throughout this paper, for the sake of simplicity, we denote A^B by $A \wedge B$. A numerical example is presented to illustrate the procedure.

II. Formulation of the Problem

Let us consider the population consists of NM elements grouped into N first-stage units and M second-stage units each. Let n and m be the corresponding sample sizes selected with equal probability and without replacement at each stage. Let y_{hrj} be the value obtained for r^{th} secondary stage unit drawn from the h^{th} primary stage unit for j^{th} character, ($h = 1, 2, \dots, N, r = 1, 2, \dots, M, j = 1, 2, \dots, p$), the relevant notations which will be used in the paper are described as follows:

$$\bar{y}_{hj} = \sum_{r=1}^m \frac{y_{hrj}}{m} = \text{Sample mean per subunit at the } h^{th} \text{ primary stage unit.}$$

$$\bar{y}_j = \sum_{h=1}^N \frac{\bar{y}_{hj}}{n} = \text{Overall sample mean per subunit (element).}$$

$$\bar{Y}_{hj} = \sum_{r=1}^M \frac{y_{hrj}}{M} = \text{Mean per element at the } h^{th} \text{ first- stage unit.}$$

$$\bar{\bar{Y}}_j = \sum_{h=1}^N \frac{\bar{Y}_{hj}}{N} = \text{Mean per element in the population.}$$

The following population variances are required.

$$S_{bj}^2 = \sum_{h=1}^N \frac{(\bar{Y}_{hj} - \bar{\bar{Y}}_j)^2}{N-1} = \text{True variance between first stage unit means.}$$

$$S_{wj}^2 = \sum_{h=1}^N \sum_{r=1}^M \frac{(y_{hrj} - \bar{Y}_{hj})^2}{N(M-1)} = \text{True variance within first stage unit.}$$

=

In case of equal first-stage units an unbiased estimate of $\bar{\bar{Y}}_j$ is $\bar{y}_j$ with its sampling variance as,

$$V(\bar{y}_j) = \left(\frac{1}{n} - \frac{1}{N} \right) S_{bj}^2 - \left(\frac{1}{nm} - \frac{1}{NM} \right) S_{wj}^2, \quad j=1, 2, \dots, p. \quad (1)$$

In two- stage sampling the total cost function may be given by:

$$C = C_1 n + C_2 nm \quad (2)$$

Where: C_1 = cost of the survey for obtaining a single primary stage unit.

C_{2j} = cost of enumerating the j^{th} character per element.

$$C_2 = \sum_{j=1}^p C_{2j} = \text{cost of enumerating all the } p \text{ characters per SSU.}$$

C = total cost for enumerating all the p characters.

Suppose that it is required to find the values of n and m , so that the total cost C is minimized, subject to the upper limits on the variances. If N and M are large, then from (1), the limits on the variances may be expressed as:

$$\frac{S_{bj}^2}{n} + \frac{S_{wj}^2}{nm} \leq v_j, \quad j=1, 2, \dots, p. \quad (3)$$

Where, v_j are the upper limits on the variances of various characters. Here S_{bj}^2 is the variance among primary stage units means, S_{wj}^2 is the variance among subunits within primary units for j^{th} characteristic respectively.

$$\left. \begin{aligned} \text{Min} \quad & \frac{S_{bj}^2}{n} + \frac{S_{wj}^2}{nm} \\ \text{Subject to} \quad & C_1 n + C_2 nm \leq C, j=1,2,\Lambda, p \text{ and } n \geq 1, m \geq 1 \end{aligned} \right\} \quad (4)$$

III. Geometric programming approach in two-stage sampling design

Geometric programming always transforms the primal problem of minimizing a posynomial subject to posynomial constraints to a dual problem of maximizing a function of the weights on each constraint. The mathematical form of problem (4) can be expressed in the following way:

The vectors $\underline{X} = (x_1, x_2)$ ($x_1 = n$ and $x_2 = nm$) can be obtained as:

$$\left. \begin{aligned} \text{Min } V(\underline{y}_j) &= \sum_{i=1}^2 \frac{a_{iq}}{x_i} \\ \text{Subject to } \sum_{i=1}^2 C_i x_i &\leq C \text{ and } x_i \geq 0, i=1,2. \end{aligned} \right\}, q=1,2,K,p \quad (5)$$

After substituting the values in the above equations we get:

$$x_1 = n, x_2 = nm, S_{bq}^2 = a_{1q}, S_{wq}^2 = a_{2q} \text{ for } q=1, \dots, p,$$

The standard GP (Primal) problem of the above Eqn. (5) can be stated as:

$$\left. \begin{aligned} \text{Min} \quad & f_{0j}(x) \\ \text{Subject to} \quad & f_q(x) \leq 1, q=1, \dots, p \text{ and } x_i > 0, i=1,2 \end{aligned} \right\} \quad (6)$$

$$\text{where } f_q(x) = \sum_{i \in [q]} d_i \left\{ \prod_{j=1}^n x_i^{\lambda_{ij}} \right\}, d_i \geq 0, x_i > 0, q=0,1,\Lambda,p. \quad (7)$$

The number of posynomial terms in the function can be denoted by q, the number of variables is denoted by n and the exponents λ_{ij} are real constants. The objective function $C(x)$ for our allocation problem that is given in 5(i) and 5(ii) has $d_i = a_{iq}$ and $d_i = c_i, i=1,2$.

The dual form of Geometric Programming problem which is stated in (6) can be given as:

$$\left. \begin{aligned} \text{Max} \quad & \left[\prod_{q=0}^p \prod_{i \in [q]} \left(\frac{d_i}{w_i} \right)^{w_i} \right] \prod_{q=1}^p \left(\sum_{i \in [q]} w_i \right)^{\sum_{i \in [q]} w_i} \quad (i) \\ \text{Subject to} \quad & \sum_{i \in [0]} w_i = 1 \quad (ii) \\ & \sum_{q=0}^p \sum_{i \in [q]} p_{ij} w_i = 0 \quad (iii) \\ \text{and} \quad & w_i \geq 0, q=0, \dots, p \end{aligned} \right\} \quad (8)$$

Step 1: For the Optimum value of the objective function, the objective function always takes the form

$$\begin{aligned} C_0(x^*) &= \left(\frac{\text{Coeffi. of first term}}{w_{01}} \right)^{w_{01}} \times \left(\frac{\text{Coeffi. of Second term}}{w_{02}} \right)^{w_{02}} \\ &\times \dots \times \left(\frac{\text{Coeffi. of last term}}{w_k} \right)^{w_k} \left(\sum w' \text{ s in the first constraint} \right)^{\sum w' \text{ s in the first constraints}} \\ &\left(\sum w' \text{ s in the last constraint} \right)^{\sum w' \text{ s in the last constraints}} \end{aligned}$$

Step 2: The equations that can be used for the geometric program for the weights are given below:

$$\sum_{i=1}^2 w_{0i} \text{ in the objective function} = 1 \quad (\text{Normality condition}) \quad (9)$$

and for each primal variable x_j given n variables and k terms

$$\sum_{i=1}^m (w_i \text{ for each trms}) \times (\text{exponent on } x_j \text{ in that term}) = 0 \quad (\text{Orthogonality Condition}) \quad (10)$$

$$w_{0i} \geq 0 \text{ and } w_i \geq 0 \quad (\text{Positivity Condition})$$

Primal-dual relationship theorem: If w_{0i}^* is a maximizing point for the dual problem (8), each minimizing points (x) for the primal problem (6) satisfies the system of equations:

$$f_0(x) = \begin{cases} w_{0i}^* v(w_{0i}^*), & i \in J[0] \\ \frac{w_{ij}}{v_L(w_{0i}^*)}, & i \in J[L] \end{cases} \quad (11)$$

where L ranges over all positive integers for which $v_L(w_{0i}^*) > 0$.

IV. Fuzzy Geometric Programming Approach

The solution procedure to solve the problem (8) consists of the following steps:

Step-1: Solve the MOGPP as a single objective problem using only one objective at a time and ignoring the others. These solutions are known as the ideal solution.

Step-2: From the results of step-1, determine the corresponding values for every objective at each solution. The pay-off matrix can be constructed with all the values of objective functions at ideal solution, where the diagonal solutions are the best solutions and the off-diagonal solutions are the worst.

Here $(x^{(1)}), (x^{(2)}), \Lambda, (x^{(j)}), \Lambda, (x^{(p)})$ are the ideal solutions for the objective functions

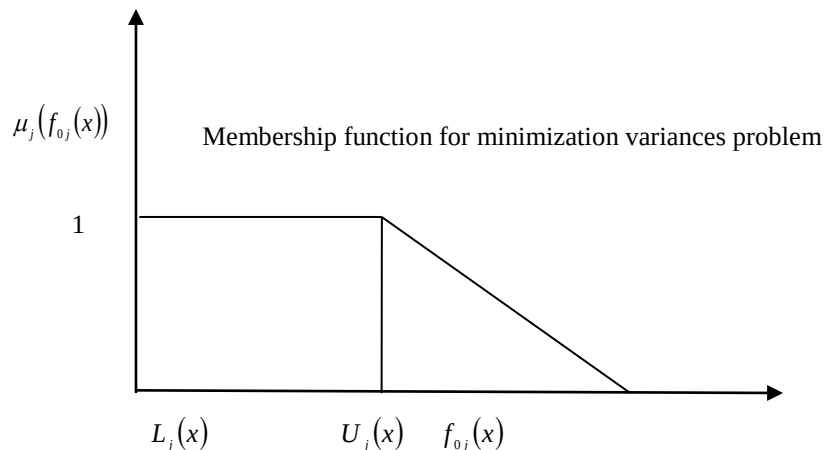
$$f_{01}(x^{(1)}), f_{02}(x^{(2)}), \Lambda, f_{0j}(x^{(j)}), \Lambda, f_{0p}(x^{(p)}). \quad U_j = \text{Max}\{f_{01}(x^{(1)}), f_{02}(x^{(2)}), \Lambda, f_{0p}(x^{(p)})\} \text{ and } L_j = f_{0j}^*(x^{(j)}), [U_j \text{ and } L_j \text{ be the upper and lower bounds of the } j^{\text{th}} \text{ objective function } f_{0j}(x), j=1,2,\dots,p.]$$

Step 3: The membership function for the given problem can be defined as:

$$\mu_j(f_{0j}(x)) = \begin{cases} 0, & \text{if } f_{0j}(x) \geq U_j \\ \frac{U_j(x) - f_{0j}(x)}{U_j(x) - L_j(x)}, & \text{if } L_j \leq f_{0j}(x) \leq U_j, j=1,2,K,p \\ 1, & \text{if } f_{0j}(x) \leq L_j \end{cases} \quad (12)$$

Here $U_j(x)$ is a strictly monotonic decreasing function with respect to $f_{0j}(x)$.

Following figure illustrates the graph of the membership function $\mu_j(f_{0j}(x))$ of equation (12) as:



Therefore the general aggregation function can be defined as $\mu_{\tilde{D}}(x) = \mu_{\tilde{D}}\{\mu_1(f_{01}(x)), \mu_2(f_{02}(x)), \dots, \mu_p(f_{0p}(x))\}$

The fuzzy multi-objective formulation of the problem can be defined as:

$$\begin{cases} \text{Max } \mu_{\tilde{D}}(x) \\ \text{Subject to } \sum_{i=1}^2 C_i x_i \leq C; x_i \geq 0 \text{ and } i=1,2 \end{cases} \quad (13)$$

The problem is to find the optimal values of (x^*) for this convex-fuzzy decision based on addition operator (like Tiwari et. al. (1987)). Therefore the problem (13) is reduced according to the max-addition operator as:

$$\begin{cases} \text{Max } \mu_{\tilde{D}}(x^*) = \sum_{j=1}^p \mu_j(f_{0j}(x)) = \sum_{j=1}^p \frac{U_j - (f_{0j}(x))}{U_j - L_j} \\ \text{Subject to } \sum_{i=1}^2 C_i x_i \leq C, 0 \leq \mu_j(f_{0j}(x)) \leq 1; x_i \geq 0 \text{ and } i=1,2. \end{cases} \quad (14)$$

The problem (12) reduces to

$$\begin{aligned} \text{Max } \mu_D(x^*) &= \sum_{j=1}^p \left\{ \frac{U_j}{U_j - L_j} - \frac{(f_{0j}(x))}{U_j - L_j} \right\} \\ \text{Subject to } f_q(x) &\leq 1; x_i \geq 0 \text{ and } i=1,2 \\ \text{where } f_q(x) &= \sum_{i=1}^2 C_i x_i \leq C \end{aligned} \quad (15)$$

The problem (15) maximizes if the function $F_{0j}(x) = \frac{(f_{0j}(x))}{U_j - L_j}$ attains the minimum values. The fuzzy multi-objective formulation of the standard primal problem can be defined as:

$$\begin{aligned} \text{Min } & \sum_{j=0}^p F_{0j}(x) \\ \text{Subject to } & f_q(x) \leq 1, q=1, \dots, p \text{ and } x_i > 0, i=1,2 \end{aligned} \quad (16)$$

The dual form of the primal GPP which is stated in (16) can be given as:

$$\begin{aligned} \text{Max } v(w) &= \prod_{q=0}^k \prod_{i \in j[q]} \left\{ \left(\frac{d_i}{w_i} \right)^{w_i} \right\} \prod_{q=1}^k \left(\sum_{i \in j[q]} w_i \right)^{\sum_{i \in j[q]} w_i} \quad (i) \\ \text{Subject } & \sum_{i \in [0]} w_i = 1 \quad (ii) \\ & \sum_{q=0}^k \sum_{i \in j[q]} p_{ih} w_i = 0 \quad (iii) \\ \text{and } & w_i \geq 0, q=0,1,\dots,k \text{ and } i=1,2,\dots,K, m_k \quad (iv) \end{aligned} \quad (17)$$

The optimal values of the sample sizes of the problems x_n^* can be calculated with the help of the primal-dual relationship theorem (11).

V. Numerical Illustrations

I have considered Chakravorthy (1955) and Maqbool *et al.* (2011) for numerical illustration.

Table 1

Dispersion due to	Degree of freedom	S.P. Matrix	Covariance Matrix
Between PSUs	19	$\begin{bmatrix} 0.5592 & 0.2993 \\ K & 1.1026 \end{bmatrix}$	$\begin{bmatrix} 0.0294 & 0.0157 \\ K & 0.0580 \end{bmatrix}$
Within PSUs Between SSUs	140	$\begin{bmatrix} 1.0872 & 0.3568 \\ K & 3.4041 \end{bmatrix}$	$\begin{bmatrix} 0.0078 & 0.0025 \\ K & 0.0243 \end{bmatrix}$
Total	159	$\begin{bmatrix} 1.6454 & 0.6561 \\ K & 4.5067 \end{bmatrix}$	

In this case the values of N and M are not known, they may be assumed to be infinite. Also the data may be taken as derived from a pilot survey and a similar survey is to be planned for which we require the best values of n and m which minimizes the total cost. Therefore from the above dispersion matrix, we have

$$S_{w1}^2 = 0.0078, S_{w2}^2 = 0.0243, S_{b1}^2 = 0.0037, S_{b2}^2 = 0.0073$$

The upper bound of v is calculated using the lower 5 percent point of distribution with 19 df. =10.117, we have, $v_1 = 0.000345$, $v_2 = 0.000681$

The upper confidence bounds of S_{wj}^2 at 95 percent confidence level are:

$$\text{Upper bound of } S_{w1}^2 = 0.009589, \text{ Upper bound of } S_{w2}^2 = 0.030025$$

The upper confidence bounds of S_{bj}^2 at 95 percent confidence level are:

$$\text{Upper bound of } S_{b1}^2 = 0.006129, \text{ Upper bound of } S_{b2}^2 = 0.011180$$

Using the above values the primal problem can be written as:

$$\left. \begin{array}{l} \text{Min} \quad \frac{0.006129}{x_1} + \frac{0.009589}{x_2} \\ \text{Subject to} \quad 0.0174x_1 + 0.005x_2 \leq 1 \text{ and } x_1, x_2 \geq 0. \end{array} \right\} \quad (18)$$

The dual GPP of the above problem (18) is as follows:

$$\left. \begin{array}{l} \text{Max } v(w_{0i}^*) = ((0.006129/w_{01})^{w_{01}}) \times ((0.009589/w_{02})^{w_{02}}) \times \\ ((0.0174)^{w_{11}}) \times ((0.005)^{w_{12}}) \times (w_{11} + w_{12})^{\wedge} (w_{11} + w_{12}) \quad (i) \\ \text{Subject to} \\ \quad w_{01} + w_{02} = 1 \quad (\text{normality condition}) \quad (ii) \\ \quad \left. \begin{array}{l} -w_{01} - w_{11} = 0 \\ -w_{02} - w_{12} = 0 \end{array} \right\} \quad (\text{orthogonality condition}) \quad (iii) \\ \quad w_{01}, w_{02}, w_{11}, w_{12} \geq 0 \quad (\text{positivity condition}) \quad (iv) \end{array} \right\} \quad (19)$$

Solving the above formulated dual problem (19), we have the corresponding solution as:

$$w_{01} = 0.5986212, \quad w_{02} = 0.4013788 \quad \text{and} \quad v(w_{01}^*) = 0.002976011$$

The optimal values x_i^* of the sample sizes of the standard primal problems can be calculated with the help of the primal–dual relationship theorem (11).

$$f_{0j}(x) = w_{0i}^* v(w^*)$$

$$f_{01}(x_1) = w_{01}^* v(w^*) \Rightarrow x_1 \cong 3, \quad f_{02}(x_2) = w_{02}^* v(w^*) \Rightarrow x_2 \cong 8 \quad \text{and value of objective function is } 0.00297601.$$

$$\left. \begin{array}{l} \text{Min} \quad \frac{0.011180}{x_1} + \frac{0.03025}{x_2} \\ \text{Subject to} \quad 0.0174x_1 + 0.005x_2 \leq 1 \text{ and } x_1, x_2 \geq 0. \end{array} \right\} \quad (20)$$

The dual GPP of the above problem (20) is as follows:

$$\left. \begin{array}{l} \text{Max } v(w_{0i}^*) = ((0.011180/w_{01})^{w_{01}}) \times ((0.03025/w_{02})^{w_{02}}) \times \\ ((0.0174)^{w_{11}}) \times ((0.005)^{w_{12}}) \times (w_{11} + w_{12})^{\wedge} (w_{11} + w_{12}) \quad (i) \\ \text{Subject to} \\ \quad w_{01} + w_{02} = 1 \quad (\text{normality condition}) \quad (ii) \\ \quad \left. \begin{array}{l} -w_{01} - w_{11} = 0 \\ -w_{02} - w_{12} = 0 \end{array} \right\} \quad (\text{orthogonality condition}) \quad (iii) \\ \quad w_{01}, w_{02}, w_{11}, w_{12} \geq 0 \quad (\text{positivity condition}) \quad (iv) \end{array} \right\} \quad (21)$$

Solving the above formulated dual problem (21), we have the corresponding solution as:

$$w_{01} = 0.5323458, \quad w_{02} = 0.4676542 \quad \text{and} \quad v(w_{01}^*) = 0.006864412$$

The optimal values x_i of the sample sizes of the standard primal problems can be calculated with the help of the primal–dual relationship theorem (11).

$$f_{0j}(x) = w_{0i}^* v(w^*)$$

$$f_{01}(x_1) = w_{01}^* v(w^*) \Rightarrow x_1 \cong 3, \quad f_{02}(x_2) = w_{02}^* v(w^*) \Rightarrow x_2 \cong 9 \quad \text{and value of objective function is } 0.00686441.$$

Now the pay-off matrix of the above problems is given below:

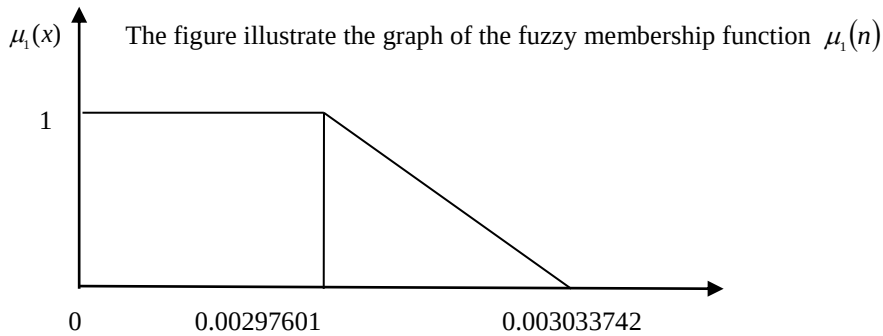
$$\begin{array}{cc} & \begin{matrix} f_{01}(x) & f_{02}(x) \end{matrix} \\ \begin{matrix} x^{(1)} \\ x^{(2)} \end{matrix} & \begin{bmatrix} 0.002976011 & 0.00698989 \\ 0.003033742 & 0.006864412 \end{bmatrix} \end{array}$$

The lower and upper bond of $f_{01}(n)$ and $f_{02}(n)$ can be obtained from the pay-off matrix

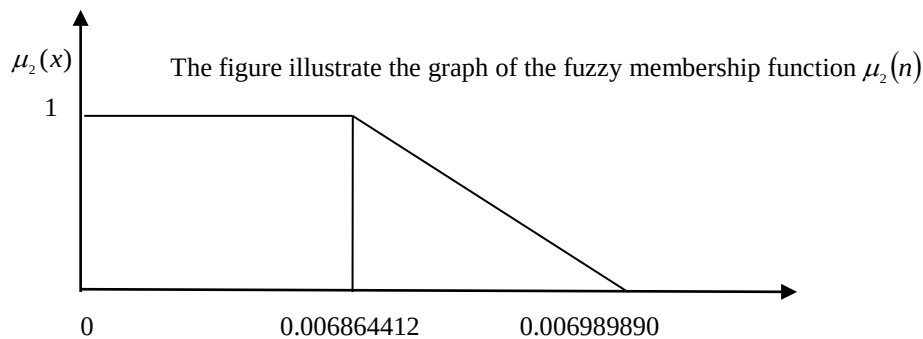
$$0.002976011 \leq f_{01}(x) \leq 0.003033742 \quad \text{and} \quad 0.00698989 \leq f_{02}(x) \leq 0.006864412.$$

Let $\mu_1(x)$ and $\mu_2(x)$ be the fuzzy membership function of the objective function $f_{01}(x)$ and $f_{02}(x)$ respectively and they are defined as:

$$\mu_1(x) = \begin{cases} 1 & , \text{if } f_{01}(n) \leq 0.002976011 \\ \frac{0.003033742 - f_{01}(x)}{0.000057731} & , \text{if } 0.002976011 \leq f_{01}(n) \leq 0.003033742 \\ 0 & , \text{if } f_{01}(n) \geq 0.003033742 \end{cases}$$



$$\mu_2(x) = \begin{cases} 1 & , \text{if } f_{02}(n) \leq 0.006989890 \\ \frac{0.006864412 - f_{02}(x)}{0.00001255} & , \text{if } 0.006989890 \leq f_{02}(n) \leq 0.006864412 \\ 0 & , \text{if } f_{02}(n) \geq 0.006864412. \end{cases}$$



On applying the max-addition operator, the MOGPP, reduces to the crisp problem as:

$$\begin{aligned} & \text{Max } (\mu_1(x) + \mu_2(x)) \\ & \text{i.e Max } \left\{ \frac{0.003033742 - f_{01}(x)}{0.000057731} + \frac{0.006864412 - f_{02}(x)}{0.00001255} \right\} \\ & \text{i.e Max } \left\{ 107.2461 - \left(\frac{f_{01}(x)}{0.000057731} + \frac{f_{02}(x)}{0.00001255} \right) \right\} \\ & \text{Subject to } 0.0174x_1 + 0.005x_2 \leq 1 \text{ and } x_1, x_2 \geq 0. \end{aligned} \quad (22)$$

In order to maximize the problem (22), we have to minimize $\left(\frac{f_{01}(x)}{0.000057731} + \frac{f_{02}(x)}{0.00001255} \right)$

So, our new problem for the solution is given as follows:

$$\begin{aligned} & \text{Min } \frac{997.1648}{x_1} + \frac{2558.5282}{x_2} \\ & \text{Subject to } 0.0174x_1 + 0.005x_2 \leq 1 \text{ and } x_1, x_2 \geq 0. \end{aligned} \quad (23)$$

Degree of Difficulty of the problem (23) is $= (4 - (2+1)) = 1$

Hence the dual problem of the above problem (23) is given as:

$$\left. \begin{aligned}
 &Max \quad v(w_{0i}^*) = \left((997.0014/w_{01})^{w_{01}} \right) \times \left((2558.5282/w_{02})^{w_{02}} \right) \times \\
 &\left(\left(\frac{0.0174}{w_{11}} \right)^{w_{11}} \right) \times \left(\left(\frac{0.005}{w_{12}} \right)^{w_{12}} \right) \\
 &((w_{11} + w_{12})^{(w_{11} + w_{12})}); \quad (i) \\
 &Subject \ to \quad w_{01} + w_{02} = 1; \text{ (normality condition)} \quad (ii) \\
 &\quad \quad \quad \left. \begin{aligned}
 &-w_{01} + w_{11} = 0 \\
 &-w_{11} + w_{12} = 0
 \end{aligned} \right\} \text{ (orthogonality condition)} \quad (iii) \\
 &\quad \quad \quad w_{01}, w_{02}, w_{11}, w_{12} \geq 0 \quad \text{ (positivity condition)} \quad (iv)
 \end{aligned} \right\} \quad (24)$$

For orthogonality condition defined in expression 24 (iii) are evaluated with the help of the payoff matrix which is defined below:

$$\begin{pmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} w_{01} \\ w_{02} \\ w_{11} \\ w_{12} \end{pmatrix} = \begin{cases} -w_{01} + w_{11} = 0 \\ -w_{02} + w_{12} = 0 \end{cases}$$

After solving the formulated dual problem (24) using lingo software we obtain the following values of the dual variables which are given as:

$$w_{01} = 0.5380269, w_{02} = 0.4619731 \text{ and } v(w_{0i}^*) = 59.92901$$

The optimal values x_i^* of the sample sizes of the standard primal problems can be calculated with the help of the primal–dual relationship theorem (11).

$$f_{0j}(x) = w_{0i}^* v(w^*), \quad f_{01}(x_1) = w_{01}^* v(w^*) \Rightarrow x_1 \cong 30 \quad f_{02}(x_2) = w_{02}^* v(w^*) \Rightarrow x_2 \cong 92$$

The optimal values and the objective function value are given below:

$$x_1^* = 30, x_2^* = 92 \text{ and the objective value of the primal problem is } 59.92901.$$

Since we have assumed that $x_1 = n$ and hence $n = 30$ and $x_2 = nm$ hence $nm = 92 \Rightarrow m \cong 3$.

Therefore the total cost (C) involved in the complete survey in these two stages can be obtained with the help of calculated values of primary (n) and secondary (m) allocations with the given values of cost of the survey for obtaining a single primary stage unit (C_1) and cost of enumerating all the p characters per SSU (C_2) as:

$$C_1 = \text{Rs}8.5, C_2 = \text{Rs}2.5, n = 30 \text{ and } m \cong 3$$

$$\text{Hence } C = C_1 n + C_2 nm \Rightarrow C = 8.5 \times 30 + 2.5 \times 92 \Rightarrow C = \text{Rs}485.$$

VI. Conclusions

This paper gives the insightful study of the problem of two-stage stratified sample surveys which is formulated as a convex MOGPP with non-linear objective function and linear constraints. The fuzzy programming approach is used for converting the (MOGPP) into Single Objective Geometric Programming Problem (SOGPP) with the help of membership function. The formulated SOGPP has been solved with the help of LINGO Software and the dual solution is obtained. The optimum allocations are obtained with the help of primal-dual relationship theorem along with corresponding dual variables. A numerical example is illustrated to establish the practical utility of the given method in multivariate two-stage stratified sample surveys. The researcher can adopt this method for obtaining the solution of very complicated convex programming problem even with multi-stage sample survey problems.

Acknowledgements

The author is grateful to the anonymous referees and the Editor in chief for their valuable suggestions, comments, necessary advice for corrections to improve the quality of presentation and solution procedure of the paper.

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