



Some Structure Rhotrices and their Properties

Isyaku Idris Isyaku¹ Muhammad Yusuf Muhammad²

¹Department of Mathematics

Kano University of Science and Technology, Wudil
P.M.B 3244, Kano State, Nigeria

²Department of Mathematics

Kano University of Science and Technology, Wudil
P.M.B 3244, Kano State
Nigeria

Abstract: The aim of this paper is to introduce some new rhotrice and to give some of their basic properties. These new rhotrices are : reflexive rhotrix, counter-identity rhotrix, centro-symmetric and skew centrosymmetrixrhotrices, centrohermitianrhotrix, skewcentrohermitianrhotrix and perhermitianrhotrix.

Keywords: Rhotrix, counteridentityrhotrix , reflexive rhotrix, centrosymmetric , skew centrosymmetrixrhotrix, centrohermitianrhotrix, perhermitianrhotrix

I. Introduction

Rhotrices and some of their basic properties where first introduce in [1]. Rhotrice could be decompose into two square matrices (Couple matrices) , (i.e $R = \langle A, C \rangle$, where $A = a_{ij}$, $C = c_{lk}$ of rhotrix R) . If $Q = \langle U, V \rangle$, then addition and subtraction is done in the following form : $R \pm Q = \langle A \pm U, C \pm V \rangle$ and multiplication is $RQ = \langle AU, CV \rangle$, the inverse , transpose and determinant are respectively $R^{-1} = \langle A^{-1}, C^{-1} \rangle$, $R^T = \langle A^T, C^T \rangle$ and $\det(R) = \det(A)\det(C)$ see ([2],[3],[4],[5]).

In next section we defined some new symmetric rhotrices and give some of their properties. This new rhotrices are : generalized reflection rhotrix, reflexive rhotrix, anti reflexive rhotrix, counter-identity rhotrix, centro-symmetric rhotrix and skew centro-symmetric rhotrix.

II. Structure Rhotrices and Properties

Definition 1 (Reflection rhotrix)

A rhotrix $R = \langle A, C \rangle$ is generalized reflection rhotrix if $R^T = R$ and $R^2 = I$ where I is an identity rhotrix and A and C are generalize reflection matrices.

Definition 2 (Reflexive and antireflexiverhotrix)

A rhotrix $R = \langle A, C \rangle$ is reflexive if and only if there exist a reflection rhotrix $Q = \langle B, D \rangle$ such that $R = QRQ$ and R said to be anti reflexive if $R = -QRQ$. Where A and C are reflexive matrices. For reflexive matrices see [6] , [7]

Proposition 1

Let R be a rhotrix, then:

- I. R is reflexive with respect to reflection rhotrix Q if and only if $QR = RQ$.
- II. R is anti reflexive with respect to reflection rhotrix Q if and only if $QR = -RQ$.

Proof 1

Suppose R is reflexive with respect to Q , then

- I. R reflexive, implies $R = QRQ$

$$\begin{aligned} QR &= \langle A, C \rangle \langle B, D \rangle \\ QR &= \langle AB, CD \rangle \end{aligned}$$

$$\begin{aligned} QR &= \langle BA, DC \rangle \\ QR &= \langle B, D \rangle \langle A, C \rangle \\ QR &= RQ \end{aligned}$$

The converse, suppose $QR=RQ$.

$$\begin{aligned} Q(QR) &= QRQ \\ Q^2R &= QRQ \\ R &= QRQ \end{aligned}$$

II. Similar to I.

Definition 2 (Counteridentityrhotrix)

A rhotrix $\hat{J} = \langle J_n, J_{n-1} \rangle$ is counter-identity rhotrix if all of its entries are zeroes except the horizontal diagonal entries which are all ones and major and minor components of $\hat{J}$ (J_n and J_{n-1}), are counter identity matrices.

See [8], [9] for details on counteridentity matrices, Centrosymmetric matrices and skew centrosymmetric matrices.

Proposition 2

- I. $\hat{J}^2 = I$,
- II. $\hat{J}^T = \hat{J}$
- III. $\hat{J}$ is generalised reflection rhotrix,
- IV. $\hat{J}^{-1} = \hat{J}$
- V. $\hat{J}^n = \begin{cases} I, & \text{if } n \text{ is odd} \\ \hat{J}, & \text{if } n \text{ is even} \end{cases}$

Proof 2

- I. $\hat{J}^2 = \langle J_n^2, J_{n-1}^2 \rangle = \langle I_n, I_{n-1} \rangle = I$,
- II. $\hat{J}^T = \langle J_n^T, J_{n-1}^T \rangle = \langle J_n, J_{n-1} \rangle = \hat{J}$,
- III. From i and ii of proposition 2,
- IV. $\hat{J}^{-1} = \langle J_n^{-1}, J_{n-1}^{-1} \rangle = \langle J_n, J_{n-1} \rangle = \hat{J}$
- V. Case 1. If n is even, $n = 2m$ where m is a natural number.

$$J^{2m} = I^m = I$$

Case 2, If n is odd, $n = 2m + 1$,

$$J^{2m+1} = J^{2m}J = IJ = J$$

Definition 3(Centrosymmetricrhotrix)

A rhotrix $R = \langle A, C \rangle$ is centrosymmetric if and only if $R = JRJ$ and A, C are centrosymmetric matrices.

Example 1

The rhotrix C below is centrosymmetric rhotrix.

$$C = \begin{pmatrix} a & c & b & d \\ f & e & h & e & f \\ d & b & c \\ a \end{pmatrix} = \left\langle \begin{bmatrix} a & d & f \\ c & h & c \\ f & d & a \end{bmatrix}, \begin{bmatrix} b & c \\ c & b \end{bmatrix} \right\rangle$$

Proposition 3

A rhotrix R is centrosymmetric if and only if R is reflexive with respect J (i.e. $JR = RJ$).

Proof 3

$$\begin{aligned} R &= JRJ \\ J^2R &= JRJ \\ J^{-1}J^2R &= J^{-1}JRJ \\ JR &= RJ \end{aligned}$$

For the converse:

$$JR = RJ$$

$$\begin{aligned} J^{-1}JR &= J^{-1}RJ \\ R &= JRJ \end{aligned}$$

Proposition 4

Let $\mathbf{C}$ be set of all centrosymmetric rhotrices, then for all $R, Q \in \mathbf{C}$

- I. $(aR + bQ) \in \mathbf{C}$, a and b are scalars.
- II. $RQ \in \mathbf{C}$,
- III. $R^{-1} \in \mathbf{C}$,
- IV. $R^T \in \mathbf{C}$,
- V. $\mathbf{C}$ is an abelian group under addition.

Proof 4

- I. $aR + bQ = aJRJ + bJQJ = J(aR + bQ)J \in \mathbf{C}$,
- II. $RQ = (\hat{J}R\hat{J})(\hat{J}Q\hat{J}) = \hat{J}RQ\hat{J} \in \mathbf{C}$,
- III. $R^{-1} = (\hat{J}R\hat{J})^{-1} = \langle J_n A^{-1} J_n, J_{n-1} C^{-1} J_{n-1} \rangle = \hat{J}R^{-1}\hat{J} \in \mathbf{C}$,
- IV. similar to III,
- V. $R + Q = \hat{J}R\hat{J} + (\hat{J}Q\hat{J}) = \hat{J}(R + Q)\hat{J} = \hat{J}(Q + R)\hat{J} = Q + R$.

Definition 4 (Bisymmetric rhotrix)

A rhotrix that is both symmetric and centrosymmetric is known as bisymmetric rhotrix. Every three dimensional centrosymmetric rhotrix is bisymmetric rhotrix.

Definition 5 (skew centrosymmetric rhotrix)

A rhotrix $R = \langle A, C \rangle$ is skew centrosymmetric if and if $R = -\hat{J}R\hat{J}$ and A, C are both skew centrosymmetric matrices.

Example 2

The rhotrix S is skew centrosymmetric rhotrix is :

$$S = \left\langle \begin{bmatrix} 0 & -a & 0 & a \\ 0 & -b & 0 & b \\ -a & 0 & a & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & a \\ -b & 0 \end{bmatrix} \right\rangle = \left\langle \begin{bmatrix} 0 & a & 0 \\ -a & 0 & a \\ 0 & -a & 0 \end{bmatrix}, \begin{bmatrix} 0 & b \\ -b & 0 \end{bmatrix} \right\rangle$$

Proposition 5

A rhotrix $R = \langle A, C \rangle$ is skew centrosymmetric if and only if $\hat{J}R = -R\hat{J}$.

Proof 5

$$\hat{J}R = \langle J_n, J_{n-1} \rangle \langle A, C \rangle = \langle J_n A, J_{n-1} C \rangle = \langle -A J_n, -C J_{n-1} \rangle = -R\hat{J}$$

Proposition 6

If R and Q are skew centrosymmetric rhotrix, then RQ is centrosymmetric rhotrix.

Proof 6

$$RQ = (-\hat{J}R\hat{J})(-\hat{J}Q\hat{J}) = \hat{J}RQ\hat{J}$$

Proposition 7

If $\mathbf{S}$ a set of skew centrosymmetric rhotrices, then for all $R, Q \in \mathbf{S}$

- a) $aR + bQ \in \mathbf{S}$, a and b are scalars,
- b) $R^{-1} \in \mathbf{S}$,

- c) $R^T \in \mathcal{S}$,
d) $\mathcal{S}$ is an abelian group under addition.

Proof 7

Similar to proof 4.

Definition 6 (centrohermitian and skew centrohermitian)

A rhotrix $R = \langle A, C \rangle$ is centrohermitian if and only if $R = \hat{f}\bar{R}\hat{f}$ where R is the complex conjugate of R and A, C are centrohermitian matrices. The rhotrix is skew centrohermitian if and only if $R = -\hat{f}\bar{R}\hat{f}$.

Centrohermitian and skewcentrohermitian rhotrices satisfy the properties of centrosymmetric rhotrices listed above. For detail on centrohermitian matrices see [10].

Definition 7 (Perhermitian rhotrix)

A rhotrix that is both hermitian and centrohermitian is perhermitian rhotrix.

The rhotrix P below is both centrosymmetric and perhermitian rhotrix.

$$P = \left\langle \begin{array}{cccc} & & 3 & \\ & -i & 5 & i \\ 2 & 4 & 6 & 4 \\ & i & 5 & -i \\ & & 3 & \end{array} \right\rangle = \left\langle \left[\begin{array}{ccc} 3 & i & 2 \\ -i & 6 & -i \\ 2 & i & 3 \end{array} \right], \left[\begin{array}{cc} 5 & 4 \\ 4 & 5 \end{array} \right] \right\rangle$$

III. References

- [1] A.O . Ajibade, "The Concept of rhotrix in mathematical enrichment", Int. J.Math. Edu. Sci. Technol. 34,(2003), pp 175 -179.
- [2] A. Mohammad, "Enrichment exercise through extension of rhotrices", Int. J.Math.Sci.Technol. 38(2007) , pp 131 - 136
- [3] B. Sani, "The row column Multiplication of Higher dimensional Rhotrix " Int . J Math. Edu Sci. Technol. 38 (2007) pp 657-662
- [4] B. Sani " An alternative method for multiplication of rhotrices" Int . J Math. Edu Sci. Technol. 35, (2004) pp. 777-81
- [5] B.Sani " ,Conversion of rhotrices to coupled matrice", Int J.Math. Edu. Sci. Technol. 39 (2008) , pp 244 - 249
- [6] H.C. Chen, "Generalized reflexive matrices: special properties and application", SIAM J. Matrix Anal. 19(1998) , pp 140 – 153.
- [7] Z.Fu-Zhao, "The solvability conditions for the inverse eigenvalue problems of reflexiv", Journal of computatinal and applied Mathematics, 188(2006) , pp 180 - 189..
- [8] I. T Abu Jeib ,Centrosymmetric matrice properties and alternative approach, Canadian Applied Mathematics Quaterly VoL. 10, 14(2002).
- [9] M. El-mikkawi, F.Atlan, On solving centrosymmetric linear systems, Applied Mathematics, 4(2013), 21-32.
- [10] I. S. Pressman, " Matrices with multiple symmetry properties : application of centrohermitian and per hermitian matrices", Linear algebra and its applications, 284 (1998) , 239 -258.