



Numerical Solution of Higher Order Singular Initial Value Problems (SIVP) by Exponentially Fitted Collocation Approximate Method (EFCAM)

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Abstract: In this work, we proposed and employed the use of exponentially fitted collocation approximate method to find numerical solutions of high order Singular Initial Value Problems. Power series form of approximation is used as basis function. Which substituted into slightly perturbed high order singular initial value equations while the resulting equations are solved using MAPLE 18. The study concluded that Exponentially Fitted Collocation Approximation Method (EFCAM) serves as alternative ways of finding numerical solutions of singular linear ordinary differential equations. In general, the method is fast, efficient and highly accurate.

Keywords: Exponentially Fitted Collocation Approximation Method (EFCAM), Power Series, Singular Initial Value Problems (SIVP) and Slightly Perturbation.

I. Introduction

Singular initial value problems occur in several model of mathematics and mathematical physics such as the theory of stellar structure, the thermal behavior of a special cloud of gas, isothermal gas sphere and theory of thermionic currents which are modeled by means of the Lane-Emden equation of the form:

$$y''(x) + \frac{\alpha}{x} y'(x) + f(x, y) = g(x), 0 < x \leq 1, \alpha \geq 0 \quad (1)$$

subject to initial conditions

$$y(x_0) = A_0, \quad y'(x_0) = B_0 \quad (2)$$

where A_0 and B_0 are constants, $f(x, y)$ and $g(x)$ are continuous real valued functions.

In recent years, there has been growing interest in the study of mathematical physics and modeling of physical phenomena often contain singular initial value problems, this equation describes the temperature variation of a spherical gas cloud under the mutual attraction of its molecules and subject to the Law of classical thermodynamics, potential problem, radiative equilibrium, elastic contact problems and many other ([1],[2],[3]) Singular initial value problems (SIVP) is usually difficult to solve analytically, so it required to obtain the approximate solution [4]. In this work, the researcher has provided a numerical method for obtaining the approximate solution of singular initial value problems. Several research works have been carried out in this area in recent year among the popular methods used by past numerical analyst are ([5], [6], [7], [8], [9]).

In this work, the researcher has investigated similar problems using exponentially fitted collocation method using Chebyshev Polynomials during the perturbation of the equations, which established in literature that Chebyshev Polynomials of the first kind possess properties that minimize errors within the region of consideration [10] The advantage of one of these properties called minimax property is in this study harnessed in such a way that the derived formula is automated in symbolic algebraic program using MAPLE 18 software.

II. Formulation of Numerical Technique

This section describes the development of exponentially fitted collocation method for the numerical solution of second and third orders singular initial value problem in ordinary differential equations of form equation (1). The whole idea of the method is to use power series as a basis function and its derivative substituted into a slightly perturbed equation with perturbation the term added to the right hand side of the equation. The addition of the perturbation term is to minimize the error of the problem in consideration

In this work, we consider approximate solutions:

(I) The Power series of the form

$$y_N(x) = \sum_{k=0}^N Y(k)x^k \quad (3)$$

(ii) The Exponentially fitted approximate solution of the form

$$y_N(x) = \sum_{k=0}^N Y(k)x^k + \tau_n e^x \quad (4)$$

where x represents the dependent variables in the problem, n represents the order of the dependent variable $y(x)$, $Y(k)$, $(k \geq 0)$ are the unknown constants to be determined, N is the computational length and τ_n is a free tau parameter to be determined.

The construction process of the EFCAM starts with the assumption of an approximate of the solution given in equation (1) with derivation of equation (3) and substitutes into the general second order singular initial value of the form

$$y_N''(x) + \alpha_1(x)y_N'(x) + \alpha_2(x)y_N(x) = g(x) \quad (5)$$

subject to the conditions

$$y(x_0) = A_0 \quad y'(x_0) = A_1 \quad (6)$$

In order to solve equation (5), we differentiated equation (3) twice, to obtain

$$y_N'(x) = \sum_{k=1}^N kY(k)x^{k-1} \quad (7)$$

and

$$y_N''(x) = \sum_{k=2}^N k(k-1)Y(k)x^{k-2} \quad (8)$$

Substituting equations (3), (7) and (8) into equation (5), we obtained

$$\sum_{k=2}^N k(k-1)Y(k)x^{k-2} + \alpha_1(x) \sum_{k=1}^N kY(k)x^{k-1} + \alpha_2(x) \sum_{k=0}^N Y(k)x^k = g(x) \quad (9)$$

The expansion of equation (9) leads to

$$\begin{aligned} & 2Y(2) + 6Y(3)x + \Lambda + N(N-1)Y(N)x^{N-2} + \alpha_1(x)Y(1) + 2\alpha_1(x)Y(2)x \\ & + 3x^2\alpha_1(x)Y(3) + \Lambda + NY(N)x^{N-1} + \alpha_2(x)Y(0) + \alpha_2Y(1)x + \alpha_2(x)Y(2)x^2 \\ & + \alpha_2Y(3)x^3 + \Lambda + \alpha_2(x)Y(N)x^N = g(x) \end{aligned} \quad (10)$$

Thus, collecting like terms in equation (10) leads to

$$\begin{aligned} & \alpha_2(x)Y(0) + (\alpha_1(x) + \alpha_2(x)x)Y(1) + (2 + 2\alpha_1(x)x + \alpha_2(x)x^2)Y(2) \\ & (6x + 3\alpha_1(x)x^2 + \alpha_2(x)x^3)Y(3) + \Lambda + (N(N-1)x^{N-2} + Nx^{N-1}\alpha_1(x) \\ & + \alpha_2(x)x^N)Y(N) = g(x) \end{aligned} \quad (11)$$

Equation (11) is slightly perturbed to give

$$\begin{aligned} & \alpha_2(x)Y(0) + (\alpha_1(x) + \alpha_2(x)x)Y(1) + (2 + 2\alpha_1(x)x + \alpha_2(x)x^2)Y(2) \\ & (6x + 3\alpha_1(x)x^2 + \alpha_2(x)x^3)Y(3) + \Lambda + (N(N-1)x^{N-2} + Nx^{N-1}\alpha_1(x) \\ & + \alpha_2(x)x^N)Y(N) = g(x) + \tau_1 T_N(x) \end{aligned} \quad (12)$$

where τ_1 is a free tau parameter to be determined and $T_N(x)$ is the Chebyshev Polynomials and is of degree N .

By collocating equation equation (12) at a point $x = x_i$, we obtained

$$\begin{aligned} & \alpha_2(x_i)Y(0) + (\alpha_1(x_i) + \alpha_2(x_i)x_i)Y(1) + (2 + 2\alpha_1(x_i)x_i + \alpha_2(x_i)x_i^2)Y(2) \\ & (6x_i + 3\alpha_1(x_i)x_i^2 + \alpha_2(x_i)x_i^3)Y(3) + \Lambda + (N(N-1)x_i^{N-2} + Nx_i^{N-1}\alpha_1(x_i) \\ & + \alpha_2(x_i)x_i^N)Y(N) - \tau_1 T_2(x_i) - \tau_2 T_1(x_i) = g(x_i) \end{aligned} \quad (13)$$

where

$$x_i = a + \frac{(b-a)i}{N+2}; \quad i = 1, 2, 3, \Lambda, N+1 \quad (14)$$

Hence, equation (14) gives rise to algebraic linear system of equations in (N+1) unknown constants. Two extra equations are obtained by fitting one tau-parameter to the initial condition as suggested by [11] to the initial conditions given as

$$y(x_0) = \sum_{k=0}^N Y(k)x_0^k + \tau_2 e^{x_0} = A_0 \quad y'(x_0) = \sum_{k=0}^N kY(k)x_0^{k-1} + \tau_2 e^{x_0} = A_1 \quad (15)$$

Altogether, we obtained (N+3) algebraic linear equations in (N+3) unknown constants. Thus, we put the (N+3) algebraic equations in Matrix form as

$$BX = p \quad (16)$$

Where

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} & K & b_{1N} & -\tau_1 T_2(x_1) & -\tau_2 T_1(x_1) \\ b_{21} & b_{22} & b_{23} & K & b_{2N} & -\tau_1 T_2(x_2) & -\tau_2 T_1(x_2) \\ M & & & & & 0 & 0 \\ M & & & & & 0 & 0 \\ b_{N1} & b_{N2} & b_{N3} & K & b_{NN} & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$X = [Y(0)Y(1), Y(2), Y(3), \dots, Y(N), \tau_1, \tau_2]^T$$

$$p = [g(x_1), g(x_2), g(x_3), \dots, g(x_{N+1}), A_0, A_1]^T$$

These (N + 3) unknown constants Y(0), Y(1), ..., Y(N), τ_1 and τ_2 are obtained using MAPLE 18 software which are then substituted into the approximate solution (4)

In order to obtain the third order Exponentially fitted collocation approximate method (EFCAM), We follow the same procedures as state from equations (5) to (15) which eventually lead to

$$CX = p \quad (17)$$

$$C = \begin{bmatrix} c_{11} & c_{12} & c_{13} & \Lambda & c_{1N} & -\tau_1 T_3(x_1) & -\tau_2 T_2(x_1) & -\tau_1 T_3(x_1) \\ c_{21} & c_{22} & c_{23} & \Lambda & c_{2N} & -\tau_1 T_3(x_2) & -\tau_2 T_2(x_2) & -\tau_1 T_3(x_2) \\ c_{31} & c_{32} & c_{33} & \Lambda & c_{3N} & -\tau_1 T_3(x_3) & -\tau_2 T_2(x_3) & -\tau_1 T_3(x_3) \\ M & M & M & \Lambda & M & 0 & 0 & 0 \\ c_{N1} & c_{N2} & c_{N3} & \Lambda & c_{1N} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$X = [Y(0)Y(1), Y(2), Y(3), \dots, Y(N), \tau_1, \tau_2, \tau_3]^T$$

$$p = [g(x_1), g(x_2), g(x_3), \dots, g(x_{N+1}), A_0, A_1, A_2]^T$$

These unknown constants Y(0), Y(1), Λ , Y(N), τ_1 , τ_2 and τ_3 are obtain using MAPLE 18 software which are then substituted into the approximate solution given (4)

III. Numerical Experiment

Example 3. 1. Considered the second order Linear singular initial value problem

$$D^2 y(x) + \frac{4}{x} Dy(x) + \frac{2}{x^2} y(x) = 12 \quad (18)$$

with the initial conditions

$$y(0) = 0, \quad Dy(0) = 0 \quad (19)$$

The exact solution is given as

$$y_e(x) = x^2 \quad (20)$$

Here $D = \frac{d}{dx}$

Example 3.2. Consider the non-homogeneous Lane Emden equation.

$$D^2 y(x) + \frac{8}{x} Dy(x) + xy(x) = x^5 - x^4 + 44x^2 - 30x \quad (21)$$

subject to the initial conditions

$$y(0) = Dy(0) = 0 \quad (22)$$

The exact solution is given as

$$y_e(x) = x^3(-1+x) \quad (23)$$

Example 3.3 Consider the non-homogeneous third order SIVP.

$$D^3 y(x) + \frac{\cos(x)}{\sin(x)} D^2 y(x) = \sin(x) \cos(x) \quad (24)$$

subject to the initial conditions

$$y(0) = 1, Dy(0) = -2, D^2 y(0) = 0 \quad (25)$$

The exact solution is given as

$$y_e(x) = \frac{x^2 + \cos^2 x - 24x + 11}{12} \quad (26)$$

Example 3.4 Consider the non-homogeneous third order SIVP.

$$D^3 y(x) + \frac{3}{x} D^2 y(x) = 12 + 8x + 3x^2 + 6x^3 \quad (27)$$

subject to the initial conditions

$$y(0) = 1, Dy(0) = -2, D^2 y(0) = 0 \quad (28)$$

The exact solution is given as

$$y_e(x) = 1 - 2x + \left(\frac{840x^3 + 224x^4 + 42x^5 + 48x^6}{1680} \right) \quad (29)$$

IV. Computational results

To obtain numerical solutions of the problems considered, Matrices Obtained from examples 1,2,3 and 4 In order to determine the numerical values of the unknown constants $Y(0), Y(1), L, Y(N), \tau_1, \tau_2$ and τ_3 .

We have the following matrices:

Example 3.1 Matrix Obtained for $N = 5$

$$B = \begin{bmatrix} 98.000000 & 42.000000 & 12.000000 & 2.867143 & 0.612248 & 0.122449 & -0.742250 \\ 24.500000 & 21.000000 & 12.000000 & 5.714286 & 2.448980 & 0.979592 & 0.799845 \\ 10.888890 & 14.000000 & 12.000000 & 8.571429 & 5.510204 & 3.306122 & 0.656929 \\ 6.125000 & 10.500000 & 12.000000 & 11.428571 & 9.795918 & 7.836735 & -0.656929 \\ 3.920000 & 8.400000 & 12.000000 & 14.285714 & 15.306122 & 15.306122 & -0.799845 \\ 2.722222 & 7.000000 & 12.000000 & 17.142857 & 22.040816 & 26.448980 & 0.742250 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$X = [Y(0), Y(1), Y(2), Y(3), Y(4), Y(5), \tau_1, \tau_2]^T, p = [12.000, 12.000, 12.000, 12.000, 12.000, 12.000, 0.000, 0.000]^T$$

Using MAPLE 18 software, we obtained the values:

$$Y(0) = 0.000, Y(1) = 0.000, Y(2) = 1.000, Y(3) = 0.000, Y(4) = 0.000, Y(5) = 0.000, \tau_1 = 0.000, \tau_2 = 0.000$$

Substitute the following into equation (4), we have

$$y_5(x) \approx Y(0)x^0 + Y(1)x + Y(2)x^2 + Y(3)x^3 + Y(4)x^4 + Y(5)x^5 + \tau_2 e^x$$

$$y_5(x) = x^2 \quad (30)$$

Example 3.2 Matrix Obtained for $N = 4$

$$B = \begin{bmatrix} 0.166667 & 48.027778 & 18.004629 & 5.000772 & 1.222351 & 0.975309 \\ 0.333330 & 24.111111 & 18.037037 & 10.012345 & 4.893099 & -0.309877 \\ 0.500000 & 16.250000 & 18.125000 & 15.062500 & 11.031250 & -1.000000 \\ 0.666670 & 16.592592 & 18.296270 & 20.197530 & 19.687240 & -0.309877 \\ 0.833333 & 10.294444 & 18.578704 & 25.482260 & 30.957436 & 0.975309 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$X = [Y(0), Y(1), Y(2), Y(3), Y(4), \tau_1, \tau_2]^T$$

$$p = [-3.778421, -5.119342, -4.031250, -0.510288, 5.475180, 0.000, 0.000]^T$$

Using MAPLE 18 software, we obtained:

$$Y(0) = 0.000000896, Y(1) = 0.000000986, Y(2) = 0.00000158, Y(3) = -1.001109,$$

$$Y(4) = 1.0000871, \tau_1 = -0.0002742, \tau_2 = 0.000000896$$

Substitute the above values into equation (4), we have

$$y_4(x) \approx (0.000000896)x^0 + (0.000000986)x + (0.00000158)x^2 + (-1.001109)x^3$$

$$+ (1.0000871)x^4 + (0.000000896)e^x \quad (31)$$

Example 3.3 Matrix Obtained for $N = 7$

$$C = \begin{bmatrix} 0.0000 & 0.0000 & 17.9259 & 11.9753 & 3.9945 & 0.9866 & 0.2056 & 0.0365 & 0.0455 & 0.3672 \\ 0.0000 & 0.0000 & 8.8514 & 11.9000 & 7.9560 & 3.9343 & 1.6407 & 0.5841 & 0.8314 & -1.0000 \\ 0.0000 & 0.0000 & 5.7761 & 11.7761 & 11.8507 & 8.8060 & 5.5141 & 2.9490 & -0.6910 & 0.0547 \\ 0.0000 & 0.0000 & 4.1997 & 11.5996 & 15.6441 & 15.5389 & 12.9930 & 9.2863 & -0.7028 & 1.0000 \\ 0.0000 & 0.0000 & 3.2218 & 11.3696 & 19.2996 & 24.0428 & 25.1797 & 22.5621 & 0.7028 & 0.0547 \\ 0.0000 & 0.0000 & 2.5418 & 11.0836 & 22.7781 & 34.1979 & 43.0868 & 46.5023 & 0.6909 & -1.0000 \\ 0.0000 & 0.0000 & 2.0307 & 10.7383 & 26.0374 & 45.8510 & 67.6080 & 85.5195 & -0.8314 & 0.3672 \\ 0.0000 & 0.0000 & 1.6236 & 10.3297 & 29.0306 & 58.8108 & 99.4843 & 144.6171 & -0.0453 & 1.0000 \\ 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 2.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}$$

$$X = [Y(0), Y(1), Y(2), Y(3), Y(4), Y(5), Y(6), Y(7), \tau_1, \tau_2, \tau_3]^T,$$

$$p = [0.1102, 0.2150, 0.3092, 0.3882, 0.4481, 0.4860, 0.4999, 0.4893, 1.0000, -2.0000, 0.0000]^T$$

Using MAPLE 18 software, we obtained:

$$Y(0) = 0.9996, Y(1) = -2.0038, Y(2) = -0.00019, Y(3) = 0.00064, Y(4) = 0.0262$$

$$Y(5) = 0.0021, Y(6) = -0.0056, Y(7) = 0.0011, \tau_1 = -0.000090, \tau_2 = -0.000087, \tau_3 = 0.000039$$

Substitute the above values into equation (4), we have

$$y_7(x) \approx Y(0)x^0 + Y(1)x + Y(2)x^2 + Y(3)x^3 + Y(4)x^4 + Y(5)x^5 + Y(6)x^6 + Y(7)x^7 + \tau_3 e^x$$

$$y_7(x) \approx (0.9996)x^0 + (-2.0038)x + (-0.00019)x^2 + (0.00064)x^3$$

$$+ (0.0262)x^4 + (0.0221)x^5 + (-0.0056)x^6 + (0.0011)x^7 + (0.00039)e^x \quad (32)$$

Example 3.4 Matrix Obtained for $N = 6$

$$C = \begin{bmatrix} 0.0000 & 0.0000 & 48.0000 & 24.0000 & 7.5000 & 1.8750 & 0.4101 & 0.3672 & -0.7423 \\ 0.0000 & 0.0000 & 24.0000 & 24.0000 & 15.0000 & 7.5000 & 3.2813 & -1.0000 & 0.7998 \\ 0.0000 & 0.0000 & 16.0000 & 24.0000 & 22.5000 & 16.8750 & 11.0742 & 0.0547 & 0.6569 \\ 0.0000 & 0.0000 & 12.0000 & 24.0000 & 30.0000 & 30.0000 & 26.2500 & 1.0000 & -0.6569 \\ 0.0000 & 0.0000 & 9.6000 & 24.0000 & 37.5000 & 46.8750 & 51.2695 & 0.0547 & -0.7998 \\ 0.0000 & 0.0000 & 8.0000 & 24.0000 & 45.0000 & 67.5000 & 88.5938 & -1.0000 & 0.7423 \\ 0.0000 & 0.0000 & 6.8571 & 24.0000 & 52.5000 & 91.8750 & 140.6836 & 0.3672 & 1.0000 \\ 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 2.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}$$

$$X = [Y(0), Y(1), Y(2), Y(3), Y(4), Y(5), Y(6), \tau_1, \tau_2, \tau_3]^T,$$

$$p = [13.0586, 14.2813, 15.7382, 17.5000, 19.6367, 22.2188, 25.3164, 1.0000, -2.0000, 0.0000]^T$$

Using MAPLE 18 software, we obtained:

$$Y(0) = 0.99980, Y(1) = -2.00020, Y(2) = -0.00011, Y(3) = 0.50040, Y(4) = 0.13265$$

$$Y(5) = 0.02550, Y(6) = 0.028400, \tau_1 = 0.000024, \tau_2 = 0.00012, \tau_3 = 0.00023$$

Substitute the above values into equation (4), we have

$$y_6(x) \approx Y(0)x^0 + Y(1)x + Y(2)x^2 + Y(3)x^3 + Y(4)x^4 + Y(5)x^5 + Y(6)x^6 + \tau_3 e^x$$

$$y_6(x) \approx 0.9998 - 2.00020x - 0.00011x^2 + 0.50040x^3 + 0.13265x^4 + 0.02550x^5 + 0.028400x^6 + 0.00023e^x \quad (33)$$

Table1. Examples 3.1& 3.2 Second Order Singular Initial Value Problems (SSVIP).

x	Exact Solution	EFCAM N=5	$E_x = x_e - x_{EFCAM} $	Exact Solution	EFCAM N=4	$E_x = x_e - x_{EFCAM} $
0.0	0.0000000	0.0000000	0.0000000	0.00000000	0.00000000	0.0000000
0.1	0.0100000	0.0100000	0.0000000	-0.0008900	-0.0008999	0.0000001
0.2	0.0400000	0.0400000	0.0000000	-0.0064000	-0.0064003	0.0000031
0.3	0.0900000	0.0900000	0.0000000	-0.0189000	-0.0189013	0.0000013
0.4	0.1600000	0.1600000	0.0000000	-0.0384000	-0.0384032	0.0000032
0.5	0.2500000	0.2500000	0.0000000	-0.0625000	-0.0625058	0.0000058
0.6	0.3600000	0.3600000	0.0000000	-0.0864000	-0.0864089	0.0000089
0.7	0.4900000	0.4900000	0.0000000	-0.1029000	-0.1029122	0.0000122
0.8	0.6400000	0.6400000	0.0000000	-0.1024000	-0.1024141	0.0000149
0.9	0.8100000	0.8100000	0.0000000	-0.0729000	-0.0729159	0.0000159
1.0	1.0000000	1.0000000	0.0000000	0.0000000	0.00000000	0.0000000

Table 2. Example 3.3& 3.4 Third Order Singular Initial Value Problems (TSVIP)

x	Exact Solution	EFCAM N=7	$E_x = x_e - x_{EFCAM} $	Exact Solution	EFCAM N=7	$E_x = x_e - x_{EFCAM} $
0.0	1.0000000	1.0000000	0.0000000	1.0000000	1.00000000	0.0000000
0.1	0.8000028	0.8000033	0.00000057	0.80051361	0.80051399	0.00000037
0.2	0.6000442	0.6000480	0.00000121	0.60422316	0.60422576	0.00000259
0.3	0.4002232	0.4002335	0.00001037	0.41466158	0.41466917	0.00000759
0.4	0.2006961	0.2007195	0.00002341	0.23578636	0.23580211	0.00001575
0.5	0.0016796	0.0017205	0.00004126	0.07206101	0.07208827	0.00002726
0.6	-0.1965684	-0.1965025	0.00006591	-0.07144297	-0.07140061	0.00004235
0.7	-0.3937514	-0.3936523	0.00990400	-0.18892352	-0.18886208	0.00006144
0.8	-0.5895499	-0.5894068	0.00014320	-0.27370484	-0.27361975	0.00008508
0.9	-0.7836334	-0.7834316	0.00020180	-0.31807372	-0.31705980	0.00101400
1.0	-0.9756727	-0.9753925	0.00071970	-0.31309523	-0.31294684	0.00014840

VI. GRAPHY PRESENTATION

Figure 1 Example 1 Second order SIVP
Exact solution vs EFCAM $N=5$ and Yabya and Liu, 2008

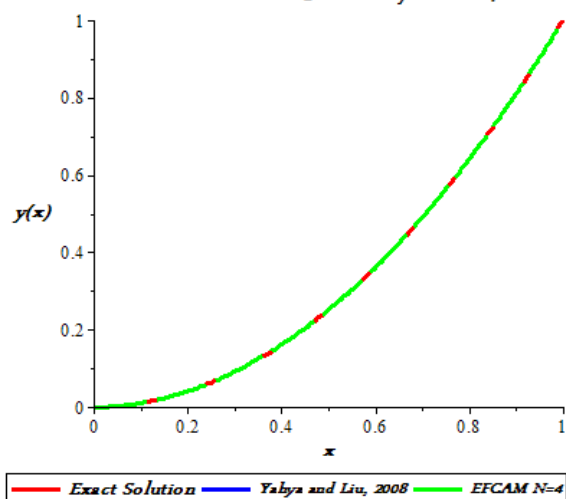


Figure 2 Example 2 Second order SIVP
Exact solution vs EFCAM $N=4$ and Ramos, 2005

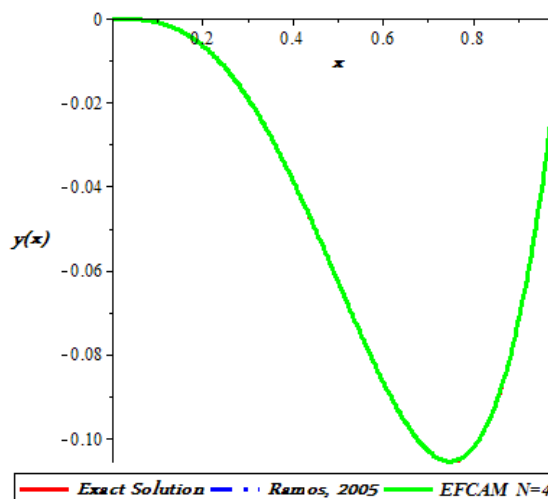


Figure 3 Example 3 Third order SIVP
Exact solution vs EFCAM $N=7$ and Hassan and Zhu, 2008).

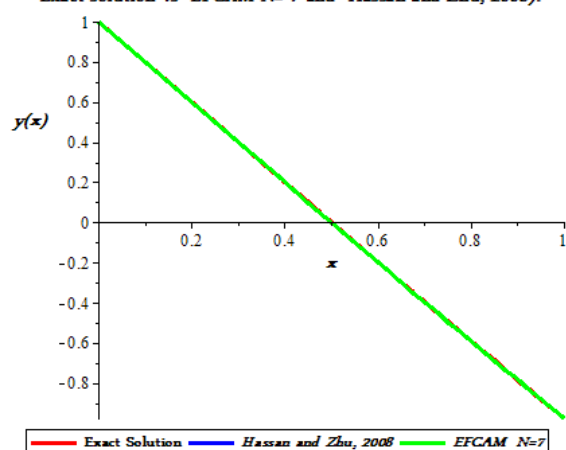
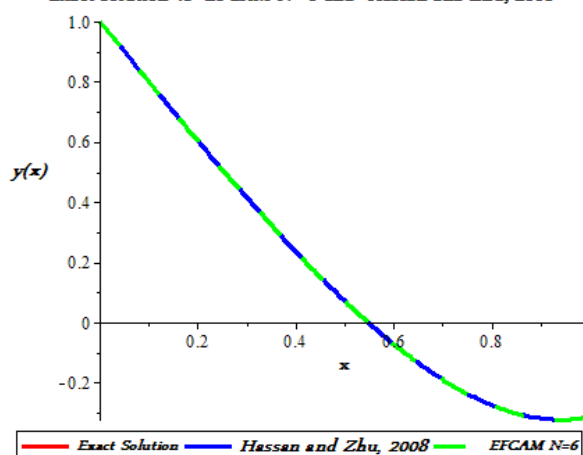


Figure 4 Example 4 Third order SIVP
Exact solution vs EFCAM $N=6$ and Hassan and Zhu, 2008



VII. Conclusion

Exponentially fitted collocation approximation method was proposed and implemented on some examples of second and third order singular initial value problems. The method is straightforward in application and very simple to apply. The comparison of the results obtained by the presented method near singular point show a good agreement with exact solution. Moreover, the result revealed that EFCAM is a powerful mathematical tool for finding the numerical solutions of singular initial value problems in terms of accuracy and efficiency.

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