



SD (WD) and ID in Fuzzy Soft Graphs

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Abstract: Let $G_{A,V}$ be a fuzzy soft graph and let x_i and x_j be any two vertices of $G_{A,V}$. If $\mu_e(x_i, x_j) \leq \rho_e(x_i) \wedge \rho_e(x_j)$ for each parameter $e \in A$ and $d_{(G_{A,V})}(x_i) \geq d_{(G_{A,V})}(x_j)$ then we say that x_i strongly dominates x_j in $G_{A,V}$. Similarly if $\mu_e(x_i, x_j) \leq \rho_e(x_i) \wedge \rho_e(x_j)$ for each parameter $e \in A$ and $d_{(G_{A,V})}(x_i) \leq d_{(G_{A,V})}(x_j)$, then we say that x_i weakly dominates x_j in $G_{A,V}$. A subset S of V is said to be a **strong(weak) fuzzy soft dominating set** if for every vertex in $V - S$ is strongly (weakly) dominated by at least one vertex in S . The minimum cardinality of a minimal strong(weak) fuzzy soft dominating set is called **strong (weak) fuzzy soft domination number** and is denoted by $\gamma_{sfs}(G_{A,V})$ ($\gamma_{wfs}(G_{A,V})$).

Key words:- Strong (weak) fuzzy soft domination, Strong (weak) fuzzy soft domination number, independent domination, independent domination number independent strong(weak)fuzzy soft domination and independent strong(weak) fuzzy soft domination number.

I. Introduction

The concept of domination in fuzzy graphs was first introduced by A Somasundaram and S Somasundaram [10]. The notion of fuzzy soft graphs and some operations in fuzzy soft graphs was introduced by Sumit Mohinta and Samanta[9]. Muhammed Akram and Saira Nawas[8] introduced different types of fuzzy soft graphs and their properties. Several authors were discussed Strong (weak) domination and independent domination in fuzzy graphs in different ways[5] &[6].

In this paper we introduce the concepts of strong (weak) domination in fuzzy soft graphs, strong (weak) domination number, independent fuzzy soft domination, independent fuzzy soft domination number, independent strong (weak) fuzzy soft domination, independent strong (weak) fuzzy soft domination number.

II. Preliminaries

Definition 2.1[7] Let G be a graph with V be the set of all vertices and E be the set of all edges. If there exist two functions $\rho: V \rightarrow [0, 1]$ and $\mu: V \times V \rightarrow [0, 1]$ such that $\mu(x, y) \leq \rho(x) \wedge \rho(y) \forall x, y \in V$. Then $G(\rho, \mu)$ is called a fuzzy graph.

Definition 2.2 [8] Let $V = \{x_1, x_2, x_3, \dots, x_n\}$ (non empty set) E (parameters set) and $A \subseteq E$. Also let $\rho_e: V \rightarrow [0, 1]$ such that each $x_i \in V$ mapped to $\rho_e(x_i)$, where $\rho_e = \rho(e)$ is the ρ image of $e \in A$ and $\mu_e: V \times V \rightarrow [0, 1]$ such that each $(x_i, x_j) \in E$ mapped to $\mu_e(x_i, x_j)$, where $\mu_e = \mu(e)$ is the μ image of $e \in A$, then the pair $((A, \rho), (A, \mu))$ is called a fuzzy soft graph if and only if $\mu_e(x_i, x_j) \leq \rho_e(x_i) \wedge \rho_e(x_j) \forall e \in A$ and $\forall i, j = 1, 2, 3, \dots, n$ is denoted by $G_{A,V}$.

Definition 2.3 [8] The underlying crisp graph of a fuzzy soft graph $G_{A,V} = ((A, \rho), (A, \mu))$ is denoted by $G^* = (\rho^*, \mu^*)$, where $\rho^* = \{x_i \in V; \rho_e(x_i) > 0 \text{ for some } e \in E\}$ and $\mu^* = \{(x_i, x_j) \in V \times V; \mu_e(x_i, x_j) > 0 \text{ for some } e \in E\}$.

Definition 2.4 [7] A fuzzy soft graph $G_{A,V}$ is called a **complete fuzzy soft graph** if $\mu_e(x_i, x_j) = \rho_e(x_i) \wedge \rho_e(x_j) \forall x_i, x_j \in \rho^*, e \in A$ and is denoted by $K_{G_{A,V}}$.

Definition 2.5 [7] Let $G_{A,V}$ be a fuzzy soft graph. Then the **order** of $G_{A,V}$ is defined as

$$O(G_{A,V}) = \sum_{e \in A} \left(\sum_{x_i \in V} \rho_e(x_i) \right) \text{ and } \text{size of } G_{A,V} \text{ is defined as } S(G_{A,V}) = \sum_{e \in A} \left(\sum_{x_i, x_j \in V} \mu_e(x_i, x_j) \right) \text{ and the}$$

$$\text{degree of a vertex } x_i \text{ is defined as } d_{(G_{A,V})}(x_i) = \sum_{e \in A} \left(\sum_{x_j \in V, x_i \neq x_j} \mu_e(x_i, x_j) \right).$$

Definition 2.6[3] **Degree** of a fuzzy soft graph $G_{A,V}$ is defined as $D_{G_{A,V}} = \max \{d_{G_{A,V}}(x_i); \forall x_i \in V\}$.

Definition 2.7[3] A fuzzy soft graph $G_{A,V}$ is said to be **regular fuzzy soft graph** if the fuzzy graph corresponding to each parameter $e \in A$ is a regular fuzzy graph.

Definition 2.8[3] A fuzzy soft graph $G_{A,V}$ without fuzzy soft loops and fuzzy soft multiple edges is called a **fuzzy soft simple graph**.

Definition 2.9 [3] An FSG is said to be **isolated** if $\mu_e(x_i, x_j) = 0 \forall (x_i, x_j) \in E, e \in A$.

Definition 2.10[3] Two fuzzy soft vertices are said to be adjacent if they are connecting by a fuzzy soft edge and two fuzzy soft edges are said to be adjacent if they are incident on a common fuzzy soft vertex.

Definition 2.11[3] The **complement** of a FSG $G_{A,V}$ is defined as $\bar{G}_{A,V} = ((A, \rho), (\bar{A}, \mu))$ where

$$\bar{\mu}_e(x_i, x_j) = \rho_e(x_i) \wedge \rho_e(x_j) - \mu_e(x_i, x_j) \forall (x_i, x_j) \in E \& e \in A\}.$$

Definition 2.12[3] **Neighborhood of a vertex** is defined as $N(x_i) = \{x_j \in V \mid \mu_e(x_i, x_j) = \rho_e(x_i) \wedge \rho_e(x_j) \forall e \in A\}$.

Definition 2.13[3] Let $G_{A,V}$ be an FSG. $\delta(G_{A,V}) = \min\{d_{(G_{A,V})}(x_i) \mid \forall x_i \in V\}$ &

$$\Delta(G_{A,V}) = \max\{d_{(G_{A,V})}(x_i) \mid \forall x_i \in V\}.$$

Definition 2.14[3] Let $G_{A,V}$ be an FSG. $d_N(x_i)$ is the fuzzy soft cardinality of $N(x_i)$.

Definition 2.15[3] Let $G_{A,V}$ be an FSG. $\delta_N(G_{A,V}) = \min\{d_N(x_i) \mid \forall x_i \in V\}$ &

$$\Delta_N(G_{A,V}) = \max\{d_N(x_i) \mid \forall x_i \in V\}.$$

Definition 2.16[4] Let $G_{A,V}$ be a fuzzy soft graph and let x_i and x_j be two vertices of $G_{A,V}$. If

$$\mu_e(x_i, x_j) \leq \rho_e(x_i) \wedge \rho_e(x_j) \text{ for each parameter } e \in A \text{ and, then we say that } x_i \text{ dominates } x_j \text{ in } G_{A,V}$$

. A subset S of V is called a **fuzzy soft dominating set** if for every $x_j \in V - S$, there exist a vertex $x_i \in S$ such that x_i dominates x_j .

The minimum scalar cardinality of S is called the fuzzy soft domination number and is denoted by $\gamma_{fs}(G_{A,V})$.

$$\text{The scalar cardinality of a fuzzy soft subset } S \text{ of } V \text{ is } \sum_{e \in A} \left(\sum_{x_i \in S} \rho_e(x_i) \right).$$

Definition 2.17[4] A fuzzy dominating set S is called an independent dominating set if S is independent.

III. Strong Fuzzy Soft Dominating (SFSD) set & Weak Fuzzy Soft Dominating (WFSD) set

Definition 3.1 Let $G_{A,V}$ be a fuzzy soft graph and let x_i and x_j be any two vertices of $G_{A,V}$. If

$$\mu_e(x_i, x_j) \leq \rho_e(x_i) \wedge \rho_e(x_j) \text{ for each parameter } e \in A \text{ and } d_{(G_{A,V})}(x_i) \geq d_{(G_{A,V})}(x_j) \text{ then we say that}$$

x_i strongly dominates x_j in $G_{A,V}$. Similarly if $\mu_e(x_i, x_j) \leq \rho_e(x_i) \wedge \rho_e(x_j)$ for each parameter $e \in A$

and $d_{(G_{A,V})}(x_i) \leq d_{(G_{A,V})}(x_j)$, then we say that x_i weakly dominates x_j in $G_{A,V}$. A subset S of V is said to be a **strong(weak) fuzzy soft dominating set** if for every vertex in $V - S$ is strongly (weakly) dominated by at least one vertex in S . The minimum cardinality of a minimal strong(weak) fuzzy soft dominating set is called **strong (weak) fuzzy soft domination number** and is denoted by $\gamma_{sfs}(G_{A,V})$ ($\gamma_{wfs}(G_{A,V})$).

Theorem 3.2 In any complete fuzzy soft graph, the following inequality holds

$$\gamma_{wfs}(G_{A,V}) \leq \gamma_{sfs}(G_{A,V}).$$

Proof : Let $G_{A,V}$ be a complete fuzzy soft graph.

Case(i): suppose for all $x_i \in V$, $\rho_e(x_i)$ are equal. Since $G_{A,V}$ is a complete fuzzy soft graph, $\mu_e(x_i, x_j) = \rho_e(x_i) \wedge \rho_e(x_j) \quad \forall e \in A \text{ and } x_i, x_j \in V$.

So $\mu_e(x_i, x_j) = \rho_e(x_i) \quad \forall e \in A \text{ and } \forall x_i \in V$.

$$\text{Hence } \gamma_{wfs}(G_{A,V}) = \gamma_{sfs}(G_{A,V}) = \sum_{e \in A} \wedge \rho_e(x_i) = \sum_{e \in A} \rho_e(x_i).$$

Case(ii): Suppose for all $x_i \in V$, $\rho_e(x_i)$ are not equal for each $e \in A$. But we know that in a complete fuzzy soft graph, any one of the vertex dominates all other vertices. If the degree of that vertex is the least one, then the dominating set with that vertex is the weak fuzzy soft dominating set and the weak fuzzy soft domination number $\gamma_{wfs}(G_{A,V}) = \sum_{e \in A} \wedge \rho_e(x_i)$ and clearly the strong fuzzy soft dominating set has a vertex other than the least degree vertex set and so the strong fuzzy soft domination number is strictly greater than the weak fuzzy soft domination number.

That is $\gamma_{wfs}(G_{A,V}) < \gamma_{sfs}(G_{A,V})$. From cases (i) and (ii), we get $\gamma_{wfs}(G_{A,V}) \leq \gamma_{sfs}(G_{A,V})$.

Corollary:3.3 (i) For a regular fuzzy soft graph and totally regular fuzzy soft graph, $\gamma_{wfs}(G_{A,V}) = \gamma_{sfs}(G_{A,V})$.

(ii) For any fuzzy soft graph $\gamma_{wfs}(G_{A,V}) \leq \gamma_{sfs}(G_{A,V})$ or $\gamma_{wfs}(G_{A,V}) \geq \gamma_{sfs}(G_{A,V})$.

Theorem: 3.4 If $G_{A,V}$ be a fuzzy soft graph of order $O(G_{A,V})$,

$$(i) \quad \gamma_{fs}(G_{A,V}) \leq \gamma_{sfs}(G_{A,V}) \leq O(G_{A,V}) - \Delta(G_{A,V}).$$

$$(ii) \quad \gamma_{fs}(G_{A,V}) \leq \gamma_{wfs}(G_{A,V}) \leq O(G_{A,V}) - \delta(G_{A,V}).$$

Proof: From the definition of fuzzy soft domination number and strong fuzzy soft domination number, we get

$$\gamma_{fs}(G_{A,V}) \leq \gamma_{sfs}(G_{A,V}). \text{ Also we have } O(G_{A,V}) = \sum_{e \in A} \left(\sum_{x_i \in V} \rho_e(x_i) \right) \quad \text{and}$$

$\Delta(G_{A,V}) = \max\{d_{(G_{A,V})}(x_i) \mid \forall x_i \in V\}$. So $O(G_{A,V}) - \Delta(G_{A,V})$ is the sum of the degrees of the vertices in $G_{A,V}$ excluding the maximum degree of a vertex. Thus we get $\gamma_{sfs}(G_{A,V}) \leq O(G_{A,V}) - \Delta(G_{A,V})$.

Combining above relations we get $\gamma_{fs}(G_{A,V}) \leq \gamma_{sfs}(G_{A,V}) \leq O(G_{A,V}) - \Delta(G_{A,V})$.

In similar way we get $\gamma_{fs}(G_{A,V}) \leq \gamma_{wfs}(G_{A,V})$ and $O(G_{A,V}) - \delta(G_{A,V})$ is the sum of the degrees of the vertices in $G_{A,V}$ excluding the minimum degree of a vertex. So $\gamma_{wfs}(G_{A,V}) \leq O(G_{A,V}) - \delta(G_{A,V})$.

Hence we get (ii).

IV. Independent Fuzzy soft domination

Definition ;3.4 A fuzzy soft dominating set S is called an **independent fuzzy soft dominating set** if S is independent. An independent fuzzy soft dominating set is said to be a maximal independent fuzzy soft dominating set if there is no proper independent fuzzy soft dominating set. An IFSD set S is said to be a maximum independent fuzzy soft dominating set if there is no IFSD set S' such that $|S'| > |S|$. The minimum scalar cardinality of a minimum independent fuzzy soft dominating set is called **independent fuzzy domination number** and is denoted by $\gamma_{ifs}(G_{A,V})$.

Remark : 3.5 For any fuzzy soft graph $\gamma_{fs}(G_{A,V}) \leq \gamma_{ifs}(G_{A,V})$.

Theorem : 3.6 For any strong fuzzy soft graph $G_{A,V}$ without isolated vertices then
$$\frac{O(G_{A,V}) - \Delta(G_{A,V})}{2} \leq \gamma_{fs} \leq \gamma_{ifs}.$$

Proof: Let S be a fuzzy soft dominating set with t elements. Since every element of $V - S$ is adjacent to some element in S , $|V - S| \leq \sum_{i=1}^t d(x_i) \leq \gamma_{fs} + \Delta(G_{A,V})$

$$\begin{aligned} &\Rightarrow O(G_{A,V}) - \gamma_{fs} \leq \gamma_{fs} + \Delta(G_{A,V}) \\ &\Rightarrow O(G_{A,V}) - \Delta(G_{A,V}) \leq 2\gamma_{fs} \\ &\Rightarrow \frac{O(G_{A,V}) - \Delta(G_{A,V})}{2} \leq \gamma_{fs} \end{aligned}$$

But we have for any fuzzy soft graph $\gamma_{fs}(G_{A,V}) \leq \gamma_{ifs}(G_{A,V})$.

Hence
$$\frac{O(G_{A,V}) - \Delta(G_{A,V})}{2} \leq \gamma_{fs} \leq \gamma_{ifs}.$$

V. Independent Strong(Weak) Fuzzy soft domination

Definition: 3.7 $V_{\delta N} = \{x_i \in V \mid d_N(x_i) = \delta_N(G_{A,V})\}$ and $V_{\Delta N} = \{x_i \in V \mid d_N(x_i) = \Delta_N(G_{A,V})\}$ are the minimum and maximum neighborhood vertex degree of a fuzzy soft graph.

Definition: 3.8 Subset S of V of a fuzzy soft graph $G_{A,V}$ is said to be independent if no two vertices of S are adjacent for each parameter $e \in A$.

Definition: 3.9 Let $G_{A,V}$ be a fuzzy soft graph and let x_i and x_j be two vertices of $G_{A,V}$. If $d_{(G_{A,V})}(x_i) \geq d_{(G_{A,V})}(x_j)$ and for each parameter $e \in A$ x_i and x_j are independently dominated, then we say that x_i independently strong dominates x_j in $G_{A,V}$. Similarly if $d_{(G_{A,V})}(x_i) \leq d_{(G_{A,V})}(x_j)$ and for each parameter $e \in A$ and x_i and x_j are independently dominated, then we say that x_i independently weak dominates x_j in $G_{A,V}$. A subset S of V is said to be an **independent strong(weak) fuzzy soft dominating** [IS(W)FSD] set if for every vertex in $V - S$ is independently strong (weak) dominated by at least one vertex in S . The minimum cardinality of a minimal independent strong(weak) fuzzy soft dominating set is called **independent strong (weak) fuzzy soft domination number** and is denoted by $\gamma_{isfs}(G_{A,V})$ ($\gamma_{iwfs}(G_{A,V})$).

Lemma: 3.10 If $G_{A,V}$ is a fuzzy soft graph and S is an IWFS set of $G_{A,V}$, then $S \cap V_{\delta N} \neq \emptyset$.

Proof: Let $x_i \in V_{\delta N} = \{x_i \in V \mid d_N(x_i) = \delta_N(G_{A,V})\}$. If $x_i \in S$ then clearly $S \cap V_{\delta N} \neq \emptyset$. Otherwise since S is an IWFS set, there exist a vertex $x_j \in S$ such that for each parameter $e \in A$ and x_i and x_j are independent and $d_{(G_{A,V})}(x_i) \geq d_{(G_{A,V})}(x_j) \Rightarrow d_N(x_i) \geq d_N(x_j)$ but $d_N(x_i) = \delta_N(G_{A,V})$.

Therefore $d_N(x_i) = d_N(x_j)$.

$\Rightarrow x_j \in V_{\delta N} \Rightarrow x_j \in S \cap V_{\delta N}$. Hence $S \cap V_{\delta N} \neq \emptyset$.

Theorem: 3.11 For a fuzzy soft graph $G_{A,V}$, $\gamma_{iwfs}(G_{A,V}) \leq O(G_{A,V}) - \delta_N(G_{A,V})$.

Proof: Let S be an IWFS set of $G_{A,V}$. Then by above lemma $S \cap V_{\delta N} \neq \emptyset$. Let $x_i \in S \cap V_{\delta N}$. Since S is independent $S \cap N(x_i) = \emptyset \Rightarrow S \subseteq V - N(x_i)$

$$\Rightarrow |S|_{fs} \leq |V - N(x_i)|_{fs}$$

$$\Rightarrow \gamma_{iwfs}(G_{A,V}) \leq |S|_{fs} \leq O(G_{A,V}) - \delta_N(G_{A,V}). \text{ Hence proved.}$$

Theorem: 3.12 Let $G_{A,V}$ be a fuzzy soft graph, then $\gamma_{iws}(G_{A,V}) = O(G_{A,V}) - \delta_N(G_{A,V})$ if and only if $V - N(x_i)$ is independent for every $x_i \in V_{\partial N}$.

Proof: Suppose $\gamma_{iws}(G_{A,V}) = O(G_{A,V}) - \delta_N(G_{A,V})$. To prove $V - N(x_i)$ is independent for every $x_i \in V_{\partial N}$. Assume on the contrary that $V - N(x_i)$ is dependent and choose $R = N[x_i]$ and $S = \{x_i\}$. If $R \neq S$, choose $x_j \in V - R$ and set $R = N[x_i] \cup N[x_j]$ and $S = \{x_i\} \cup \{x_j\}$. Repeat this process until $R = S$. From this process we get an IWS set of cardinality atmost $O(G_{A,V}) - \delta_N(G_{A,V}) - \sum_{e \in A} \rho_e(x_i)$.

But this is a contradiction to our assumption that $\gamma_{iws}(G_{A,V}) = O(G_{A,V}) - \delta_N(G_{A,V})$. Therefore $V - N(x_i)$ must be an independent set.

Conversely suppose that $V - N(x_i)$ is independent for every $x_i \in V_{\partial N}$.

To prove $\gamma_{iws}(G_{A,V}) = O(G_{A,V}) - \delta_N(G_{A,V})$. Assume that S be a minimum IWS set. Then $S \cap V_{\partial N} \neq \emptyset$, by above theorem. Let $x_i \in S \cap V_{\partial N}$. Since $x_i \in S$, $S \cap N(x_i) = \emptyset$. But we know that $V - N(x_i)$ is independent for every $x_i \in V_{\partial N}$. This implies that $S = V - N(x_i) \Rightarrow \gamma_{iws}(G_{A,V}) = O(G_{A,V}) - \delta_N(G_{A,V})$. Hence the theorem.

VI. Conclusion

In this paper we defined new concepts such as Strong (weak) fuzzy soft domination, Strong (weak) fuzzy soft domination number, independent domination, independent domination number, independent strong(weak)fuzzy soft domination, independent strong(weak) fuzzy soft domination number and proved some theorems related to this. These concepts are very useful for solving wide range of problems.

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