



Right Derivation of Ternary Γ – Semiring

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Abstract: In this paper we are introduce the notion of derivation, kerd, k - TF -ideal, $d\Gamma$ -ideal. It is proved that let T be a commutative ternary Γ -semiring and idempotent with respect to addition. Then for a fixed element t in T , the mapping $d_t: T \rightarrow T$ is given by $d_t(x) = xat\beta t$ for all $x \in T$ and $a, \beta \in \Gamma$ is a right derivation of T . Further it is proved that let d be a right derivation of a ternary Γ -semiring T , then (i) $d(0) = 0$ (ii) $d(xay\beta z) \leq d(x+y+z)$ (iii) If $d^2(x) = d(d(x)) = d(x)$ then $d(xax\beta d(x)) = d(x)$, $a, \beta \in \Gamma$ and for all $x \in T$. It is proved that let T be an additive commutative cancellative prime ternary Γ -semiring and d be a non zero right derivation of T . Then T is a commutative ternary Γ -semiring

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I. Introduction

A Γ -semiring is an algebraic structure with two binary operations addition and multiplication. As well as ternary Γ -semiring is also a algebraic structure with operations addition and ternary multiplication. Ternary Γ -semirings introduced by D. MadhusudhanaRao and SajaniLavanyain the year 2015 and they developed the literature of ternary Γ -semiring. In this paper we are mainly discussed about right derivations of ternary Γ -semiring.

II. Preliminaries :

Definition 2.1 : The non empty sets T and Γ together with a binary operation called addition and ternary multiplication denoted by juxtaposition is said to be a **ternary Γ - semiring** if T and Γ be two additive commutative semigroups satisfying the following conditions.

$$(i) \quad [[x_1ax_2\beta x_3]\gamma x_4\delta x_5] = [x_1a[x_2\beta x_3\gamma x_4]\delta x_5] = [x_1ax_2\beta[x_3\gamma x_4\delta x_5]]$$

$$(ii) \quad [(x_1+x_2)ax_3\beta x_4] = [x_1ax_3\beta x_4] + [x_2ax_3\beta x_4]$$

$$(iii) \quad [x_1a(x_2+x_3)\beta x_4] = [x_1ax_2\beta x_4] + [x_1ax_3\beta x_4]$$

$$(iv) \quad [x_1ax_2\beta(x_3+x_4)] = [x_1ax_2\beta x_3] + [x_1ax_2\beta x_4] \quad \text{for all } x_1, x_2, x_3, x_4, x_5 \in T \text{ and } a, \beta, \gamma, \delta \in \Gamma.$$

Definition 2.2 : An additive subsemigroup A of T is said to be a **TF -ideal of T** if $xay\beta a \in A$, $xa\alpha\beta y \in A$ and $a\alpha x\beta y \in A$.

Definition 2.3: A proper TF -ideal P of a ternary Γ -semiring T is said to be a **prime TF -ideal** of T if for any three TF -ideals A, B, C of T , $A\Gamma B\Gamma C \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$ or $C \subseteq P$.

III. Right Derivation of Ternary Γ -Semiring:

Definition 3.1: Let T be a ternary Γ -semiring. A function $d: T \rightarrow T$ is said to be a **right derivation of T** if (i) $d(x+y) = d(x) + d(y)$

$$(ii) \quad d(xay\beta z) = d(x)\alpha y\beta z + d(y)\alpha z\beta x + d(z)\alpha x\beta y \text{ for all } x, y, z \in T \text{ and } \alpha, \beta \in \Gamma.$$

Example 3.2 : Let $T = \left\{ \begin{pmatrix} a & 0 \\ b & a \end{pmatrix} \mid a, b \in Q \right\}$ where Q is the set of all rational numbers and $\Gamma = \left\{ \begin{pmatrix} a & 0 \\ b & a \end{pmatrix} \mid a, b \in N \right\}$ where N is the set of all natural numbers. Then T and Γ are additive abelian semigroups with respect to the usual matrix addition of 2×2 matrices and ternary operation is defined as $T \times \Gamma \times T \times \Gamma \times T$ to T by $(x, \alpha, y, \beta, z) \rightarrow xay\beta z$ using the usual matrix multiplication for all $x, y, z \in T$ and $\alpha, \beta \in \Gamma$. Then T is a ternary Γ -semiring.

Define a mapping $d: T \rightarrow T$ by $d\left(\begin{pmatrix} a & 0 \\ b & a \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 \\ 2b & o \end{pmatrix}$. Then d is a right derivation but not a derivation of T .

Theorem 3.3 : Let T be a commutative ternary Γ - semiring and $(T, +)$ be idempotent. Then for a fixed element $t \in T$, the mapping $d_t : T \rightarrow T$ is given by $d_t(x) = xat\beta t$ for all $x \in T$ and $\alpha, \beta \in \Gamma$ is a right derivation of T .

Proof: Let $t \in T$. Suppose $x, y \in T$ and $\alpha, \beta \in \Gamma$.

$$d_t(x + y) = (x + y)at\beta t = xat\beta t + yat\beta t = d_t(x) + d_t(y) \text{ and}$$

$$d_t(x\alpha y\beta z) = (x\alpha y\beta z)at\beta t = (xat\beta t)\alpha y\beta z + (yat\beta t)\alpha z\beta x + (zat\beta t)\alpha x\beta y \\ = d_t(x)\alpha y\beta z + d_t(y)\alpha z\beta x + d_t(z)\alpha x\beta y. \text{ Hence } d_t \text{ is a right derivation of } T.$$

Theorem 3.4 : Let d be a right derivation of a ternary Γ - semiring T then $d(0) = 0$.

Proof: Let $\alpha, \beta \in \Gamma$. We have $d(0) = d(0\alpha 0\beta 0) = d(0)\alpha 0\beta 0 + d(0)\alpha 0\beta 0 + d(0)\alpha 0\beta 0 \\ = 0 + 0 + 0 = 0$. Therefore $d(0) = 0$.

Theorem 3.5 : Let d be a right derivation of the idempotent ternary Γ - semiring T . Then $d(x) = x$ for all $x \in T$.

Proof: Let $x \in T$. Then there exist $\alpha, \beta \in \Gamma$ such that $x = x\alpha x\beta x$ implies $d(x) = d(x\alpha x\beta x) = d(x)\alpha x\beta x + d(x)\alpha x\beta x + d(x)\alpha x\beta x = d(x)\alpha x\beta x$. Therefore $d(x) = x$.

Hence the theorem is proved.

Theorem 3.6 : Let d be a right derivative ternary Γ - semiring of T . Then $d(x\alpha y\beta z) \leq d(x+y+z)$.

Proof: Let $x, y, z \in T$ and $\alpha, \beta \in \Gamma$. We have $d(x)\alpha y\beta z \leq d(x)$, $d(y)\alpha z\beta x \leq d(y)$, $d(z)\alpha x\beta y \leq d(z)$. Therefore $d(x\alpha y\beta z) = d(x)\alpha y\beta z + d(y)\alpha z\beta x + d(z)\alpha x\beta y \leq d(x) + d(y) + d(z) = d(x + y + z)$.

Theorem 3.7 : Let d be a right derivative of the idempotent ternary Γ - semiring T . If

$$d^2(x) = d(d(x)) = d(x) \text{ then } d(x\alpha x\beta d(x)) = d(x), \alpha, \beta \in \Gamma \text{ and for all } x \in T.$$

Proof: let $x \in T$. Then there exist $\alpha, \beta \in \Gamma$ such that $x = x\alpha x\beta x$.

$$d(x\alpha x\beta d(x)) = d(x)\alpha x\beta d(x) + d(x)\alpha d(x)\beta x + d(d(x))\alpha x\beta x = d(x) + d(x) + d(x) = d(x). \text{ Therefore } d(x\alpha x\beta d(x)) = d(x).$$

Theorem 3.8 : Let T be a commutative ternary Γ - semiring and d_1, d_2 be right derivatives of T . Define $d_1 d_2(x) = d_1(d_2(x))$ for all $x \in T$. If $d_1 d_2 = 0$ then $d_2 d_1$ is a right derivation of T .

Proof: Supposed $d_1 d_2 = 0$ for $x, y, z \in T$ and $\alpha, \beta \in \Gamma$. Then $d_1 d_2(x\alpha y\beta z) = 0$

$$\Rightarrow d_1(d_2(x\alpha y\beta z)) = 0 \Rightarrow d_1(d_2(x)\alpha y\beta z + d_2(y)\alpha z\beta x + d_2(z)\alpha x\beta y) = 0$$

$$\Rightarrow d_1 d_2(x)\alpha y\beta z + d_1(y)\alpha z\beta d_2(x) + d_1(z)\alpha d_2(x)\beta y + d_1 d_2(y)\alpha z\beta x + d_1(z)\alpha x\beta d_2(y) + d_1(x)\alpha d_2(y)\beta z + d_1 d_2(z)\alpha x\beta y + d_1(x)\alpha y\beta d_2(z) + d_1(y)\alpha d_2(z)\beta x = 0$$

$$\text{and } d_2 d_1(x\alpha y\beta z) = d_2(d_1(x\alpha y\beta z)) = d_2(d_1(x)\alpha y\beta z + d_1(y)\alpha z\beta x + d_1(z)\alpha x\beta y) = d_2 d_1(x)\alpha y\beta z + d_2(y)\alpha z\beta d_1(x) + d_2(z)\alpha d_1(x)\beta y + d_2 d_1(y)\alpha z\beta x + d_2(z)\alpha x\beta d_1(y) + d_2(x)\alpha d_1(y)\beta z + d_2 d_1(z)\alpha x\beta y + d_2(x)\alpha y\beta d_1(z) + d_2(y)\alpha d_1(z)\beta x = d_2 d_1(x)\alpha y\beta z + d_2 d_1(y)\alpha z\beta x + d_2 d_1(z)\alpha x\beta y.$$

obviously, $d_2 d_1(x+y) = d_2 d_1(x) + d_2 d_1(y)$. Hence $d_2 d_1$ is a right derivation of commutative ternary Γ - semiring T .

Definition 3.9 : Let d be a right derivation of a ternary Γ - semiring T . Then $\ker d = \{x \in T / d(x) = 0\}$

Definition 3.10 : A Γ - ideal A of T is called a k - Γ - ideal provided $a + b \in A$, $a \in T$, $b \in A$ implies $a \in A$.

Theorem 3.11 : Let d be a right derivation of a ternary Γ - semiring T . Then $\ker d$ is a k - Γ - ideal of a ternary Γ - semiring T .

Proof: Suppose $x, y, z \in \ker d$ and $\alpha, \beta \in \Gamma$. Then $d(x) = 0$, $d(y) = 0$ and $d(z) = 0$ implies $d(x\alpha y\beta z) = 0$ and $d(x + y) = 0$. Therefore $x + y \in \ker d$ and $x\alpha y\beta z \in \ker d$. Hence $\ker d$ is a ternary Γ - subsemiring of T . Let $x \in T$, $x + y \in \ker d$ and $y \in \ker d$. Then $d(x + y) = d(y) = 0 \Rightarrow d(x) + d(y) = 0 \Rightarrow d(x) = 0$. Therefore $x \in \ker d$. Hence $\ker d$ is a k - Γ ideal of T .

Theorem 3.12: Let T be a ternary Γ - semiring. If d_1, d_2 be right derivatives of T and

$$(d_1 + d_2)(x) = d_1(x) + d_2(x) \text{ for all } x \in T, \text{ then } (d_1 + d_2) \text{ is also a right derivation of } T.$$

Theorem 3.13: Let d be a right derivation of additively cancellative commutative idempotent ternary Γ - semiring T . Define a set $\text{Fix}_d(T) = \{x \in T / d(x) = x\}$ is a k - Γ -ideal of T .

Proof: Suppose $x, y, z \in \text{Fix}_d(T)$ and $\alpha, \beta \in \Gamma$. Then $d(x) = x$, $d(y) = y$ and $d(z) = z$ implies

$$d(x + y) = d(x) + d(y) = x + y. \text{ Therefore } x + y \in \text{Fix}_d(T).$$

$$d(x\alpha y\beta z) = d(x)\alpha y\beta z + d(y)\alpha z\beta x + d(z)\alpha x\beta y = x\alpha y\beta z + y\alpha z\beta x + z\alpha x\beta y = x\alpha y\beta z + x\alpha y\beta z + x\alpha y\beta z = x\alpha y\beta z. \text{ Therefore } \text{Fix}_d(T) \text{ is a ternary } \Gamma\text{-subsemiring of } T. \text{ Suppose } x + y \in \text{Fix}_d(T) \text{ and } y \in \text{Fix}_d(T). \text{ Then } d(x + y) = x + y \text{ and } d(y) = y. \text{ That implies } d(x) + d(y) = x + y. \text{ Therefore } d(x) = x. \text{ Hence } \text{Fix}_d(T) \text{ is a } k\text{-}\Gamma\text{- ideal of } T.$$

Theorem 3.14: Let d be a right derivation of a ternary Γ - semiring T and A, B be two Γ -ideals of T and $A \subseteq B$, then $d(A) \subseteq d(B)$.

Proof: Suppose A, B be any two Γ - ideals of a ternary Γ - semiring T and $A \subseteq B$. Let $x \in d(A)$. Then there exist $y \in A$ such that $x = d(y)$. Since $y \in A$ and $A \subseteq B \Rightarrow y \in B$. Therefore $x \in d(B)$. Hence $d(A) \subseteq d(B)$.

Definition 3.15 : Let T be a ternary Γ - semiring and d be a right derivation of T . A Γ -ideal A of ternary Γ - semiring T is said to be a $d\Gamma$ - ideal if $d(A) = A$.

Example 3.16 : The zero Γ - ideal $\{0\}$ is a $d\Gamma$ - ideal of ternary Γ - semiring. Since $d(0) = 0$. If d is onto derivation, then $d(T) = T$. Hence T is a $d\Gamma$ - ideal.

Theorem 3.17: Let T be a ternary Γ - semiring and A, B be $d\Gamma$ - ideals of T then $A + B$ is also a $d\Gamma$ - ideal of T .

Proof: A and B are subsets of $A + B$. Therefore $A = d(A) \subseteq d(A + B)$, $B = d(B) \subseteq d(A + B)$ and $(A + B) \subseteq d(A + B) \subseteq d(A) + d(B) = A + B$. Therefore $d(A + B) = A + B$. Hence $A + B$ is also a $d\Gamma$ - ideal of T .

Theorem 3.18: Let T be a ternary Γ - semiring and d be a right derivation then for an element $a \in T$, $d((aa)^n a) = nd(a)a(aa)^{n-1}a$ for all $a \in \Gamma$ and for some odd natural number n .

Proof: We prove this result by mathematical induction. Let d be a right derivation of T , $a \in \Gamma$ and $a \in T$. Suppose $n = 1$

$$d(aaaaa) = d(a)aaaa + d(a)aaaa + d(a)aaaa = 3d(a)aaaa.$$

Suppose that the statement is true for $n + 1$, for all $n \in \mathbb{N}$.

That is $d((aa)^{n+1}a) = (n + 1)d(a)a(aa)^na$. Then $d((aa)^{n+2}a) = d(aa(aa)^{n+1}a) = d(a)a(aa)^{n+1}a + d((aa)^{n+1}aaaa) + d(a)aaa(aa)^na = (n + 2)d(a)a(aa)^{n+1}a$.

Hence by mathematical induction, the theorem is true for all odd $n \in \mathbb{N}$.

Definition 3.19: A ternary Γ -semiring T is said to be a *prime ternary Γ -semiring* if $a\Gamma T\Gamma b\Gamma T\Gamma c = 0$ then $a = 0$ or $b = 0$ or $c = 0$ for all $a, b, c \in T$.

Theorem 3.20: let T be an additive commutative cancellative prime ternary Γ -semiring and d be a non zero right derivation of T . Then T is a commutative ternary Γ -semiring.

IV. Conclusion

In this paper we introduced the notion of a right derivation of a ternary Γ -semiring and we studied some of the properties of right derivations of ternary Γ -semiring.

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References:

- [1] P.J. Allen, A fundamental theorem of homomorphism for semirings, Proc. Amer.MathSoc. 21 (1969) 412 - 416. doi:10.1090/S0002-9939-1969-0237575-4
- [2] M. Bresar and J. Vukman, On the left derivation and related mappings, Proc. Amer.MathSoc. 10 (1990) 7 - 16. doi:10.2307/2048234
- [3] T.K. Dutta and S. Kar, On regular ternary semirings, in: Advances in Algebra, Proceedings of the ICM Satellite Conference in Algebra and Related Topics (Ed(s)), (World Scientific, 2003) 343 - 355.
- [4] H. Lehmer, A ternary analogue of abelian groups, Amer. J. Math. 59 (1932) 329 - 338. doi:10.2307/2370997
- [5] W.G. Lister, Ternary rings, Tran. Amer. Math. Soc. 154 (1971) 37 - 55. doi:10.2307/1995425
- [6] K.H. Kim, On right derivation of incline algebras, J. Chungcheong Math. Soc. 26(2013) 683 - 690. doi:10.14403/jcms.2013.26.4.683
- [7] K.H. Kim, On generalized right derivation of incline algebras, Gulf J. Math. 3 (2015) 36 - 46.
- [8] D. Madhusudhana Rao, Theory of Γ -ideal in Γ -semigroups - Thesis, ANU (2011).
- [9] D. Madhusudhana Rao, A. Anjaneyulu and A. Gangadhara Rao, Prime Γ - Radicals in Γ -Semigroups, International e-journal of Mathematics and Engineering, 138 (2011), 1250-1259.
- [10] D. Madhusudhana Rao and G. Srinivasa Rao, Prime Radicals and Completely Prime Radicals in Ternary Semirings, International Journal of Basic Science and Applied Computing (IJBSAC) Volume-1, Issue 4, February 2015.
- [11] D. Madhusudhana Rao and G. Srinivasa Rao, Structure of P-Prime and P-Semiprime Ideals in Ternary Semirings- International Journal of Innovation in Science and Mathematics (IJISM) Vol 3, Issue 2, February 2015, pp 24-31.
- [12] M. Murali Krishna Rao, Γ -semirings-I, Southeast Asian Bull. Math. 19 (1995) 49-54.
- [13] M. Murali Krishna Rao, Γ -semirings-II, Southeast Asian Bull. of Math. 21 (1997) 281 - 287.
- [14] M. Murali Krishna Rao and B. Venkateswarlu, Regular Γ -semirings and field Γ -semirings, Novi Sad J. Math. 45(2015) 155 - 171.
- [15] V. Neumann, On regular rings, Proc. Nat. Acad. Sci. 22 (1936) 707 - 713. doi:org/10.1073/pnas.22.12.707
- [16] N. Nobusawa, On a generalization of the ring theory, Osaka. J. Math. 1(1964) 81-89. ir.library.osaka-u.ac.jp/dspace/bitstream/11094/12354/1/ojm 01-01-08.pdf
- [17] E.C. Posner, Derivations in prime rings, Proc. Amer. Math. Soc. 8 (1957) 1093-1100. doi:10.2307/2032686
- [18] M. Sajani Lavanya, D. Madhusudhana Rao, and V. Syam Julius Rajendra, On Lateral Ternary Γ -Ideals of Ternary Γ -Semirings- American International Journal of Research in Science, Technology, Engineering & Mathematics (AIJRSTEM), 12(1), September- November, 2015, pp: 11-14.
- [19] M. Sajani Lavanya, D. Madhusudhana Rao, and V. Syam Julius Rajendra, Prime Bi-ternary Γ -Ideals in Ternary Γ -Semirings- British Journal of Research, Volume 2, Issue 6, November-December, 2015, pp: 156-166.
- [20] Y. Sarala, A. Anjaneyulu and D. Madhusudhana Rao, Prime radicals in ternary semigroups, International Organization Scientific Research Journal of Mathematics (IOSR- JM) ISSN: 2278-5728 Volume 4, Issue 5 (Jan.-Feb. 2013), pp 43-53.
- [21] M.K. Sen, On Γ -semigroup, in: Proc. of International Conference of algebra and its application, Decker Publication (Ed(s)), (New York, 1981) 301-308.
- [22] K. Sri Latha, D. Madhusudhana Rao, G. Shobha Latha, Fully Idempotent Ternary Γ semirings - International Journal of Development Research (IJDR) ISSN: 2230-9926, Volume 07, Issue 09, September, 2017, pp: 15110-15114.
- [23] K. Sri Latha, G. Shobha Latha, D. Madhusudhana Rao, On Weakly Regular Ternary Γ semirings - Asian Journal of Science and Technology, ISSN : 0976-3376, Volume 08, Issue 10, October, 2017, pp : 6046-6049.
- [24] K. Sri Latha, D. Madhusudhana Rao, G. Shobha Latha, Prime Γ - Radical and Radical $\Gamma\Gamma$ -ideal in Ternary Γ - semirings - International Journal on Future Revolution in Computer Science and Communication Engineering, (IJFRCSCE) ISSN: 2454-4248, Volume 03, Issue 10, October, 2017, pp: 198-201.

- [25] K.SriLatha, G.ShobhaLatha, D.Madhusudhana Rao, Generalized Semi $T\Gamma$ -ideals in Ternary Γ -semirings- Accepted in International Journal of Advanced Research Trends in Engineering and Technology (IJARTET)
- [26] H.S. Vandiver, Note on a simple type of algebra in which the cancellation law of addition does not hold, Bull. Amer. Math. 40(1934) 914-921. doi:10.1090/s0002-9904-1934-06003-8