



A Study on the Performance of Capability Indices using Laplace Distribution

J.Livingston Thiraviya Kumar¹, S.Devaraj Arumainayagam²

¹Department of Economics and Statistics, Government of Tamil Nadu, India.

²Department of Statistics, Govt. Arts College,
Coimbatore, Tamil Nadu, India.

Abstract: Quality of the product is enhanced by controlling the process using Statistical process control techniques. Process capability indices are used to assess the capability of the process and C_p , C_{pk} , C_{pm} and C_{pmk} are used when the process is normal and in statistically control. In practical, most of the processes are non-normal. The Laplace distribution is also a symmetric distribution and is widely used in industries. In this paper, performance of the four basic process capability indices were tested using the Laplace distributed data.

Keywords: Capability indices, Laplace Distribution, Process Control.

I. Introduction

Process capability indices were developed to test whether the process is capable. The four basic process capability indices C_p , C_{pk} , C_{pm} and C_{pmk} are used for measuring the capability of a process. The index C_p measures the variability of the process only. To deal with violations of the centering assumptions, the index C_{pk} was developed. Chan *et al.* (1988) introduced so called Taguchi capability index C_{pm} that is measurable & directly related to the quadratic loss of the measured feature. Pearn *et al.* [8] proposed the capability index C_{pmk} . The capability index C_p measures the variability alone, where

$$C_p = \frac{USL - LSL}{6\sigma} \quad (1)$$

Kane [13] discussed the index C_{pk} , which attempts to measure the variability and shift in process mean simultaneously and it is defined as

$$C_{pk} = \min \left\{ \frac{USL - M}{3\sigma}, \frac{M - LSL}{3\sigma} \right\} \quad (2)$$

Chan *et al.* [1] introduced so called Taguchi capability index C_{pm} considering specification range, process variation and variation of mean from the target, given by

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}} \quad (3)$$

Pearn *et al.* [8] proposed the index C_{pmk} by modifying the numerator and denominator of the index C_{pm} as

$$C_{pmk} = \frac{\min(USL - \mu, \mu - LSL)}{3\sqrt{\sigma^2 + (\mu - T)^2}} \quad (4)$$

For the process with two-sided specification limit and if the process is centered, then all the process capability indices coincide each other, i.e.,

$$C_p = C_{pk} = C_{pm} = C_{pmk} \quad (5)$$

Steven E.Somerville & Douglas C.Montgomery (1996) examined the error in calculation of C_p or C_{pk} for a non-normal distribution and making inference about process fallout with normality assumption and also stated that the Process being analyzed must be in statistical control, otherwise the indices are meaningless. Many attempts were made in developing the process capability index for non-normal process. Johnson (1949) proposed a system of distributions based on moment method called Johnson transformation. Box and Cox (1964) used transformation method for non-normal data by presenting family of power transformations. Clements (1989)

proposed a capability index based on percentiles. Ahmad *et al.* (2007) compared the Cumulative Distribution Function (CDF) method with the latest proposed process capability evaluation methods Burr percentile method and Clements percentile method when the underlying distribution is non-normal and concluded that the CDF method is less sensitive to distribution of process. Hosseinifard *et al.* (2009) compared the efficiency of root transformation technique with other techniques Box-Cox, Burr and Clements method under non-normal assumption. The root transformation technique provides a best estimate of process capability for non-normal process. In this paper we will review and compare the Clements method and robust estimator (median absolute deviation) based process capability index by conducting a simulation study using Gamma distribution.

II. Laplace distribution

The Laplace distribution is named after Pierre-Simon Laplace (1749-1827), who obtained the likelihood of the Laplace distribution is maximized when the location parameter is set to be the median. The Laplace distribution is also known as the law of the difference between two exponential random variables. Consequently, it is also known as double-exponential distribution, as well as the two tailed distribution. It is also known as the bilateral exponential law. The classical Laplace probability distribution is denoted by $L(\theta, \phi)$ and is defined by the probability density function:

$$f(x; \theta, \phi) = \frac{1}{2\phi} \exp\left(-\frac{|x - \theta|}{\phi}\right), \quad -\infty < x < \infty \quad (6)$$

where $\theta \in (-\infty, \infty)$ and $\phi > 0$ are location and scale parameters respectively. It is a symmetric distribution whose tails fall off less sharply than the Gaussian distribution but faster than the Cauchy distribution. The basic properties of Laplace distribution are tabulated in Table-1.

Table-1

Distribution	E(X)	V(X)	Sk(X)	Kur(X)
Normal	μ	σ^2	0	3
Laplace	θ	$2\phi^2$	0	6

III. Performance study of Laplace distribution:

For the purpose of comparing the performance of Laplace distribution with Normal distribution on the calculation of capability index, location values of 0, 1, 1.5, 2 and 2.5 along with a fixed scale value of 1 were considered.

Table-2

Laplace Distribution	mean	0	1	1.5	2	2.5
	scale	1	1	1	1	1
	variance	2	2	2	2	2
	SD	1.41421356	1.41421356	1.41421356	1.41421356	1.41421356
	$\mu + 2\sigma$	2.82842712	3.82842712	4.32842712	4.82842712	5.32842712
Laplace	LCDF	0.970447	0.970447	0.970447	0.970447	0.970447
	UCDF	0.029553	0.029553	0.029553	0.029553	0.029553
	PPM	29553	29553	29553	29553	29553
Normal	LCDF	0.97724987	0.97724987	0.97724987	0.97724987	0.97724987
	UCDF	0.02275013	0.02275013	0.02275013	0.02275013	0.02275013
	PPM	22750.13	22750.13	22750.13	22750.13	22750.13
Absolute Error		6802.87	6802.87	6802.87	6802.87	6802.87
Relative Error		23.0192197	23.0192197	23.0192197	23.0192197	23.0192197

Table-2 summaries the probability of confirming area (LCDF), probability of non-confirming area (UCDF), PPM, Absolute and relative error of both the Normal and Laplace distribution for the specified values of location and scale parameter. From the table-2 it is clear that at the spread of $\mu + 2\sigma$, the parts per million defective of Normal distribution is 22750.13 and the parts per million defective of Laplace distribution is 29553. The relative error in all the matches is 23.0192197.

Table-3

Laplace Distribution	mean	0
	scale	1
	variance	2
	SD	1.414214
$\mu+3\sigma$		4.242641
Laplace	LCDF	0.992815
	UCDF	0.007185
	PPM	7185
Normal	LCDF	0.99865
	UCDF	0.00135
	PPM	1349.87
Absolute Error		5835.13
Relative Error		81.21267

Table-4

Laplace Distribution	mean	0
	scale	1
	variance	2
	SD	1.414214
$\mu+4\sigma$		5.656854
Laplace	LCDF	0.998253
	UCDF	0.001747
	PPM	1747
Normal	LCDF	0.999968
	UCDF	0.0000317
	PPM	31.67
Absolute Error		1715.33
Relative Error		98.18718

Table-5

Laplace Distribution	mean	0
	scale	1
	variance	2
	SD	1.414214
$\mu+5\sigma$		7.071068
Laplace	LCDF	0.999575
	UCDF	0.000425
	PPM	425
Normal	LCDF	1
	UCDF	0.0000003
	PPM	0.29
Absolute Error		424.71
Relative Error		99.93176

Table-6

Laplace Distribution	mean	0
	scale	1
	variance	2
	SD	1.414214
$\mu+6\sigma$		8.485281
Laplace	LCDF	0.999897
	UCDF	0.000103
	PPM	103
Normal	LCDF	1
	UCDF	0
	PPM	0
Absolute Error		103
Relative Error		100

Table-3, Table-4, Table-5 and Table-6 summaries the probability of confirming area (LCDF), probability of non-confirming area (UCDF), PPM, Absolute and relative error of both the Normal and Laplace distribution with spread of $\mu + 3\sigma$, $\mu + 4\sigma$, $\mu + 5\sigma$, $\mu + 6\sigma$ respectively. For the process spread of $\mu + 3\sigma$, the parts per million defective of Normal distribution is 1349.87 and the parts per million defective of Laplace distribution is 7185. The relative error in all the matches is 81.21267. For the process spread of $\mu + 4\sigma$, the parts per million defective of Normal distribution is 31.67 and the parts per million defective of Laplace distribution is 1747. The relative error in all the matches is 98.18718. For the process spread of $\mu + 5\sigma$, the parts per million defective of Normal distribution is 0.29 and the parts per million defective of Laplace distribution is 425. The relative error in all the matches is 99.93176. For the process spread of $\mu + 6\sigma$, the parts per million defective of Normal distribution is 0 and the parts per million defective of Laplace distribution is 103. The relative error in all the matches is 100.

IV. Conclusion

The errors occur in calculating the capability index and PPM for a non-normal distribution and making inference with the normality assumption are quite clear. The error associated with making the normality assumption if the process really follows Laplace distribution is quite severe. To deal with the non normal distribution in calculating the capability index, a better alternative should be identified.

References

- [1]. Box GEP Cox DR (1964). "An analysis of transformation", J Roy Stat Soc B, 26:211-243
- [2]. Chan L.K, Chen S.W & Spring F.A (1988), A New Measure of Process Capability C_{pm} , J. Qual. Technol, 20(3), 162-175.
- [3]. Clements (1989). "Process capability calculations for non-normal distributions" Quality Progress 22: 95-100.
- [4]. Douglas C. Montgomery, Introduction to Statistical Quality Control, 4th Edition, Wiley India.
- [5]. Johnson NL (1949). "System of frequency curves generated by methods of translation", Biometrika, 36, 149-176.
- [6]. Kayode S. Adekeye (2013). "Process capability indices based on median absolute deviation", International Journal of Applied Science and Technology, Vol 3 No. 4.

- [7]. Somerville S, Montgomery D (1996). "Process capability indices and non-normal distributions", Quality Engineering, 19(2): 305-316.
- [8]. S.Ahmad, M.Abdollahian, P.Zeepongsekul and B.Abbasi (2007). "Performance analysis for skewed data", Ubiquitous Computing and Communication Journal.
- [9]. S.Z.Hosseinfard, BabakAbbasi, S.Ahmad and M.Abdollahian (2009). "A transformation technique to estimate the process capability index for non-normal processes" Int J AdvManufTechnol, 40, 512-517.
- [10]. Victor E. Kane (1986). "Process Capability Indices", Journal of Quality Technology, Vol. 18, No.1.
- [11]. S.Z.Hosseinfard, Babak Abbasi, S.Ahmad and M.Abdollahian (2009). "A transformation technique to estimate the process capability index for non-normal processes" Int J Adv Manuf Technol, 40, 512-517.
- [12]. Victor E. Kane (1986). "Process Capability Indices", Journal of Quality Technology, Vol. 18, No.1.