



Analysis of Two Phase Blood Flow in Human Pulmonary Artery during Lung Cancer by Mathematical Modeling

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Abstract: Present paper examined a mathematical model of two phase blood flow in human pulmonary artery; keeping in view the nature of pulmonary blood circulation during Lung Cancer. Some previous researchers already assumed the blood flow to be two phase. Here power law model applied which transformed into biofluid mechanical set up. For the purpose, blood has been assumed to be constituted by plasma and blood cells which is realistic so far. In present study overall presentation is in tensorial form and the solution technique adopted analytical as well as numerical. The role of Hematocrit explicit in determination of blood pressure drop.

Key Words: Pulmonary blood flow, lung cancer, power law model, hematocrit, pressure drop etc.

I. Introduction

The human lung is an elastic structure that collapses like a balloon and expels all its air through the trachea, there is no external force to keep it inflated. In human being respiration is responsible for supply for oxygen to the tissues and remove Carbon dioxides to achieve this respiration can be further divided in to Pulmonary ventilation, Diffusion of oxygen and carbon dioxides between the alveoli and the blood, Transport of oxygen and carbon dioxides in the blood and body fluids and Regulation of ventilation four major function [6].

In order of human blood flow, the human pulmonary arteries start as the pulmonary trunk or main pulmonary artery. The main pulmonary artery begins at the base of the right ventricle. It is short and wide approximately 5 centimeters in length and 3 centimeters in diameter. The pulmonary artery carries deoxygenated blood from the right ventricle to the lungs. The blood here passes through capillaries adjacent to alveoli and becomes oxygenated as part of the process of respiration. In contrast to the pulmonary arteries, the bronchial arteries supply nutrition to the lungs themselves [2]. The human pulmonary artery pressure is a measure of the blood pressure found in the main pulmonary artery. This is measured by inserting a catheter into the main pulmonary artery [9].

The human pulmonary artery is relevant in a number of clinical states. Pulmonary hypertension is used to describe an increase in the pressure of the pulmonary artery, and may be defined as a mean pulmonary artery pressure of greater than 25mmHg [10]. Pulmonary embolism refers to an embolus that lodges in the human pulmonary circulation. This may arise from a deep venous thrombosis, especially after a period of immobility. A pulmonary embolus is a common cause of death in patients with cancer and stroke [11]. A large pulmonary embolus which becomes lodged in the bifurcation of the pulmonary trunk with extensions into both the left and right main pulmonary arteries is called a saddle embolus [7].

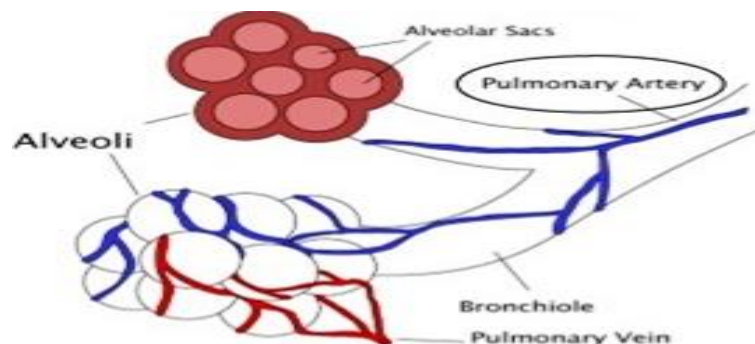


Fig. 01 Pulmonary artery

Lung Cancer is the leading cause of death worldwide. According to **Worldwide, 2015 report** there were an estimated 14.1 million cancer cases around the world in 2012, of these 7.4 million cases were in men and 6.7 million in women. This number is expected to increase to 24 million by 2035. Lung cancer was the most common cancer worldwide contributing 17% of the total number of new cases diagnosed in 2012. The top three, lung, prostate and colorectal cancers, contributed nearly 42% of all cancers. There were 268 cases of cancer diagnosed per 100,000 in more developed regions, compared to 148 in less developed regions in 2012 [3]. In human lung cancer develops when normal lung cells sustain genetic damage that eventually leads to uncontrolled cell proliferation. Like all cancers, lung cancer cells have the ability to invade neighboring tissues and spread or metastasize to distant parts of the body. Left untreated, lung cancer eventually causes death [8].

II. Real Model

A. Choice of frame of reference

A frame of reference was selected for mathematical modeling of two phase blood flow of the state of a moving blood. It was observed in view the difficulty and generality of the problem of blood flow and selected generalized three-dimensional orthogonal curvilinear co-ordinate system, briefly prescribed as E3, called as 3-dim Euclidean space. It was interpreted the quantities related to blood flow in tensorial form which was comparatively more realistic, the biophysical laws thus expressed fully hold good in any co-ordinate system, which was compulsion for the truthfulness of the law. Now, let the co-ordinate axes be OX^i , O denotes origin and superscript $i = 1, 2, 3$ let X^i be the co-ordinates of any point P in space. Mathematical description of the state if a moving blood is affected by means of functions which give the distribution of the blood velocity $v^k = v^k(X^i, t)$, $k = 1, 2, 3$ and of any two thermodynamic quantities pertaining to the blood, for instance the pressure $p = p(X^i, t)$ and the density $\rho = \rho(X^i, t)$. As was well known, all the thermodynamic quantities are determined by the values of any two of them, together with the equation of state. Hence, if we are given five quantities, namely the three components of velocity v^k , the pressure p and the density ρ , the state of moving blood was completely determined.

All these quantities are functions of the co-ordinates X^i , $i = 1, 2, 3$ and of the time t . It emphasized that $v^k(X^i, t)$ was the velocity of the blood at a given point X^i in space and at a given t , i.e. it refers to fixed points in space and not to fixed particles of the blood; in the course of time, the latter move about in space. The same remarks apply to p and ρ . Blood was a mixed fluid and uses many symbols [14].

B. Boundary Conditions are as follows

1. The velocity of blood flow on the axis of arteries at $r = 0$ will be maximum and finite, say $V_0 = \text{maximum velocity}$, $V = V_0$ then $A = 0$.
2. The velocity of blood flow on the wall of pulmonary artery at $r = R$, where, R is the radius of pulmonary artery, will be zero. This condition is well known as no-slip condition. $V = 0$ at $r = R$

C. Constitution of two phase blood volume

The first and foremost reason is that the blood is not an ideal fluid but it is a mixture of the two phases one is of plasma and other one is of blood cells. These blood cells, semi permeable packages of liquid of a density greater than that of plasma, are capable of changing their shape and size while flowing through different blood vessels [15]. Plasma is a liquid containing semi permeable packages of RBCs.

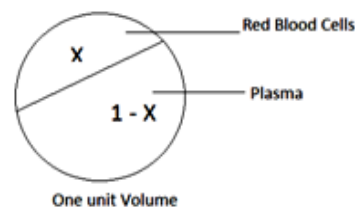


Fig. 02 Blood Volume

The behavior of blood is almost Newtonian at high shear rate, while at low shear rate the blood exhibits yield stress and non-Newtonian behavior. The flow of blood is affected by the presence of blood cells. This effect is directly proportional to the volume occupied by blood cells. Let the volume portion covered by blood cells in unit volume be X , X is replaced by $H/100$, where H is the hematocrit the volume percentage of blood cells. The hematocrit is normally about three times the hemoglobin concentration (reported as grams per deciliter) [1]. Then the volume portion covered by the plasma will be $(1-X)$.

If mass ratio of cells to plasma is r then clearly: $r = \frac{X\rho_c}{(1-X)\rho_p}$.

Where ρ_c and ρ_p are densities of blood cells and plasma respectively. Usually this mass ratio is not a constant. Even then this may be supposed to constant in present context [14].

III. Mathematical modeling / Formulation

According to Upadhyay and Pandey [13], we take the blood flow in pulmonary arteries remote from heart to be non-Newtonian power law model. Equations for power law flow will be as follows:

$$\frac{1}{\sqrt{g}(\sqrt{g}v^i)_{,i}} = 0 \dots \dots (3.1)$$

$$\rho_m \frac{\partial v^i}{\partial t} + (\rho_m v^i) v_{,j}^i = T_{,j}^{ij} \dots \dots \dots (3.2) \text{ Where, } T^{ij} \text{ is taken from constitutive equation of power law flow.}$$

$\rho_m = X\rho_c + (1 - X)\rho_p$ is the density of blood and $\eta_m = X\eta_c + (1 - X)\eta_p$ is the viscosity of mixture of the blood. $X = H/100$ is volume ratio of the blood cell; H is the Hematocrit. Other symbols have their usual meanings.

Since the blood vessels are cylindrical, the above governing equations have to be transformed into cylindrical co-ordinates. As we know earlier:

Now we have to transform the equations (3.1) and (3.2) in cylindrical form. As we know. For cylindrical $x^1 = r, x^2 = \Theta, x^3 = z$

Matrix of metric tensor in cylindrical co-ordinates is as follows:-

$$[g_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

While matrix of conjugate matrix tensor is as follow-

$$[g^{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Whereas the chritoffel's symbol of 2nd kind are as follow:-

$$\left\{ \begin{matrix} 1 \\ 2 \end{matrix} \right\} \left\{ \begin{matrix} 1 \\ 2 \end{matrix} \right\} = -r, \left\{ \begin{matrix} 1 \\ 2 \end{matrix} \right\} \left\{ \begin{matrix} 1 \\ 2 \end{matrix} \right\} = \left\{ \begin{matrix} 1 \\ 2 \end{matrix} \right\} \left\{ \begin{matrix} 1 \\ 2 \end{matrix} \right\} = \frac{1}{r}$$

Remaining others is zero.

Relation between contra variant physical components of the blood flow will be as follows:

$$\begin{aligned} \sqrt{g_{11}}v^1 &= v_r \Rightarrow v_r = v^1 \\ \sqrt{g_{22}}v^2 &= v_\theta \Rightarrow v_\theta = rv^2 \\ \sqrt{g_{33}}v^3 &= v_z \Rightarrow v_z = v^3 \end{aligned}$$

Again the physical components of $p_{,j}g^{ij}$ is $-\sqrt{g_{ii}}p_{,j}g^{ij}$ and the matrix of the physical components of shearing stress-tensor -

$T^{ij} = \eta_m (e^{ij})^n = \eta_m (g^{ik}v_{,k}^i + g^{jk}v_{,k}^j)^n$ Will be as follows -

$$\begin{bmatrix} 0 & 0 & \eta_m \left(\frac{dv}{dr}\right)^n \\ 0 & 0 & 0 \\ \eta_m \left(\frac{dv}{dr}\right)^n & 0 & 0 \end{bmatrix}$$

The covariant derivative of T^{ij}

$$T_{,j}^{ij} = \frac{1}{\sqrt{g}} \frac{\partial(\sqrt{g}T^{ij})}{\partial x^j} + \left\{ \begin{matrix} i \\ j \end{matrix} \right\} T^{ij}$$

Keeping in view the above fact, the governing tensorial equation can be transformed into cylindrical form which as follows

Equation of continuity

$$\frac{\partial v}{\partial z} = 0 \dots \dots \dots (3.3)$$

Equation of motion

r – Component

$$-\frac{\partial p}{\partial r} = 0 \dots \dots \dots (3.4)$$

Θ – Component

$$0 = 0 \dots \dots \dots (3.5)$$

Z - Component

$$0 = -\frac{\partial p}{\partial z} + \frac{\eta_m}{r} \left[r \left(\frac{\partial v_z}{\partial r} \right)^n \right] \dots \dots \dots (3.6)$$

Here, this fact has been taken in view that the blood flow is axially Symmetric in arteries concerned, i.e.

$$v_\theta = 0 \text{ And } v_r = 0, \dots \dots \dots (3.7)$$

v_z and p do not depend upon Θ . Also the blood flow steadily, i.e.

$$\frac{\partial p}{\partial t} = \frac{\partial v_r}{\partial t} = \frac{\partial v_\theta}{\partial t} = \frac{\partial v_z}{\partial t} = 0 \dots \dots \dots (3.8)$$

IV. Mathematical Analysis

On integrating equation (3.3) we get- $v_z = v(r)$ because v does not depend upon Θ

The integrating of equation of motion (3.5) yields: $P = p(z)$ since p does not depend upon Θ

Now, with the help of equation (3.7) and (3.8) the equations of motion (3.6) convert in the following form

$$0 = -\frac{dp}{dz} + \frac{\eta_m}{r} \frac{d}{dr} \left\{ r \left(\frac{dv}{dr} \right)^n \right\} \dots \dots \dots (4.1)$$

The pressure-gradient $-\frac{\partial p}{\partial z} = P$ of blood flow in the arteries remote the heart can be supposed to be constant and hence the equation (3.8) takes the following form

$$\frac{d}{dr} \left\{ r \left(\frac{dv}{dr} \right)^n \right\} = -\frac{Pr}{\eta_m} \dots \dots \dots (4.2)$$

On integrating equation (3.8), we get

$$r \left(\frac{dv}{dr} \right)^n = \frac{Pr}{2\eta_m} + A \dots \dots \dots (4.3)$$

We know that the velocity of the blood flow on the axis of cylindrical arteries is maximum and constant. So that.

We apply the boundary condition at $r = 0$, $v = v_0$ (constant), on equation (4.2) takes the following form

$$r \left(\frac{dv}{dr} \right)^n = \frac{Pr}{2\eta_m} \Rightarrow \frac{-dv}{dr} = \left[\frac{Pr}{2\eta_m} \right]^{\frac{1}{n}} \dots \dots \dots (4.4)$$

Integrating equation (4.4) once again, we get

$$v = -\left[\frac{P}{2\eta_m} \right]^{\frac{1}{n}} \frac{r^{\frac{1}{n}+1}}{\frac{1}{n}+1} + B \dots \dots \dots (4.5)$$

To determine the arbitrary constant B , we will apply the non-slip condition on the inner wall of the arteries at $r = R$,

$v = 0$, where R = radius of vessel, on equation (4.5) so as to get

$$B = \left[\frac{P}{2\eta_m} \right]^{\frac{1}{n}} \frac{nR^{\frac{1}{n}+1}}{n+1}$$

Hence the equation (4.5) takes the following form

$$v = \left[\frac{P}{2\eta_m} \right]^{\frac{1}{n}} \frac{n}{n+1} [R^{\frac{1}{n}+1} - r^{\frac{1}{n}+1}] \dots \dots \dots (4.6)$$

Which determine the velocity of the blood flow in the artery remote from heart where, P is gradient of blood pressure. And η_m is the velocity of blood mixture.

V. Bio- Physical Interpretation

The total flow of blood through the transverse section of the arteries is-

$$\begin{aligned} Q &= \int_0^R v \cdot 2\pi r dr = \int_0^R \left[\frac{P}{2\eta_m} \right]^{\frac{1}{n}} \frac{n}{n+1} (R^{\frac{1}{n}+1} - r^{\frac{1}{n}+1}) \\ Q &= \left[\frac{P}{2\eta_m} \right]^{\frac{1}{n}} \frac{n2\pi}{n+1} \left[\frac{R^{\frac{1}{n}+1} \cdot r^2}{2} - \frac{nr^{\frac{1}{n}+3}}{3n+1} \right]_0^R \\ Q &= \left[\frac{P}{2\eta_m} \right]^{\frac{1}{n}} \frac{n2\pi}{n+1} \frac{(n+1) \cdot R^{\frac{1}{n}+3}}{2(3n+1)} \end{aligned}$$

$$Q = \left[\frac{P}{2\eta_m} \right]^{\frac{1}{n}} \frac{n\pi R^{\frac{1}{n}+3}}{3n+1} \dots \dots \dots (5.1)$$

The pattern of blood flow can be shown by fig.

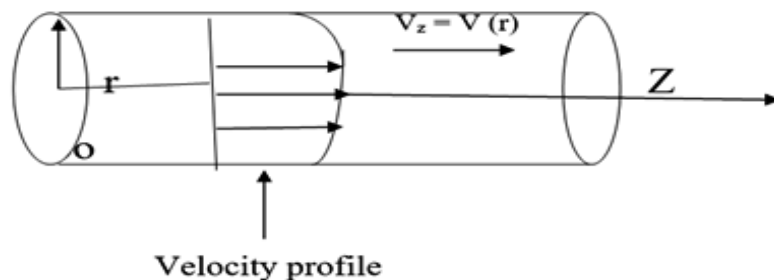


Fig.03 Blood flow velocity profile

VI. Result and Discussion

Examination: Hematocrit v/s blood pressure in during Lung Cancer patient.

Patient name: L (Male)

Age: 58 years old

Diagnosis: Lung cancer (Pulmonary disease)

Table 1: Blood pressures v/s Hemoglobin in Clinical data

Date	HB(Hemoglobin) in (gram/dl)	Hematocrit in ($3 \times HB$) (kg/m^3)	Blood Pressure (BP) in (mmhg)	Arteries Pressure Drop in Pascal -second $\left(\frac{S+D}{2}\right) - S$
8/2/2012	12.8	0.036227	140/90	-3328.3
11/9/2016	12.2	0.034529	110/70	-2662.64
26/3/2017	10.16	0.028755	100/60	-2662.64
12/5/2017	10.2	0.028868	130/80	-3328.3
17/8/2017	10.0	0.028302	110/70	-2662.64

Now, we have, $Q = 425 \frac{ml}{min}$, $Q = 0.00708333 m^3/second$

Approximately Radius of pulmonary artery $R = 1.5 cm$ or $0.015m$

According to Gustafson, Daniel R. [5], $\eta_p = 0.0013$ pascal second

According to Glenn Elert [4], $\eta_m = 0.0271$ pascal second and $H = 0.034529$

Pressure drop $(P_f - P_i) = 2662.64 \text{ Pascalsecond}$

Approximately pulmonary artery length $(z_f - z_i) = 5 cm$ or $0.05m$

$P(z) = \frac{P_f - P_i}{z_f - z_i}$ And by using relation $\eta_m = \eta_c X + \eta_p (1 - X)$ Where, $X = \frac{H}{100}$, and we get η_c

$$\eta_m = \eta_c X + \eta_p (1 - X), \Rightarrow 0.0271 = \eta_c (0.00034529) + 0.0013(0.99965471)$$

$$\Rightarrow \eta_c = 74.7209592 \text{ Pascal second}$$

Again using this relation and change in to the hematocrit-

$$\eta_m = \eta_c X + \eta_p (1 - X) \Rightarrow \eta_m = 0.747209592H + 0.0012996$$

From equation (5.1)

$$P = -\frac{d_p}{d_z}$$

We get

$$Q = \left[\frac{\Delta P}{2\eta_m \Delta z} \right]^{\frac{1}{n}} \frac{n\pi R^{\frac{1}{n}+3}}{3n+1} \dots\dots\dots (6.1)$$

Put the values of Q, ΔP , Δz and R in equation (6.1)

$$0.0070833 = \left[\frac{2662.64}{2 \times 0.0271 \times 0.05} \right]^{\frac{1}{n}} \frac{n \times 3.14 \times (0.015)^{\frac{1}{n}+3}}{3n+1}$$

$$668.3963 = \left(\frac{n}{3n+1} \right) (14737.86)^{\frac{1}{n}}$$

By using trial method, we get the value of n is $n = 1.22359$

Again using equation (6.1) and putting $n = 1.22359$

$$Q = \left[\frac{\Delta P}{2\eta_m \Delta z} \right]^{\frac{1}{n}} \frac{n\pi R^{\frac{1}{n}+3}}{3n+1}$$

$$0.0070833 = \left[\frac{\Delta P}{2\eta_m \Delta z} \right]^{\frac{1}{n}} \frac{n \times 3.14 \times (R)^{\frac{1}{n}+3}}{3n+1}$$

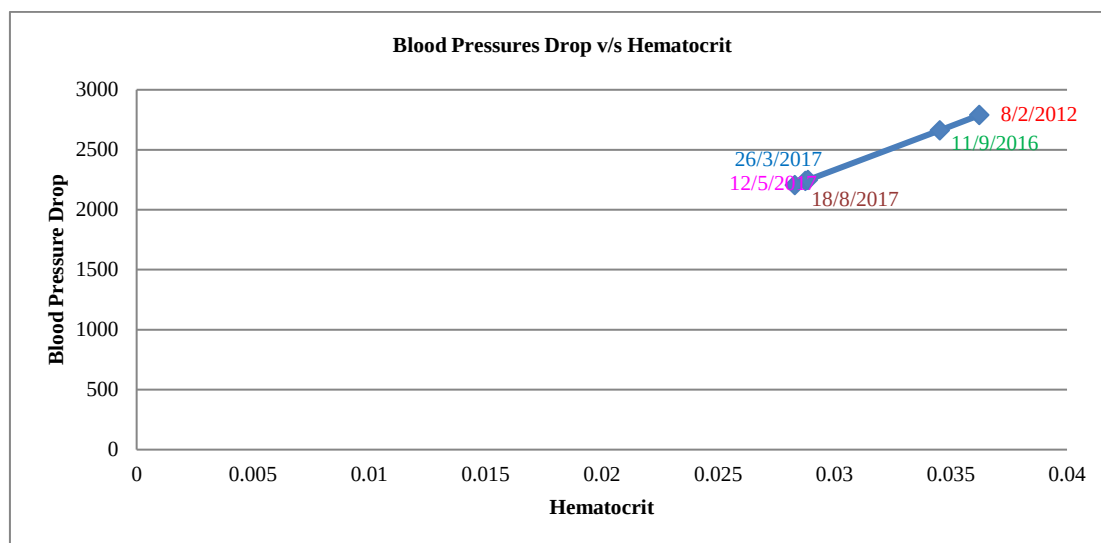
$$0.0070833 = \left[\frac{\Delta P}{2\eta_m \Delta z} \right]^{\frac{1}{n}} \frac{1.22359 \times 3.14 \times (0.015)^{\frac{1}{1.22359}+3}}{(3 \times 1.22359) + 1}$$

$$\Delta P = \eta_m (98269.04)$$

$$\Delta P = (73427.56929H + 127.71045) (98269.04)$$

Table 2: Blood pressures drop v/s Hematocrit in Clinical data

Date	8/2/2012	11/9/2016	26/3/2017	12/5/2017	18/8/2017
H (Hematocrit) In kg/m^3	0.036227	0.034529	0.028755	0.028868	0.028302
BPD (Blood Pressure drop) In Pascal-second	2787.77101	2663.09091	2239.12021	2247.41753	2205.8575



Observation: A mathematical investigation of the graph from 8/2/2012 via 11/9/2016 to 26/3/2017 shows decreasing sense. Whereas from 26/3/2017 to 12/5/2017 shows increasing sense. Again shows graph from 12/5/2017 to 18/8/2017 decreasing sense.

Conclusion: When blood pressure drop is increased then we cannot suggest for operation but when blood pressure drop is decreased we suggest for successful operation. Between 26/3/2017 to 18/8/2017 successful operation is suggested otherwise not.

VII. Acknowledgment

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