



A Study of quasi C-reducible Finsler space

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Abstract: The main aim of this paper is to study the theory of quasi C-reducible Finsler space and T tensor in a quasi C-reducible Finsler space. In this paper we have obtained some important results.

Keywords: Quasi C-reducibility, Finsler space, T tensor, Landsberg space, hv-torsion tensor.

I. Introduction:

Let n-dimensional Finsler space be $F^n(M^n, L)$ with fundamental function is $L(x, y)$ and underlying manifold is M^n .

The metric tensor g_{ij} of Finsler space F^n is defined as

$$(1.1) \quad g_{ij} = \frac{1}{2} \dot{\partial}_i \dot{\partial}_j L^2$$

$$(1.2) \quad h_{ij} = g_{ij} - l_i l_j$$

II. Quasi C-Reducibility In Finsler Space:

Definition 2.1:

If the C-tensor C_{ijk} of any two dimensional Finsler space can be written the form

$$(2.1) \quad C_{ijk} = (1/C^2) C_i C_j C_k$$

Then the Finsler space $F^n_{(n \geq 2)}$ is termed as C-2 like Finsler space.

Definition 2.2:

A Finsler space F^n is said to be C-reducible, if the following three conditions holds good

- (i) Finsler space F^n is not Riemannian,
- (ii) The dimension is more than two.

- (iii) the (h)hv-torsion tensor C_{ijk} of F^n may be written as

$$(2.2) \quad C_{ijk} = \{1/(n+1)\} \left(h_{ij} C_k + h_{jk} C_i + h_{ki} C_j \right)$$

Wherein

$$(2.3) \quad h_{ij} = g_{ij} - l_i l_j$$

and C_i is the contracted torsion tensor.

Definition 2.3:

An n-dimensional Finsler Space F^n is said to be S-3 like Finsler space, if the v-curvature tensor S_{ijkl} is of the form

$$(2.4) \quad S_{ijkl} = \{1/(n-1)(n-2)\} S \left(h_{il} h_{jk} - h_{ik} h_{jl} \right)$$

Definition 2.4:

An n-dimensional Finsler Space F^n is said to be S-4 like Finsler space, if the v-curvature tensor S_{ijkl} is written as

$$(2.5) \quad S_{ijkl} = \left(h_{il} Q_{jk} + h_{jk} Q_{il} - h_{ik} Q_{jl} - h_{jl} Q_{ik} \right)$$

Definition 2.5:

A Finsler space F^n is said to be quasi C-reducible, if a Finsler space of n-dimensional whose hv-torsion tensor C_{ijk} is written in the form

$$(2.6) \quad C_{ijk} = \left(B_{ij} C_k + B_{jk} C_i + B_{ki} C_j \right)$$

Wherein C_i is a covariant vector and B_{ij} is a symmetric tensor.

Contracting equation (2.6) with g^{jk} yields

$$(2.7) \quad (1 - B) C_i = 2 B_i^j C_j$$

Wherein

$$(2.8) \quad B = B_{ij} g^{ij}$$

If we take $(1 - B)/2 = \mu$ then equation (2.7) becomes

$$(2.9) \quad B_i^j C_j = \mu C_i$$

If an n-dimensional Finsler space is C-reducible then B_{ij} becomes [10]

If an n-dimensional Finsler space is C-2 like then B_{ij} becomes in the form [10]

$$(2.11) \quad B_{ij} = \left\{ 1/3C^2 \right\} C_i C_j$$

Contracting equation (2.9) by g_{jk} , we get

$$(2.12) \quad B_{ik} C_j = \mu g_{jk} C_i$$

Contracting equation (2.9) by g^{ik} , we get

$$(2.13) \quad B^{jk} C_j = \mu C^k$$

Contracting equation (2.10) by g^{ij} we obtain

$$(2.14) \quad g^{ij} h_{ij} = (n+1)B$$

Contracting equation (2.11) by g^{ij} , we get

(2.15)

$$\text{From } \boxed{C_i C^i = 3BC^2} \text{ equations (2.14) and (2.15), we obtain}$$

$$(2.16) \quad C_i C^i = \{3/(n+1)\} C^2 g^{ij} h_{ij}$$

Contracting equation (2.16) by g_{ij} yields

(2.17)

$$\boxed{C_i C_j = \{3n/(n+1)\} C^2 h_{ij}}$$

III. T TENSOR IN A QUASI C-REDUCIBLE FINSLER SPACE:

The T tensor in quasi C-reducible Finsler space F^n is defined as

$$(3.1) \quad C_{ijk,l} = B_{jk,l} C_i + B_{ki,l} C_j + B_{ij,l} C_k$$

Contracting equation (3.1) by g^{jk} , we get

$$(3.2) \quad C_{i,l} = B_{,l} C_i + B_{i,l} C_j + B_{i,l} C_k$$

Using the relation $C_{i,l} = 0$, then the equation (3.2) reduces in the form

$$(3.3) \quad B_{,l} C_i + B_{i,l} C_j + B_{i,l} C_k = 0$$

Contracting equation (3.3) with C^i and using the equation (2.15), we get

$$\text{equation (3.5)} \quad \boxed{3BC^2 B_{,l} + 2B_{i,l} C^i C_j = 0} \quad \text{Contracting equation (3.1) by } g_{ik} \text{ and using (2.17) yields}$$

$$\boxed{Bg_{ik} B_{,l} + 2\{n/(n+1)\} h_{jk} B_{i,l} = 0}$$

IV. THE LANDSBERG SPACES IN A QUASI C-REDUCIBLE FINSLER SPACE:

Quasi C-reducible Finsler space $C_{ijk,l}$ is the h-covariant derivative of C_{ijk} and it is expressed as

$$(4.1) \quad C_{ijk,l} = \left(B_{ij,l} C_k + B_{jk,l} C_i + B_{ki,l} C_j + B_{ij} C_{k,l} + B_{jk} C_{i,l} + B_{ki} C_{j,l} \right)$$

We have by Berwald's space

$$(4.2) \quad C_{ijk,l} = 0$$

And

$$(4.3) \quad C_{i,l} = 0$$

In views of equations (4.2) and (4.3), equation (4.1) assumes the form

$$(4.4) \quad B_{ij,l} C_k + B_{jk,l} C_i + B_{ki,l} C_j = 0$$

Contracting equation (4.4) by g^{kn} yields

$$(4.5) \quad B_{ij,l} C^n + B_{j,l}^n C_i + B_{ki,l} C_j = 0$$

Next, contracting equation (4.5) with C^i and using equation (2.15), we obtain

$$(4.6) \quad B_{ij,l} C^i C^k + 3B C^2 B_{j,l}^k + B_{i,l}^k C^i C_j = 0$$

In view of relation $B_{ij,l} C^i = -(1/2) B_{,l} C_j$, the equation (4.6) assumes the form

$$(4.7) \quad B_{j,l}^i = (1/3BC^2) B_{,l} C^i C_j = 0$$

Contracting equation (4.7) by g_{ik} yields

$$(4.8) \quad B_{jk,l} = (1/3BC^2) B_{,l} C_j C_k$$

Theorem 4.1:

If the n-dimensional Finsler space F^n is C-reducible as well as C-2 like then the following relation

$$B_{jk,l} = \{n/(n+1)\} B^{-1} B_{,l} h_{jk}$$

Holds good.

Proof:

From equations (2.17) and (4.8)

(4.9)

$$B_{jk,l} = \{n/n+1\} B^{-1} h_{jk} B_{,l}$$

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