



The Application of Expanded Rank-Size Model in Tamilnadu City Size Distribution

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Abstract: The structure of city size in TamiNadu state is analysed for the census year 1991 to 2011 by using expanded rank-size model. Earlier studies related to rank size model in Tamilnadu have pointed out rank-size relationships. In this study changes in city- size distributions over time studies a note of differences in slope and intercept at the different points in time and results are inferred to indicate growth or decline in entire system of cities and the relative rates of growth or decline of large and small areas. In this paper, the expanded rank-size model is applied in all cities of Tamilnadu and region wise.

I. Introduction

The general aim of this paper is to discuss expanded rank-size distribution to evaluate city size that under going changes in population. The rank-size distribution model is applied by Zipf to many of the problems that stick with in empirical rank-size research. Berr and Garrison and Beckman studied rank-size rule. Malecki, Stephens and Holly, Strickland and Aiken's have said that rank-size model is not sufficient in explaining statistically significant changes in the coefficients and the particular dynamics of city system change. In city system direction of slope and intercept value in rank size rule indicates growth or decline within urban system. Rank size model doesn't explain the slope and intercept value that can fluctuate because larger cities have the most rapid growth with constant growth in smaller cities or intercept value can vary while slope remains constant because of a uniform rate of growth or decline in the city system. Cassetti improved the expanded rank-size model to examine strictly the changes in coefficients over time. This model reformulated the rank-size model to include a time parameter to assess change in rank-size relations. An attempt has been made to study the expanded rank-size model. Empirical analysis about the expanded rank-size model has been made using the population of cities of Tamil Nadu state in 1991, 2001 and 2011 census.

II. Objective:

To examine the changes in coefficient over time in Rank-size model of cities in Tamil Nadu state and it is classified into five regions and expanded rank size rule is applied for the census years 1991-2011.

THE STANDARD LOGARITHMIC RANK SIZE AND EXPANDED RANKSIZE MODEL OF CITIES OVER TIME

The rank-size model is expressed as

$$\text{Log } P_i = \text{Log } \alpha - \beta \text{Log } r_i \quad (1)$$

Where, P_i = Population of city i , r_i = rank of center i ;

α = a constant approximately equal to the population of the center of rank i ;

β = the slope coefficient.

The plot of rank against size on double logarithmic should give a straight line with a slope of $-\beta$. The slope coefficient expresses the degree of concentration or dispersion of population within the urban system. The standard logarithmic rank-size function is fit to the data for the census year data 1991, 2001 and 2011.

Table 1: Regression Equations for Tamil Nadu City for Census of 1991-2011

Year	Log α	β	R	R ²
1991	Log $P_i=14.446$ (101.429)*	-0.923logr (16.322)*	0.958	0.917
2001	Log $P_i=14.821$ (149.817)*	-1.004 logr (25.324)*	0.982	0.964
2011	Log $P_i=14.992$ (187.971)*	-1.014 logr (-34.118)*	0.987	0.975

*Significant at 5% level of significance. t-values given in bracket.

The above table1 shows the results of regression and it has an increasing intercept values, for census years, from 14.446 in 1991, 14.821 in 2001 and 14.992 in 2011. In addition to the slope of rank-size distribution has increased in all three census years. From this analysis it is impossible to tell apart whether the change because of increasing population in larger cities faster than smaller ones or to an increase in populations of larger cities alone or decrease in the population of smaller cities.

Cassetti developed a time expanded model which is examined for changes in rank-size coefficients over time. The method expresses the parameters of the logarithmic form of the rank-size function to account for changes in the rank-size structure over time. The values for log α and β remain constant only for given point in time. Since log α and β vary temporally, they can be expressed as functions of time.

The expanded linear function for the intercept becomes

$$\text{Log } \alpha = \text{Log } \alpha_0 + \text{Log } \alpha_1 t \quad (2)$$

$$\beta = \beta_0 + \beta_1 t \quad (3)$$

Where

α_0 = constant at the initial point in time; β_0 = slope at the initial point in time;

t = time

Incorporating these linear functions with standard rank-size model, the expanded rank-size model becomes;

$$\text{Log } P_i = \text{Log } \alpha_0 + \text{Log } \alpha_1 t - \beta_0 \log r_i + \beta_1 t \log r_i$$

Where

log α_1 = significant growth or decline over time in the system of cities.

β_1 = significant difference in the rates of growth or decline between larger and smaller city areas.

Inclusion of the time aspect of the urban rank-size distribution allows more accurate in stability and change over time, and, in the case of change, a more accurate estimation of the locus of such city system change.

Table 2: Expanded Regression Equations for Tamil Nadu state City for Census of 1991-2011

Year	Log α_0	Log α_1	β_0	β_1	R	R ²
1991-2011	14.480 (146.768)*	0.272 (3.649)*	-0.935 (23.880)*	-0.045 (1.559)*	0.977	0.955

Table 2 shows the results of the expanded logarithmic rank-size model and it determines source of change in the Tamilnadu city system. According to the results, on the basis of t-test (0.05 level), log α is significantly greater than zero but β is not. It can be concluded that for Tamilnadu state rank-size distribution, all city centres have increased at more or less at same rate and change in population is statistically Significant.

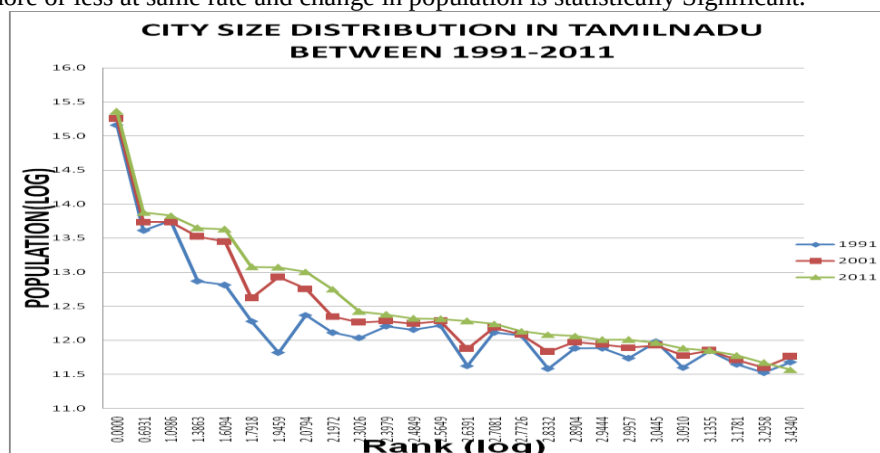


Figure 1 shows parallel rank-size curves in over this time and cities have no shift in population rank at all.

III. THE STANDARD LOGARITHMIC RANK SIZE AND EXPANDED RANK-SIZE MODEL OF CITIES ACCORDING TO REGIONS

Metro Region

Table 3

Regression Equations for Metropolitan city of Tamil Nadu Census of 1991-2011

Year	Log α	β	R	R ²
1991	Log $P_i=14.930$ (55.931)*	-1.089logr (-10.530)*	0.978	0.957
2001	Log $P_i=15.109$ (81.319)*	-1.119logr (-14.847)*	0.989	0.978
2011	Log $P_i=15.235$ (119.267)*	-1.122logr (-21.493)*	0.994	0.987

The data is fitted for the census year 1991-2001 using standard logarithmic rank-size function. Table 3 shows the t- values of each regression coefficient. The regression results show an increasing intercept values, from 14.930 in1991 to 15.235 in 2011. In addition to the slope of rank-size distribution has increase from 1.089 in 1991 to –1.122 in 2011.From this analysis it is unclear whether the change because of increasing population in larger cities faster than smaller ones or to an increase in populations of larger cities alone or decrease in the population of smaller cities.

Table 4

Expanded Regression Equations for Metro regions for Census of 1991-2011

Year	Log α_0	Log α_1	β_0	β_1	R	R ²
1991-2011	14.938 (87.498)*	0.153 (1.171)*	-1.093 (-16.398)*	-0.017 (-0.322)*	0.987	0.974
2001-2011	15.109 (94.986)*	0.126 (0.569)*	-1.119 (-17.342)*	-0.003 (-0.028)*	0.991	0.983

When the same data fitted to the expanded rank-size model according to two periods, it can be concluded that for Metro region rank-size distribution, all city centres have increased at approximately at the same rate and change in population is statistically significant.

North Region

Table 5: Regression Equations for North Region city of Tamil Nadu Census of 1991-2011

Year	Log C	Q	R	R ²
1991	Log $P_i=13.978$ (97.654)*	-0.738logr (-15.097)*	0.993	0.987
2001	Log $P_i=13.724$ (58.923)*	-0.621logr (-7.817)*	0.976	0.953
2011	Log $P_i=14.607$ (147.457)*	-0.885logr (-27.886)*	0.997	0.994

The data is fitted for the census year 1991-2001 using standard logarithmic rank-size function. The result given in Table 5 with t- values shows each regression coefficient. The regression results exhibit an increasing intercept values, from 13.978 in1991 to 14.607 in 2011. In addition to the slope of rank-size distribution has decreased from 0.738 in 1991 to 0.621 in 2001 and increased to 0.885 in 2011.

Table 6: Expanded Regression Equations for North Region city for Census of 1991-2011

Year	Log C	Log C_1	Q	Q_1	R	R ²
1991-2011	13.822 (81.758)*	0.327 (2.779)*	-0.685 (-11.888)*	-0.079 (-2.019)*	0.988	0.976
2001-2011	13.724 (77.203)*	0.883 (3.997)*	-0.621 (-10.242)*	-0.264 (-3.573)*	0.993	0.986

When the same data fitted to the expanded rank-size model according to two periods, it can be concluded that for North region rank-size distribution, all city centres have increased at approximately at the same rate and change in population is statistically significant.

West Region

Table 7: Regression Equations for West Region city of Tamil Nadu Census of 1991-2011

Year	Log C	Q	R	R ²
1991	Log $P_i=14.058$ (50.398)*	-0.795logr (-5.327)*	0.967	0.934
2001	Log $P_i=14.489$ (42.366)*	-0.853logr (-4.865)*	0.960	0.922
2011	Log $P_i=14.749$ (51.183)*	-0.895logr (-7.223)*	0.972	0.946

The result given in Table 7 with t- values shows each regression coefficient. The regression results exhibit an increasing intercept values, from 14.058 in1991 to 14.749 in 2011. In addition to the slope of rank-size distribution has increase from 0.795 in 1991 to 0.895 in 2011.

Table 8

Expanded Regression Equations for West Region city for Census of 1991-2011

Year	Log C	Log C ₁	Q	Q ₁	R	R ²
1991-2011	14.213 (69.751)*	0.240 (1.778)*	-0.871 (-7.397)*	0.011 (0.092)*	0.966	0.934
2001-2011	14.489 (43.958)*	0.260 (0.587)*	-0.853 (-5.048)*	-0.042 (-0.199)*	0.968	0.938

When the same data fitted to the expanded rank-size model according to two periods, it can be concluded that for west region rank-size distribution, all city centres have increased at approximately at the same rate and change in population is statistically significant.

Delta Region

Table 9: Regression Equations for Delta Region city of Tamil Nadu Census of 1991-2011

Year	Log C	Q	R	R ²
1991	Log P _i =13.732 (68.545)*	-0.670logr (-8.446)*	0.986	0.973
2001	Log P _i =14.856 (63.792)*	-1.008logr (-11.365)*	0.992	0.985
2011	Log P _i =15.036 (113.585)*	-1.028logr (-22.154)*	0.997	0.994

The result given in Table 9 with t- values shows each regression coefficient. The regression results exhibit an increasing intercept values, from 13.732 in1991 to 15.036 in 2011. In addition to the slope of rank-size distribution has increase from 0.670 in 1991 to 1.028 in 2011.

Table 10

Expanded Regression Equations for Delta Region city for Census of 1991-2011

Year	Log C	Log C ₁	Q	Q ₁	R	R ²
1991-2011	13.862 (72.373)*	0.693 (4.565)*	-0.713 (-9.472)*	-0.197 (3.378)*	0.990	0.979
2001-2011	14.621 (34.474)*	0.235 (0.872)*	-0.961 (-5.972)*	-0.047 (-0.465)*	0.995	0.990

When the same data fitted to the expanded rank-size model according to two periods, it can be concluded that for Delta region rank-size distribution, all city centres have increased at approximately at the same rate and change in population is statistically significant.

South Region

Table 11: Regression Equations for South Region city of Tamil Nadu Census of 1991-2011

Year	Log C	Q	R	R ²
1991	Log P _i =14.058 (50.398)*	-0.795logr (-5.327)*	0.967	0.934
2001	Log P _i =14.489 (42.366)*	-0.853logr (-4.865)*	0.960	0.922
2011	Log P _i =14.749 (51.183)*	-0.895logr (-7.223)*	0.972	0.946

The result given in Table 11 with t- values shows each regression coefficient. The regression results exhibit an increasing intercept values, from 14.058 in1991 to 14.749 in 2011. In addition to the slope of rank-size distribution has increase from 0.795 in 1991 to 0.895 in 2011.

Table 12: Expanded Regression Equations for south Region city for Census of 1991-2011

Year	Log C	Log C ₁	Q	Q ₁	R	R ²
1991-2011	14.636 (54.579)*	0.131 (0.636)*	-0.980 (-8.840)*	0.002 (0.030)*	0.970	0.942
2001-2011	14.704 (67.301)*	0.173 (0.512)*	-0.903 (-9.052)*	-0.080 (-0.563)*	0.977	0.955

When the same data fitted to the expanded rank-size model according to two periods, it can be concluded that for south region rank-size distribution, all city centres have increased at approximately at the same rate and change in population is statistically significant.

IV. CONCLUSION

Logarithmic rank-size rule and expanded rank-size rule are examined for cities in Tamilnadu state and five regions between the census year 1991-2011 and it shows that city size distribution adjusted more to the logarithmic straight-line. When the same data is fitted to the expanded rank-size model, for Tamilnadu cities and five regions rank-size distribution, all city centres have increased at approximately at the same rate and change in population is statistically significant. The rank size distributions of cities maintain same level in a state scale and regional scale.

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