



PERTURBATION SOLUTION OF COMPRESSIBLE TWO- PHASE JET FLOW AND HEAT TRANSFER

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Abstract: The laminar compressible plane wall jet has been studied by reducing the compressible boundary layer equations to incompressible form. The velocity and Temperature distribution are obtained by using the Schlichting's model. The zeroth order perturbation solution for velocity and temperature has been obtained by assuming the particle does not effect the velocity and temperature field. The first order solutions have been obtained by using finite difference technique. The effects of various parameters have been discussed with figures and table.

Key words: Particulate suspensions, boundary layer characteristics, heat transfer, particle loading, diffusion, CFD.

Nomenclature: Specific heats of fluid and solid particles (C_p, C_s), Eckret number E_c , Thermal conductivity(k), Prandtl number(pr), Temperatures of fluid and particle phase (T, T_p), Temperature of the plate at $\eta=0(T_0)$, Temperature of the plate at $\eta=1(T)$, Fluid phase velocities(u, v), particle phase velocities (u_p, v_p), Binary diffusion coefficient(D), Particle loading(α), Diffusion parameter($\epsilon=v/v_p$), Kinematic coefficients of viscosity of fluid and particle phase respectively (ν, ν_p), Density of fluid and particle phase respectively(ρ, ρ_p), Ratibetween C_s and C_p (γ), Volume fraction of the suspended particles(ϕ),Coefficient of viscocity of fluid and particle phase(μ, μ_p), Thermal equilibrium time(τ_T), Velocity equilibrium time (τ_p), Root mean square (r.m.s)

I. Introduction

The problem of mixing of a Jet of compressible fluid discharged through a narrow slit in a wall affords an example of boundary layer type motion of fluid in the absence of solid boundary. In case of two phase Jet flow the governing equations does not admit a similar solution. The velocity and temperature distribution can be studied by a perturbation on the Schlichting's [1] model. By following the analysis of pozzi[2,3] & Binachini[2,3]. The laminar Jet mixing have been studied by Soo[5],Panda et.e[6,7]., Ryhming[9], Dutta & Mishra [10], Dutta & Das [11],by the case of dilute suspension. Dutta & Das[12] have also considered the laminar axi-symmetric Jet mixing of compressible dusty fluid under full expansion by using the flow variables of the Jet differ only slighting from that of surrounding stream. In the present paper the flow of a laminar compressible plane free Jet by using a perturbation techniqe on Schlichting's model and using Dorodnitsyn Transformation to reduce the governing compressible equations to incompressible form.

II. Mathematical Modeling and Formulation of the problem

An initially heated compressible fluid be discharged through a narrow slit in a wall and then mix with the same surrounding fluid which is at rest and has a temperature T_∞ . Taking the origin at the slit and the coordinates axes x and y along the axis of the jet and normal to it. The boundary layer equations of the steady flow of compressible fluid containing dust particles in Cartesian coordinates after introducing the non-dimensional form of variables are

$$\bar{x} = \frac{x}{\lambda}, \bar{y} = \frac{y}{(\tau_p \nu_\infty)^{\frac{1}{2}}}, \bar{u} = \frac{u}{U}, \bar{v} = v \left(\frac{\tau_p}{\nu} \right)^{\frac{1}{2}}, \bar{\rho} = \frac{\rho}{\rho_\infty}, \bar{\mu} = \frac{\mu}{\mu_\infty}, \bar{u}_p = \frac{u_p}{U}, \bar{v}_p = v_p \left(\frac{\tau_p}{\nu_\infty} \right)^{\frac{1}{2}},$$

$$\bar{\rho}_p = \frac{\rho_p}{\rho_{p\infty}}, \bar{\mu}_p = \frac{\mu_p}{\mu_{p\infty}}, \bar{h} = \left(\frac{h-h_\infty}{h_\infty} \right), \bar{h}_p = \left(\frac{h_p-h_\infty}{h_\infty} \right), \bar{K}_s = \frac{K_s}{K_{s\infty}}, \tau_p = \frac{2}{3} \frac{C_{p\infty}}{C_{s\infty}} \frac{1}{Pr} \tau_T,$$

$$Pr = \frac{\mu_\infty C_{p\infty}}{k_\infty}, \lambda = \tau_p U, Ec = \frac{U^2}{c_p T_\infty}, \epsilon = \frac{\nu_{p\infty}}{\nu_\infty}, D_p \approx \nu_p, \alpha = \frac{\phi \rho_{s\infty}}{(1-\phi) \rho_\infty}$$

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \quad (1)$$

$$\frac{\partial}{\partial x}(\rho_p u_p) + \frac{\partial}{\partial y}(\rho_p v_p) = 0 \quad (2)$$

$$\rho \left[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) - \frac{\alpha}{1-\phi} \rho_p (u - u_p) \quad (3)$$

$$\left[u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} \right] = \epsilon \frac{\partial}{\partial y} \left(\frac{\partial u_p}{\partial y} \right) + (u - u_p) \quad (4)$$

$$\left(\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} \right) = \frac{1}{Pr} \frac{\partial}{\partial y} \left(\mu \frac{\partial h}{\partial y} \right) + \frac{\alpha}{1-\phi} \rho_p (h_p - h) + \frac{\alpha}{1-\phi} \rho_p Ec (u - u_p)^2 + Ec \mu \left(\frac{\partial u}{\partial y} \right)^2 \quad (5)$$

$$\left[u_p \frac{\partial h_p}{\partial x} + v_p \frac{\partial h_p}{\partial y} \right] = \frac{1}{Pr} \frac{\partial}{\partial y} \left(\frac{\partial h_p}{\partial y} \right) - (h_p - h) - Ec (u - u_p)^2 + Ec \epsilon \frac{\partial}{\partial y} \left(u_p \frac{\partial u_p}{\partial y} \right) \quad (6)$$

And the diffusion equation is given by

$$u_p \frac{\partial \rho_p}{\partial x} + v_p \frac{\partial \rho_p}{\partial y} = \epsilon \frac{\partial^2 \rho_p}{\partial y^2} \quad (7)$$

The non-dimensional boundary conditions are as follows

$$y = 0, \frac{\partial u}{\partial y} = 0 = \frac{\partial u_p}{\partial y}, v = 0, v_p = 0, \frac{\partial h}{\partial y} = 0, \frac{\partial h_p}{\partial y} = 0, \frac{\partial \rho_p}{\partial y} = 0$$

$$y = \pm\infty, u = u_p = 0, h = h_p = 0, \rho_p = 1$$

and the integral conditions

$$\frac{d}{dx} \int_{-\infty}^{+\infty} \rho u^2 dy + \frac{\alpha}{(1-\phi)} \int_{-\infty}^{+\infty} \rho_p (u - u_p) dy = 0$$

$$\frac{d}{dx} \int_{-\infty}^{+\infty} \rho u \left(h + \frac{u^2}{2} - h_\infty \right) dy - \frac{\alpha}{(1-\phi)} \int_{-\infty}^{+\infty} \rho_p (h_p - h) dy - \frac{\alpha}{(1-\phi)} Ec \int_{-\infty}^{+\infty} \rho_p (u - u_p)^2 dy = 0$$

III. Method of Solution

By taking a perturbation on the Schlichting [1] model by writing $u = u_0 + u_1$, $v = v_0 + v_1$, $u_p = u_{p0} + u_{p1}$, $v_p = v_{p0} + v_{p1}$, $\rho_p = \rho_{p0} + \rho_{p1}$, $\rho = \rho_0 + \rho_1$ where $u_1, v_1, u_{p1}, v_{p1}, \rho_{p1}$ are perturbation quantities.

We get two sets of equations as follows

$$\frac{\partial}{\partial x}(\rho_0 u_0) + \frac{\partial}{\partial y}(\rho_0 v_0) = 0 \quad (8)$$

$$\frac{\partial}{\partial x}(\rho_{p0} u_{p0}) + \frac{\partial}{\partial y}(\rho_{p0} v_{p0}) = 0 \quad (9)$$

$$\rho_0 \left[u_0 \frac{\partial u_0}{\partial x} + v_0 \frac{\partial u_0}{\partial y} \right] = \frac{\partial}{\partial y} \left(\mu \frac{\partial u_0}{\partial y} \right) - \frac{\alpha}{1-\phi} \rho_{p0} (u_0 - u_{p0}) \quad (10)$$

$$\left[u_{p0} \frac{\partial u_{p0}}{\partial x} + v_{p0} \frac{\partial u_{p0}}{\partial y} \right] = \epsilon \frac{\partial}{\partial y} \left(\frac{\partial u_{p0}}{\partial y} \right) + (u_0 - u_{p0}) \quad (11)$$

Neglecting the drag force the equation (9) becomes

$$u_{p0} \frac{\partial \rho_{p0}}{\partial x} + v_{p0} \frac{\partial \rho_{p0}}{\partial y} = \epsilon \frac{\partial^2 \rho_{p0}}{\partial y^2} \quad (12)$$

The boundary condition's are:

$$y = 0: \frac{\partial u_0}{\partial y} = 0 = \frac{\partial u_{p0}}{\partial y}, v = 0, v_{p0} = 0, \frac{\partial \rho_{p0}}{\partial y} = 0$$

$$y = \pm\infty: u_0 = u_{p0} = 0, \rho_{p0} = \rho_{p\infty}$$

Together with Integral condition

$$\frac{d}{dx} \int_{-\infty}^{+\infty} \rho_0 u_0^2 dy + \frac{\alpha}{(1-\phi)} \int_{-\infty}^{+\infty} \rho_{p0} (u_0 - u_{p0}) dy = 0$$

The solution of

$$\frac{\partial}{\partial x}(\rho_0 u_0) + \frac{\partial}{\partial y}(\rho_0 v_0) = 0 \quad \& \quad \rho_0 u_0 \frac{\partial u_0}{\partial x} + \rho_0 v_0 \frac{\partial u_0}{\partial y} = \frac{\partial}{\partial y} \left(\mu \frac{\partial u_0}{\partial y} \right)$$

is given by Dorodnitsyn [14]

We have solved equation (9),(11),(12) by using finite difference technique with non-uniform grid for the interpretation of the result.

$$\frac{\partial}{\partial x}(\rho_0 u_1 + \rho_1 u_0) + \frac{\partial}{\partial y}(\rho_0 v_1 + \rho_1 v_0) = 0 \quad (13)$$

$$\frac{\partial}{\partial x}(\rho_{p0} u_{p1} + \rho_{p1} u_{p0}) + \frac{\partial}{\partial y}(\rho_{p0} v_{p1} + \rho_{p1} v_{p0}) = 0 \quad (14)$$

$$\rho_0 u_0 \frac{\partial u_1}{\partial x} + \rho_0 u_1 \frac{\partial u_0}{\partial x} + \rho_1 u_0 \frac{\partial u_0}{\partial x} + \rho_0 v_0 \frac{\partial u_1}{\partial y} + \rho_0 v_1 \frac{\partial u_0}{\partial y} + \rho_1 v_0 \frac{\partial u_0}{\partial y} = \frac{\partial}{\partial y} \left(\mu \frac{\partial u_1}{\partial y} \right) - \frac{\alpha}{1-\phi} [\rho_{p0} u_1 + \rho_{p1} u_0 - \rho_{p0} u_{p1} - \rho_{p1} u_{p0}] \quad (15)$$

$$u_{p0} \frac{\partial u_{p1}}{\partial x} + u_{p1} \frac{\partial u_{p0}}{\partial x} + v_{p0} \frac{\partial u_{p1}}{\partial y} + v_{p1} \frac{\partial u_{p0}}{\partial y} = \epsilon \frac{\partial}{\partial y} \left(\frac{\partial u_{p1}}{\partial y} \right) + (u_1 - u_{p1}) \quad (16)$$

$$u_{p0} \frac{\partial \rho_{p1}}{\partial x} + u_{p1} \frac{\partial \rho_{p0}}{\partial x} + v_{p0} \frac{\partial \rho_{p1}}{\partial y} + v_{p1} \frac{\partial \rho_{p0}}{\partial y} = \epsilon \frac{\partial^2 \rho_{p1}}{\partial y^2} \quad (17)$$

The boundary conditions are

$$y = 0, v_1 = 0, \frac{\partial u_1}{\partial y} = 0, v_{p1} = 0, \frac{\partial u_{p1}}{\partial y} = 0, \frac{\partial \rho_{p1}}{\partial y} = 0, y = \infty, u_1 = 0, u_{p1} = 0, \rho_{p1} = 0$$

and the integral condition is

$\frac{d}{dx} \int_{-\infty}^{\infty} (\rho_0 u_1^2 + 2\rho_0 u_0 u_1 + \rho_1 u_0^2) dy + \frac{\alpha}{1-\phi} \int_{-\infty}^{\infty} (\rho_{p0} u_1 - \rho_{p0} u_{p1} + \rho_{p1} u_0 - \rho_{p1} u_{p0}) dy = 0$ is identically satisfied.

The equation (13),(15),(16),(17) together with boundary conditions have been solved by using finite difference technique with non-uniform grid for the interpretation of the result.

Taking $h = h_0 + h_1$ and $h_p = h_{p0} + h_{p1}$ where h_1 and h_{p1} are perturbation quantities. We get two sets of equation.

$$\rho_0 u_0 \frac{\partial h_0}{\partial x} + \rho_0 v_0 \frac{\partial h_0}{\partial y} = \frac{1}{Pr} \frac{\partial}{\partial y} \left[\mu \frac{\partial h_0}{\partial y} \right] + \frac{\alpha}{1-\phi} \rho_{p0} [h_{p0} - h_0] + \frac{\alpha}{1-\phi} Ec \rho_{p0} [u_0 - u_{p0}]^2 + Ec \mu \left(\frac{\partial u_0}{\partial y} \right)^2 \quad (18)$$

$$u_{p0} \frac{\partial h_{p0}}{\partial x} + v_{p0} \frac{\partial h_{p0}}{\partial y} = \frac{1}{Pr} \frac{\partial}{\partial y} \left(\frac{\partial h_{p0}}{\partial y} \right) - (h_{p0} - h_0) - Ec (u_0 - u_{p0})^2 + Ec \epsilon \frac{\partial}{\partial y} \left(u_{p0} \frac{\partial u_{p0}}{\partial y} \right) \quad (19)$$

Subject to the boundary conditions

$$y = 0, \frac{\partial h_0}{\partial y} = 0, \frac{\partial h_{p0}}{\partial y} = 0, \quad y = \infty, h_0 = h_{\infty}, h_{p0} = h_{\infty}$$

And the following integral condition must satisfied

$$\frac{d}{dx} \int_{-\infty}^{\infty} \rho_0 u_0 \left(h_0 + \frac{u_0^2}{2} - h_{\infty} \right) dy - \frac{\alpha}{1-\phi} Ec \int_{-\infty}^{\infty} \rho_{p0} (h_{p0} - h_0) dy - \frac{\alpha}{1-\phi} \int_{-\infty}^{\infty} \rho_{p0} (u_0 - u_{p0})^2 dy = 0$$

Taking dilute suspension the equation (18) becomes

$$\text{So } \rho_0 u_0 \frac{\partial h_0}{\partial x} + \rho_0 v_0 \frac{\partial h_0}{\partial y} = \frac{1}{Pr} \frac{\partial}{\partial y} \left(\mu \frac{\partial h_0}{\partial y} \right) + Ec \mu \left(\frac{\partial u_0}{\partial y} \right)^2 \quad (20)$$

The solution of (20) is obtained in by taking $h_0 - h_{\infty} = \theta_1 - \theta_2$

Where $\theta_1 = C \xi^{-\frac{1}{3}} \text{sech}^{2Pr} \zeta$, $C = \frac{H_0}{6\nu_{\infty} a h_{\infty}} \left(\int_0^{\infty} \text{sech}^{2+2Pr} \zeta d\zeta \right)^{-1}$ and

$$\theta_2 = -18\nu_{\infty}^2 \alpha^4 \zeta^{-\frac{2}{3}} \theta_0(\zeta)$$

and θ_0 is the solution of

$$(1 - s^2) \frac{d^2 \theta_0}{ds^2} + 2(Pr - 1)s \frac{d\theta_0}{ds} + 4Pr\theta_0 = 8Prs^2(1 - s^2) \quad (21)$$

With the boundary conditions

$$s = 0: \frac{d\theta_0}{ds} = 0; \quad s = \pm 1: \theta_0 = 0$$

The solutions of (21) is carried out by Bansal & Tak[4],

$$\rho_0 u_0 \frac{\partial h_1}{\partial x} + \rho_0 u_1 \frac{\partial h_0}{\partial x} + \rho_1 u_0 \frac{\partial h_0}{\partial x} + \rho_0 v_0 \frac{\partial h_1}{\partial y} + \rho_0 v_1 \frac{\partial h_0}{\partial y} + \rho_1 v_0 \frac{\partial h_0}{\partial y} = \frac{1}{Pr} \frac{\partial}{\partial y} \left(\mu \frac{\partial h_1}{\partial y} \right) + \frac{\alpha}{1-\phi} [\rho_{p0} h_{p1} - \rho_{p0} h_1 + \rho_{p1} h_{p0} - \rho_{p1} h_0] + \frac{\alpha}{1-\phi} Ec [2\rho_{p0} (u_0 - u_{p0})(u_1 - u_{p1}) + \rho_{p1} (u_0 - u_{p0})^2] + Ec \mu \left(\frac{\partial u_0}{\partial y} + \frac{\partial u_1}{\partial y} \right)^2 \quad (22)$$

$$u_{p0} \frac{\partial h_{p1}}{\partial x} + u_{p1} \frac{\partial h_{p0}}{\partial x} + v_{p0} \frac{\partial h_{p1}}{\partial y} + v_{p1} \frac{\partial h_{p0}}{\partial y} = \frac{1}{Pr} \frac{\partial}{\partial y} \left(\frac{\partial h_{p1}}{\partial y} \right) - (h_{p1} - h_1) - Ec [2(u_0 - u_{p0})(u_1 - u_{p1})] + Ec \epsilon \frac{\partial}{\partial y} \left[u_{p0} \frac{\partial u_{p1}}{\partial y} + u_{p1} \frac{\partial u_{p0}}{\partial y} \right] \quad (23)$$

Further h_1 , & h_{p1} are the solutions of (22) & (23) subject to the boundary condition.

$$y = 0: v_1 = 0, \frac{\partial u_1}{\partial y} = 0, v_{p1} = 0, \frac{\partial u_{p1}}{\partial y} = 0, \frac{\partial \rho_{p1}}{\partial y} = 0, \frac{\partial h_{p1}}{\partial y} = 0, \frac{\partial h_1}{\partial y} = 0, y = \infty: u_1 = 0, u_{p1} = 0, \rho_{p1} = 0, T_1 = 0, T_{p1} = 0$$

and the following integral condition is identically satisfied

$$\frac{d}{dx} \int_{-\infty}^{\infty} (\rho_0 u_0 h_1 + \rho_0 u_0^2 u_1 + \rho_0 u_1 h_0 + \rho_0 u_1 \frac{u_0^2}{2} - \rho_0 u_1 h_{\infty} + \rho_1 u_0 h_0 + \rho_1 \frac{u_0^3}{2} - \rho_1 u_0 h_{\infty}) dy - \frac{\alpha}{1-\phi} Ec \int_{-\infty}^{\infty} (\rho_{p0} h_{p1} - \rho_{p0} h_1 + \rho_{p1} h_{p0} - \rho_{p1} h_0) dy - \frac{\alpha}{1-\phi} Ec \int_{-\infty}^{\infty} [\rho_{p1} (u_0 - u_{p0})^2 - 2\rho_{p0} (u_0 - u_{p0})(u_1 - u_{p1})] dy = 0$$

The equation (19),(22), & (23) have been solved numerically using non uniform grid for the interpretation of the result.

IV. Discussion Of The Results and conclusion

For numerical computation we have considered $= 0.913 \text{ kg/m}^3$; $\rho_p = 800, 2403, 8010 \text{ kg/m}^3$; $D = 0.5, 50 \mu\text{m}$; $U = 60.96, 160.00, 260.00 \text{ m/sec}$; $Pr = 0.71, 1.0, 7.0$; $\mu = 1.5415 \times 10^{-5} \text{ kg/m sec}$; $\phi = 0.001, 0.0001, 0.00001$; $Ec = 0.1$ to study the velocity field, temperature field, concentration parameter and the influence of the dimensionless parameters entering into the problem. To guarantee the best result the program has been executed for several values of r_y , the grid growth ratio. The computational results for the flow of fluids with SPM are obtained. It is observed that the result obtained for $r_y = 0.52$ is acceptable as r.m.s. error. The velocity,

temperature and particle density depend upon the parameters $\alpha, \epsilon, \phi, Ec$ & Pr . Figures have been presented to shown the influence of these parameters on the above physical quantities. Fig-1: shows the profile of the velocity u_0 , when it is not affected significantly by the presence of the particles. Clearly the velocity at the center of the Jet is maximum and continuously decreases to attain the velocity of the ambient fluid. Fig-2: depicts the temperature profile with and without consideration of viscous heating for different Prandtl's number. It shows that temperature increases with the increase of Prandtl number. The values of Prandtl number Pr are taken as 0.71 and 1.0 which physically corresponds to air and electrolyte solution respectively. Here the heat is diffused away from the center of the Jet more rapidly for higher values of Pr . The maximum temperature is observed at the center of the Jet and asymptotically approaches towards the temperature of the ambient fluid. Fig-3: shows the velocity distribution of the particle phase u_{p0} . Here also the velocity of the particles goes on decreasing along the axis of the Jet. This approaches asymptotically from maximum value at the center of the Jet towards free stream value. It also shows that the spread of the Jet continuously increasing along the axis of the Jet. From Fig-4: it is observed that the velocity of the particles u_{p0} at first increases with the diffusion parameter ϵ and then decreases towards the downstream. Here also the Jet broadens as the diffusion parameter ϵ increases. Fig-5: depicts the perturbation velocity u_{p1} with and without viscous heating. The magnitude of u_{p1} at the center is more in case of viscous heating. Fig-6: shows the dependence of u_{p1} on ϵ . The diffusion parameter which shows that the magnitude of the velocity u_{p1} increases as diffusion of the particles is more. Fig-7: shows the density ρ_{p1} of SPM at different station of the center of the Jet as we go along the x-axis the density of the particles become thinner and thinner as the Jet goes on broadening. Fig-8: shows the temperature h_{p0} at the center of the Jet which goes on decreasing asymptotically towards the downstream station of the Jet-axis. Table-1: it is observed that the density ρ_{p0} at the center of the jet goes on decreasing with x i.e along the Jet axis, this indicating that as the Jet broadens the density distribution becomes thinner and thinner. Table-2: gives the magnitude of u_{p1} at different stations x along the Jet axis which shows that u_{p1} goes on decreasing towards the downstream indicating the settlement of particles. From Table-3 it is observed that there is no change in h_{p1} with or without viscous heating but h_{p1} decreases with the increase of diffusion parameter ϵ . The variation of h_{p1} is Table-4 it is observed that h_1 increases with the increase of Pr with or without viscous heating.

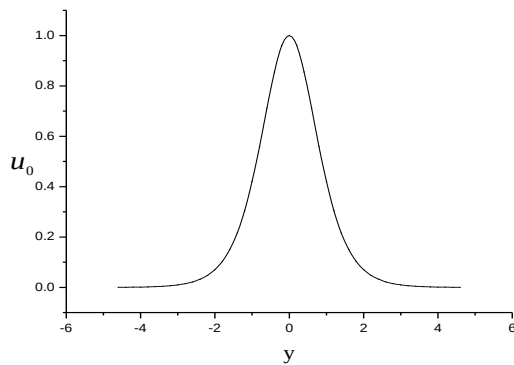


Fig.1: Velocity distribution (u_0) with y

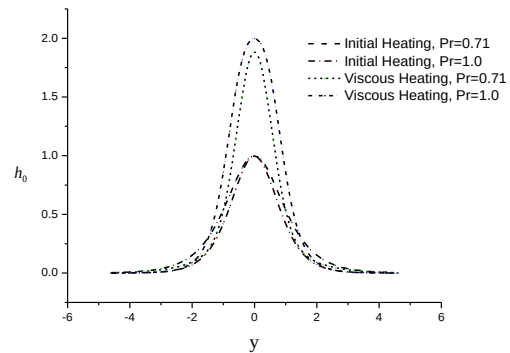


Fig.2: Variation of h_0 with y for different Pr

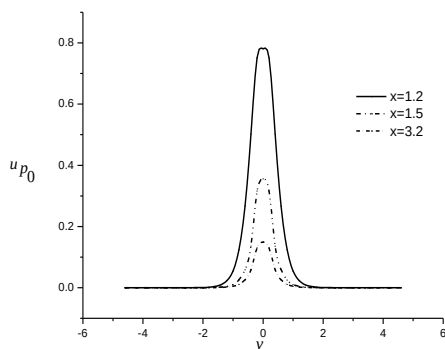


Fig.3: Variation of u_{p0} with y at for different ϵ

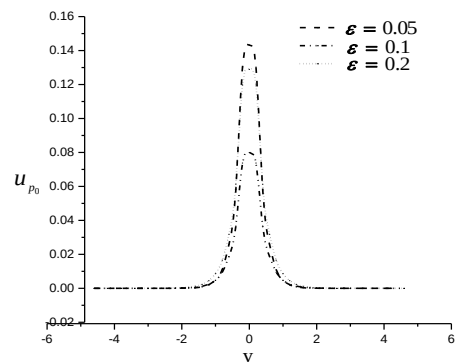


Fig.4: Variation of u_{p0} with y

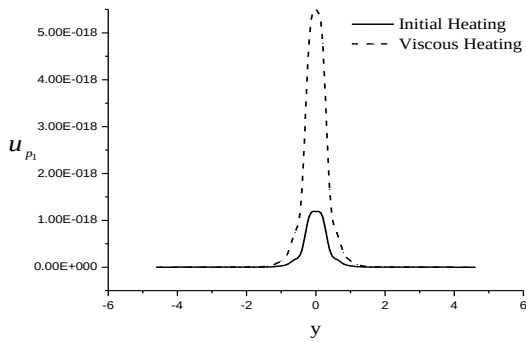


Fig. 5: Variation of u_{p1} with y

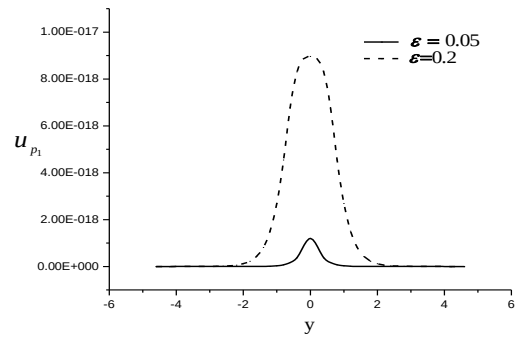


Fig. 6: Variation of u_{p1} with y for different ϵ

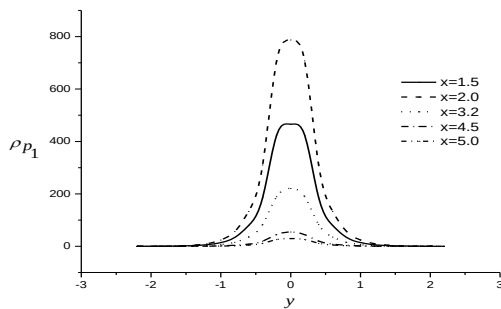


Fig 7: Variation of ρ_{p1} with y at different stations x along the axis

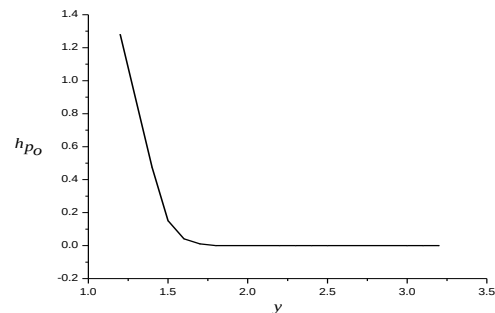


Fig 8: Variation of h_{p0} along the axis of the jet

Table-1: Variation of ρ_{p0} with y for different x

y	$x=1.2$	$x=1.9$	$x=3.2$
-3.2	2.49E-06	8.62E-10	1.82E-13
-2.4	8.64E-05	2.98E-08	6.31E-12
-1.6	2.78E-03	9.61E-07	2.03E-10
-0.8	8.87E-02	3.07E-05	6.50E-09
0	8.41E-01	3.87E-04	8.19E-08
0.8	8.87E-02	3.07E-05	6.50E-09
1.6	2.78E-03	9.61E-07	2.03E-10
2.4	8.64E-05	2.98E-08	6.31E-12
3.2	2.49E-06	8.62E-10	1.82E-13

Table-2: Variation of u_{p1} with y for different x

y	$x=1.3$	$x=2.1$	$x=3.1$
-4.6	0.00E+00	0.00E+00	0.00E+00
-2.8	3.44E-11	1.35E-18	5.28E-23
-1.6	6.34E-09	2.50E-16	9.75E-21
-0.8	2.03E-07	7.96E-15	3.11E-19
0	2.64E-06	1.30E-13	4.78E-18
0.8	2.03E-07	7.96E-15	3.11E-19
1.6	6.34E-09	2.50E-16	9.75E-21
2.8	3.44E-11	1.35E-18	5.28E-23
4.6	0.00E+00	0.00E+00	0.00E+00

Table-3: Variation of h_{p1} with y for different ϵ

y	Initial Heating			Viscous Heating		
	$\epsilon = 0.05$	$\epsilon = 0.1$	$\epsilon = 0.2$	$\epsilon = 0.05$	$\epsilon = 0.1$	$\epsilon = 0.2$
-4.6	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
-2.8	4.81E-05	3.91E-08	9.84E-10	4.81E-05	3.91E-08	9.84E-10
-1.6	4.83E-05	3.92E-08	9.88E-10	4.83E-05	3.92E-08	9.88E-10
-0.8	4.84E-05	3.95E-08	1.01E-09	4.84E-05	3.95E-08	1.01E-09
0	5.00E-05	4.18E-08	1.15E-09	5.00E-05	4.18E-08	1.15E-09
0.8	4.84E-05	3.95E-08	1.01E-09	4.84E-05	3.95E-08	1.01E-09
1.6	4.83E-05	3.92E-08	9.88E-10	4.83E-05	3.92E-08	9.88E-10
2.8	4.81E-05	3.91E-08	9.84E-10	4.81E-05	3.91E-08	9.84E-10
4.6	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

Table-4: Variation of h_1 with y for different Pr

y	Initial Heating			Viscous Heating		
	$Pr = 0.71$	$Pr = 1.0$	$Pr = 7.0$	$Pr = 0.71$	$Pr = 1.0$	$Pr = 7.0$
-4.6	1.00E-05	1.00E-05	1.00E-05	1.00E-05	1.00E-05	1.00E-05
-2.8	8.09E-10	1.24E-10	8.00E-11	8.60E-11	8.92E-11	3.89E-09
-1.6	1.79E-15	2.56E-15	7.34E-15	5.77E-15	6.30E-15	1.03E-12
-0.8	1.45E-13	1.34E-13	6.25E-13	2.38E-13	2.68E-13	1.18E-14
0	1.13E-12	1.17E-12	3.48E-12	2.22E-12	2.42E-12	5.03E-13
0.8	1.45E-13	1.34E-13	6.25E-13	2.38E-13	2.68E-13	1.18E-14
1.6	1.79E-15	2.56E-15	7.34E-15	5.77E-15	6.30E-15	1.03E-12
2.8	8.09E-10	1.24E-10	8.00E-11	8.60E-11	8.92E-11	3.89E-09
4.6	1.00E-05	1.00E-05	1.00E-05	1.00E-05	1.00E-05	1.00E-05

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