



## Fuzzification of Some Tree Related Graphs

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**Abstract:** Let  $G=(V,E)$  be a crisp graph. Then  $G$  has an evidence labeling if there exist two injections  $m$  and  $\rho$  such that  $m: V^* \rightarrow [0,1]$  defined by  $m(v_i) = \frac{i}{N}$ , where  $V^*=(v_1, v_2, \dots, v_N)$ ,  $v_i \in V$  and

$\rho: E \rightarrow [0,1]$  defined by

$$\rho(v_i, v_j) = \frac{m(v_i) \wedge m(v_j)}{N} \quad \text{if} \quad (v_j, v_k) \in E \text{ and } m(v_k) = \bigwedge_{(v_j, v_p) \in E} m(v_p) \text{ and } i < j < k$$

$$\text{and } \rho(v_i, v_j) = \frac{m(v_i) \wedge m(v_j)}{N^3} \quad \text{if} \quad (v_j, v_k) \notin E \text{ for any } k, \text{ where } 1 \leq i \leq N-1, 1 \leq j \leq N.$$

The graph  $G$  which admits evidence labeling is called evidently labeled graph. In this paper we introduce some tree related graphs which admits evidence labeling.

**Keywords:** Evidence Labeling, Tree graph, Twig graph, Centipede graph, Star graph.

### I. Introduction

All graphs we discuss in this paper are simple and undirected.

Graphs are used to explain relations between objects. To include fuzziness in such relations fuzzy graphs were introduced. The first definition of fuzzy graph by Kaufmann was based on Zadeh's fuzzy relations. After that Rosenfield considered fuzzy relation on fuzzy sets and developed the structure of fuzzy graphs. After the work of Rosenfield, Yeh and Bang introduced various connectedness concepts of graphs and digraphs into fuzzy graphs. In [6], A Nagoor Gani and D Rajalaxmi (a) Subahashini presents a new concept called fuzzy labeling and in [2] we introduced a type of labeling called fuzzy digraph labeling in fuzzy digraphs. In [5] we introduced evidence labeling.

In this paper we present some tree related graphs which admits evidence labeling.

### Definition

A fuzzy graph  $G=(V, \mu, \rho)$  is a non empty set  $V$  together with a pair of functions  $\mu: V \rightarrow [0,1]$  and  $\rho: V \times V \rightarrow [0,1]$  such that for all  $x, y$  in  $V$ ,  $\rho(x, y) \leq \mu(x) \wedge \mu(y)$ . We call  $\mu$  the fuzzy vertex set of  $G$  and  $\rho$ , the fuzzy edge set of  $G$  respectively.

We denote the underlying graph of the fuzzy graph  $G=(\mu, \rho)$  by  $G^*=(\mu^*, \rho^*)$ , where  $\mu^*=\{x \in V : \mu(x) > 0\}$  and  $\rho^*=\{(x,y) \in V \times V : \rho(x,y) > 0\}$ .

### II. Main Results

#### Theorem 1

The twig graph  $T_w(n)$  admits evidence labeling.

#### Proof

Consider the twiggraph  $T_w(n)$ , where  $|V(G)| = 3n-4$ ,  $|E(G)| = 3n-5$ .

Let  $V^*=(v_1, v_2, \dots, v_{3n-4})$ , where  $V(P_n) = \{v_1, v_2, \dots, v_n\}$  and  $v_{n+1}, v_{n+2}, \dots, v_{3n-4}$  are the newly added vertices.

Define  $m: V^* \rightarrow [0,1]$  as  $m(v_i) = \frac{i}{3n-4}$ ,  $1 \leq i \leq 3n-4$  and

$$\rho: E \rightarrow [0,1] \text{ as } \rho(v_i, v_{i+1}) = \frac{i(i+2)}{(3n-4)^3}, \quad 1 \leq i \leq n-2.$$

$$\rho(v_{n-1}, v_n) = \frac{(n-1)}{(3n-4)^4}.$$

$$\rho(v_i, v_{i+(n-1)}) = \frac{i}{(3n-4)^4}, \quad 2n-2 \leq i \leq 3n-5$$

$$\rho(v_i, v_{i+(2n-3)}) = \frac{i}{(3n-4)^4}, 1 \leq i \leq n-2$$

If possible, suppose that  $m$  is not injective.

Then  $m(v_i) = m(v_j)$  for some  $i \neq j$

$$\Rightarrow i = j.$$

If possible, suppose that  $\rho(v_i, v_{i+1}) = \rho(v_j, v_{j+1})$  for some  $i \neq j$

$$\Rightarrow \frac{i(i+2)}{n^3} = \frac{j(j+2)}{n^3}$$

$$\Rightarrow i^2 - j^2 = 2(j-i)$$

$$\Rightarrow i + j = -2, \text{ contradiction.}$$

Therefore,  $\rho$  are injective if  $1 \leq i \leq n-2$ .

Similarly we can show for all values of  $i$ . Hence  $\rho$  is injective.

### Theorem 2

The centipede graph  $C_n$  admits evidence labeling.

#### Proof

Consider the  $G = (V, E)$  be a centipede graph, where  $|V(G)| = 2n$ ,

$|E(G)| = 2n-1$ ,  $V(P_2) = \{v_i, v_{i+1}\}$ ,  $i = 1, 2, \dots, n-1$ .

Let  $V^* = (v_1, v_2, \dots, v_n, v_{n+1}, v_{n+2}, \dots, v_{2n})$ .

Define  $m: V^* \rightarrow [0, 1]$  as  $m(v_i) = \frac{i}{2n}$ ,  $1 \leq i \leq 2n$

$\rho: E \rightarrow [0, 1]$  as  $\rho(v_i, v_{i+1}) = \frac{i(i+2)}{(2n)^3}$ ,  $i = 1, 3, 5, \dots, 2n-3$ .

$$\rho(v_{n-1}, v_n) = \frac{(n-1)}{(2n)^3}.$$

$$\rho(v_i, v_{i+2}) = \frac{i(i+4)}{(2n)^3}, 2 \leq i \leq n-4.$$

$$\rho(v_{n-2}, v_n) = \frac{n-2}{(2n)^4}.$$

Clearly,  $m$  and  $\rho$  are injective.

### Theorem 3

The star graph  $K_{1,n}$  admits evidence labeling.

#### Proof

Consider the star graph  $K_{1,n} = G(V, E)$ , where  $|V| = n+1$ ,  $|E| = n$ .

Let  $v_{n+1}$  be the internal vertex

Let  $V^* = (v_1, v_2, \dots, v_n, v_{n+1})$ ,  $(v_i, v_{n+1})$  be the edge,  $i = 1, 2, \dots, n$ .

Define  $m: V^* \rightarrow [0, 1]$  as  $m(v_i) = \frac{i}{n+1}$ ,  $1 \leq i \leq n+1$  and

$\rho: E \rightarrow [0, 1]$  as  $\rho(v_i, v_{n+1}) = \frac{i}{(n+1)^4}$ ,  $1 \leq i \leq n$ .

Clearly,  $m$  and  $\rho$  are injective.

### Theorem 4

The bistar graph  $B_{n,n}$  admits evidence labeling.

#### Proof

Consider the two copies of the star graph  $K_n$ . Let  $\{v_1, v_2, \dots, v_n\}$  and  $\{v_{n+2}, v_{n+3}, \dots, v_{2n+1}\}$  be the corresponding vertices of each copy with apex vertex  $v_{n+1}$  and  $v_{2n+2}$ .

Let  $B_{n,n} = G(V, E)$  where  $|V(G)| = 2n+2$ ,  $|E(G)| = 2n+1$ . Let  $V^* = (v_1, v_2, \dots, v_n, v_{n+1}, v_{n+2}, \dots, v_{2n+2})$ .

Define  $m: V^* \rightarrow [0, 1]$  as  $m(v_i) = \frac{i}{2n+2}$ ,  $1 \leq i \leq 2n+2$  and

$\rho: E \rightarrow [0, 1]$  as  $\rho(v_i, v_{i+n}) = \frac{i(2n+2)}{(2n+2)^3}$ ,  $i = 1, 2, 3, \dots, n$ .

$$\rho(v_{n+1}, v_{2n+2}) = \frac{n+1}{(2n+1)^4}.$$

$$\rho(v_i, v_{2n+2}) = \frac{i}{(2n+2)^4}, i = n+2, n+3, \dots, 2n+2.$$

Clearly  $m$  and  $\rho$  are injective.

### Theorem 5

The double star graph  $K_{1,n,n}$  admits evidence labeling.

**Proof**

Consider the double star graph  $K_{1,n,n}$ , where  $|V(G)| = 2n+1$ ,  $|E(G)| = 2n$ .

Let  $V^* = (v_1, v_2, \dots, v_n, v_{n+1}, v_{n+2}, \dots, v_{2n+1})$ , where  $v_1, v_2, \dots, v_n$  are the newly added vertices internal vertex as  $v_{2n+1}$

Define  $m: V^* \rightarrow [0,1]$  as  $m(v_i) = \frac{i}{2n+1}$ ,  $1 \leq i \leq 2n+1$  and

$\rho: E \rightarrow [0,1]$  as  $\rho(v_i, v_{i+n}) = \frac{i(2n+1)}{(2n+1)^3}$ ,  $i = 1, 2, 3, \dots, n$ .

$\rho(v_{n+i}, v_{2n+1}) = \frac{n+i}{(2n+1)^4}$ ,  $i = 1, 2, \dots, n$ .

Clearly  $m$  and  $\rho$  are injective.

**Theorem 6**

The coconut tree graph  $CT(m,n)$  admits evidence labeling.

**Proof**

Consider the coconut tree graph  $CT(m,n)$ , where  $|V(CT(m,n))| = n+m$ ,

$|E(CT(m,n))| = n+m-1$  and  $v_1, v_2, \dots, v_n$  are the newly appended vertices to the path  $P_m$  with vertices  $v_{n+1}, v_{n+2}, \dots, v_{n+m}$ .

Let  $V^* = (v_1, v_2, \dots, v_n, v_{n+1}, v_{n+2}, \dots, v_{n+m})$ .

Define  $m: V^* \rightarrow [0,1]$  as  $m(v_i) = \frac{i}{n+m}$ ,  $1 \leq i \leq n+m$  and

$\rho: E \rightarrow [0,1]$  as  $\rho(v_i, v_{i+k}) = \frac{i(n+2)}{(m+n)^3}$ ,  $1 \leq i \leq n$ ,  $1 \leq k \leq n$ ,  $i+k = n$ .

$\rho(v_i, v_{i+1}) = \frac{i(i+2)}{(m+n)^3}$ ,  $n+1 \leq i \leq n+m-2$ .

$$\rho(v_{n+m-1}, v_{n+m}) = \frac{n+m-1}{(n+m)^4}$$

Clearly  $m$  and  $\rho$  are injective.

**Theorem 7**

The graph obtained by duplicating a pendent vertex in a path graph  $P_n$  admits evidence labeling.

**Proof**

Let  $V(P_n) = \{v_1, v_2, \dots, v_n\}$ . Without loss of generality, assume that  $v_1$  is the duplicated vertex and its copy be  $v_{n+1}$ .

Let  $V^* = (v_1, v_2, \dots, v_n, v_{n+1})$ .

Define  $m: V^* \rightarrow [0,1]$  as  $m(v_i) = \frac{i}{n+1}$ ,  $1 \leq i \leq n+1$  and

$\rho: E \rightarrow [0,1]$  as  $\rho(v_i, v_{i+1}) = \frac{i(i+2)}{(n+1)^3}$ ,  $1 \leq i \leq n-2$

$\rho(v_{n-1}, v_n) = \frac{n-1}{(n+1)^4}$ .

$$\rho(v_2, v_{n+1}) = \frac{2}{(2n+1)^4}$$

Clearly  $m$  and  $\rho$  are injective.

**Theorem 8**

The Hurdle graph admits evidence labeling.

**Proof**

Let  $V(P_n) = \{v_1, v_2, \dots, v_n\}$ . Let  $v_2, v_3, \dots, v_{n-1}$  are the internal vertices and Let  $V^* = (v_1, v_2, \dots, v_n, v_{n+1}, \dots, v_{2n-2})$ .

Define  $m: V^* \rightarrow [0,1]$  as  $m(v_i) = \frac{i}{2n-2}$ ,  $1 \leq i \leq 2n-2$  and

$\rho: E \rightarrow [0,1]$  as  $\rho(v_i, v_{i+1}) = \frac{i(i+2)}{(2n-2)^3}$ ,  $1 \leq i \leq 2n-2$

$\rho(v_{n-1}, v_n) = \frac{n-1}{(2n-2)^4}$ .

$$\rho(v_i, v_{i+n-1}) = \frac{i}{(2n-2)^4}$$

Clearly  $m$  and  $\rho$  are injective

## I. References

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