



A non- dominated weighted solution for multilevel linear Fractional programming problems

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Abstract : Multi-level programming techniques are developed to solve decentralized planning problems with multiple decision makers in a hierarchical organization. These become more important for contemporary decentralized organizations where each unit or department seeks its own interests. In this paper a compromise weighted solution is presented for solving multi- level linear fractional programming problems , where a non-dominated solution set is obtained. In weighting approach, the relative weights represent the relative importance of the objective functions of each DM. The hierarchical system is converted into Scalar Optimization Problem (SOP) by finding proper weights using the Analytic Hierarchy Process (AHP) so that objective functions can be combined into a single objective. The presented approach achieves an efficient and feasible solution in an appropriate time which has been evaluated by comparing to references and test problems. A numerical example is solved to illustrate the potential use of the approach.

Keywords: Multi level linear fractional programming problems, hierarchical structures; weighting approach, scalar optimization problem, analytic hierarchy process.

I. Introduction

There are many planning and/or decision making situations that can be properly represented by a multi- level programming model. In the real world, we often encounter decision making situations involving multiple decision makers (DMs). Especially in industrial or governmental decision making situations, those DMs have different interest and decision priority. A multi-level programming problem [MLPP] is one of mathematical optimization models for them. Most of the developments in MLP problems focus on bi-level linear programming as a class of MLPP. Bi-level nonlinear or fractional programming was studied in [3,5,9,10,14,16,18,19,26,27,30]. Three-level programming (TLP) is another class of MLP problems in which there are three independent decision-makers (DMs) . Each DM attempts to optimize his objective function and is affected by the actions of the other DMs.

In this paper, we focus on multi-level linear fractional Programming problems (MLLFPPs), in which there exist a DM with linear fractional objective function at each level. MLP problems are characterized that a DM at a certain level of the hierarchy may have his objective function and decision space determined partially by other levels where each DM controls over some decision variables. So, the followers can take part of the system decision which be concerned by their control variables because they always try to optimize their objective functions but they must take the goal or preference of the leader into consideration. DM1 defines his objective function and decision variables, this information then constrains the DM2's feasible space and so on. So, the preference information is delivered from the upper-levels to the lower-levels sequentially.

Various approaches for MLPPs could exist according to situations which the DMs are placed in. Under the assumption that the DMs do not have motivation to cooperate mutually, a Stackelberg solutions is adopted as a reasonable solution for the situation. It is assumed that the DM at the upper level (leader) and the DM at the lower level (follower) completely know their objective functions and the constraints of the problem and they do not have any motivation to cooperate with each other, and the leader first makes a decision and then the follower specifies a decision so as to optimize the objective function of itself with full knowledge of the decision of the leader. Under this assumption, the leader also makes a decision such that the objective function of the leader is optimized. Then, a solution defined as the above mentioned procedure is called a Stackelberg (equilibrium) solutions, which has been employed as a solution concept for two-level mathematical programming problems [7, 16,22, 23].

During the past few decades, many methodological developments have been reported for multi-level programming problem (MLPP). However, these methods are proven to be computationally ineffective and can handle only simple MLLPs. Most of these methods are based on concepts of vertex enumeration and transformation approaches. The former is to seek a compromise vertex by simplex algorithm based on adjusting higher level control variables. It is rather inefficient, especially for large size problems. The later

involves transforming the lower level programming problem to be the constraints of the higher-level by its Kuhn-Thucker (K-T) conditions or penalty functions, because non-linear terms appear in constraints, the auxiliary problems become complex and sometimes unmanageable.

To overcome the shortcomings of the traditional methods, the concept of membership function of fuzzy set theory was incorporated for large and complex hierarchical optimization problems. In a hierarchical decision making context, it has been realized that each DM should have a motivation to cooperate with the other, and a minimum level of satisfaction of the DM at a lower-level must be considered for the overall benefit of the organization. The use of the concept of membership function of fuzzy set theory to MLPPs for satisfactory decisions was first introduced by Lai [17]. Thereafter, Lai's satisfactory solution concept was extended by Shih et al. [29] and a supervised search procedure with the use of max-min operator was proposed. Abo-Sinna [1] extended the fuzzy approach for multi-level programming problems of Shih et al. [29] for solving bi-level and three-level non-linear multi-objective programming problems. The basic concept of these fuzzy programming (FP) approaches is the same as implies that each lower level decision maker optimizes his/her objective function, taking a goal or preference of the first level decision makers into consideration. In the decision process, the membership functions of the fuzzy goals for the decision variables of all the decision makers are taken into consideration and an FP problem is solved with a constraint on an overall satisfactory degree of any upper levels. If the proposed solution is not satisfactory to any upper levels, the solution search is continued by redefining the elicited membership functions until a satisfactory solution is reached.

The main difficulty arises with the fuzzy programming approaches developed so far is that there is a possibility of rejecting the solution again and again by the leader and re-evaluation of the problem is repeatedly needed to reach the satisfactory decision, when the objectives of DMs are over conflicting. Unfortunately, there is a possibility that their method leads to an undesirable final solution because of inconsistency between the fuzzy goals of the objective function and the decision variables. A bibliography of the related references on bi-level and multi-level programming in both linear and non-linear cases, which is updated biannually, can be found in [34].

However, compared to linear MLPPs, only few methodological developments have been reported for non-linear or fractional MLPPs (or MLLFPP).

Fractional programming gains significant stature since many of the real world problems represented as fractional function are often encountered in the following situation: return on investment, current ratio, risk-assets to capital, actual capital to required capital, foreign loans to total loans, residential mortgages to total mortgages, for finance or corporate planning, debt-to-equity ratio etc. for production planning, inventory to sales, actual cost to standard cost, output per employee, and so forth. A cross-section of the hierarchical optimization problems is financial control, economical analysis, government regulation, organizational management, conflict resolution, network design, traffic assignment, planning for resource management, defense, transportation, etc. Large scale optimization problems can be described by static or dynamic systems. This system can be represented by either linear/non linear algebraic equations/ differentials equations.

Fractional programming (FP) is a special case of nonlinear programming. Guzel and Sivri[15] presented Taylor series solution for multi-objective linear fractional programming problem (MOLFPP) in crisp environment. Toksari [33] proposed Taylor series approach to fuzzy MOLFPP. Thirwani and Arora [31] studied linear fractional bilevel programming problem (LFBLPP) in 1993. Thirwani and Arora [30] proposed an algorithm for the integer LFBLPP. Mathur and Puri [19] stated that the optimal solution of the LFBLPP might occur at a non-extreme point. Calvete and Galé [11] showed that the example posed by Mathur and Puri [19] is not well posed because the second level problem has multiple optima. Calvete and Galé [10] developed an enumerative algorithm, which offers a global optimal solution to the bilevel linear/linear fractional programming problem. Arora et al. [5] presented an algorithm for LFBLPP when the follower controls few variables. Malhotra and Arora [18] used goal programming (GP) approach for solving LFBLPP. Alemayehu and Arora [3] discussed integer LFBLPP. Calvete and Galé [8, 9] discussed optimality conditions for LFBLPP. Mishra and Ghosh [21] discussed interactive fuzzy programming (IFP) approach to bi-level quadratic fractional programming problem. Ahlatcioglu and Tiriyaki [2] developed two different IFP algorithms for decentralized LFBLPP based on the technique of MOLFPP due to Chakraborty and Gupta [12] and Charnes and Cooper [13]. Mishra [20] discussed weighting method for LFBLPP by using analytical hierarchy process [28]. Abo-Sinna and Baky [1] discussed fuzzy goal programming procedure to bi-level MOLFPP by using the concept of MOLFPP due to Pal et al. [25].

Toksari [32] developed Taylor series approach for solving LFBLPP due to Guzel and Sivri [15] and Toksari [33]. Toksari [32] obtained the optimal solution which is the individual best solution of the lower level DM. In general, the individual best solution of LDM cannot be acceptable to the UDM. To overcome such problems, we use GP approach to LFBLPP by considering that DMs provide their preference bounds on the decision variables under their control. To compare the efficiency of our proposed approach distance function [30] is used. The performance of the proposed GP approach is experimentally validated by an example.

When the objective functions of each level DM of a MLPP are linear fractional in nature, then the MLPP is called multi-level linear fractional programming problem (MLLFPP). A usual linear fractional programming problem is a special case of a non linear programming problem, but it can be transformed into a linear programming problem

by using the variable transformation method by Charnes and Cooper [13]. Mishra [20] presented weighting approach to solve bi-level linear fractional programming problem. We have extended and modified the weighting method [20,35] for solving MLLFPP in this paper. In Weighting method decision makers (DMs) provide their preference bound to the decision variables that is the upper and lower bounds to the decision variables they control. We convert the hierarchical system into scalar optimization problem (SOP) by finding proper weights using the analytic hierarchy process (AHP) [28,20,35] so that objective functions of both levels can be combined into one objective function. Here the relative weights represent the relative importance of the objective functions. A non-dominated solution set is obtained by this method. Perhaps the most creative task in making a decision is to choose the factors that are important for that decision. In the AHP we arrange these factors, once selected, in a hierarchical structure descending from an overall goal to criteria, sub-criteria and alternatives in successive levels.

II. Multi-level linear fractional programming problems (MLLFPP)

MLLFPP can be defined as a p-person, nonzero sum game with perfect information in which each player moves sequentially from top to bottom and having linear fractional objective functions. Consider a p-level programming problem of maximization-type objective function at each level.

Mathematically, MLLFPP can be formulated as follows:

$$\begin{aligned} \max_{x_1} z_1(x) = \max_{x_1} \text{imize} \quad & z_1(x_1, x_2, \dots, x_p) = \max_{x_1} \frac{a_1 x + r_1}{b_1 x + s_1} \\ \max_{x_2} z_2(x) = \max_{x_2} \text{imize} \quad & z_2(x_1, x_2, \dots, x_p) = \max_{x_2} \frac{a_2 x + r_2}{b_2 x + s_2} \\ & \vdots \\ \max_{x_p} z_p(x) = \max_{x_p} \text{imize} \quad & z_p(x_1, x_2, \dots, x_p) = \max_{x_p} \frac{a_p x + r_p}{b_p x + s_p} \end{aligned} \quad (1)$$

Subject to

$$x \in S = \{A_1 x_1 + A_2 x_2 + \dots + A_p x_p \ (\geq, =, \leq) B, x \geq 0\}.$$

Where,

$$z_i(x_1, x_2, \dots, x_p) = \frac{a_i x + r_i}{b_i x + s_i} = \frac{a_{i1} x_1 + a_{i2} x_2 + \dots + a_{ip} x_p + r_i}{b_{i1} x_1 + b_{i2} x_2 + \dots + b_{ip} x_p + s_i}, \quad (i=1,2,\dots, p)$$

Here, S is the non empty convex constraint set; M is the total number of constraints.

A_i is the $M \times N_i$ matrix ($i=1,2,\dots, p$);

B is M component column vector;

$a_{i1}, b_{i1} \in R^{N_1}, a_{i2}, b_{i2} \in R^{N_2}, \dots, a_{ip}, b_{ip} \in R^{N_p}; r_i, s_i, (i=1,2,\dots, p)$ are scalars.

We take, $x = x_1 \cup x_2 \cup \dots \cup x_p$; $N_1 + N_2 + \dots + N_p = N$.

We also assume that $b_{i1} x_1 + b_{i2} x_2 + \dots + b_{ip} x_p + s_i > 0$, ($i=1,2,\dots, p$) for all $x \in S$.

This system has interacting decision making units within a hierarchical structure where each level performs its policies after knowing completely the decisions of superior levels.

III. Weighting method

Decision-making is the process of selecting a possible course of action from all the available alternatives. Although few optimization tools are available for MLLFPP, the efficiency of these techniques depends to a great extent on the nature of the mathematical formulation of the problem. Some of these traditional techniques, which give accurate results are computationally expansive and become inefficient for a large domain. Weighting method has been widely studied, experimented and applied in many fields in engineering worlds. Not only does Weighting method provide an alternate method to solving problem, it consistently outperforms other traditional methods in the most of the problem link. Weighting method has no special requirement for the characters and differentiability of the function. Perhaps the most creative task in making a decision with the hierarchical decision making with the hierarchical decision making situations is to choose the factors that are important for that decision. The basic idea of assigning weights to the various objective functions, combining these into a single objective function and

parametrically varying the weights to generate the non-dominated set was first proposed by Zadeh in 1963. Mathematically, the weighting method can be stated as follows:

$$\max/\min \quad w_1 z_1(\bar{x}) + w_2 z_2(\bar{x}) + \dots + w_p z_p(\bar{x}) \quad (2)$$

Subject to $\bar{x} \in X$ where X is the feasible region. Thus, a multiple objective problem has been transformed into a single objective optimization problem for which solution methods exist. The coefficient w_p operating on the p^{th} objective function, $z_p(\bar{x})$, is called the weight and can be interpreted as “the relative weight or worth” of that objective function when compared to the other objectives. These weights can be obtained by Analytic Hierarchy Process (AHP) [20,24,28,35]. AHP is a mathematical technique developed for incorporating multi criteria decision making and designed to solve its complex problems. AHP and similar methods often use pairwise comparison matrices for determining the scores of alternatives with respect to a given criterion, or determining values of a weight vector. AHP can be conducted in three steps:

- *perform pairwise comparisons,
- * assess consistency of pairwise judgments,
- * compute the relative weights and then, it enables DM to make pairwise comparisons of importance between objectives according to the scale [24].

Because human is not always consistent, the theory of AHP does not demand perfect consistency and allows some small inconsistency in judgment and provides a measure of inconsistency. Before computing the weights based on pairwise judgments, the degree of inconsistency is measured by the Consistency Index (CI)[24]. Perfect consistency implies a value of zero for CI.

If the weights of the various objectives are interpreted as the representing the relative preference of some DM, then the solution to (2) is equivalent to the best compromise solution, i.e., the optimal solution relative to a particular preference structure. Moreover, the optimal solution to (2) is a non-dominated solution provided all the weights are positive. Allowing negative weights would be equivalent to transforming the maximizing problem to a minimizing one, for which a different set of non-dominated solutions will exist. The trivial case where all the weights are zero will simply identify $\bar{x} \in X$ as an optimal solution and will not distinguished between dominated and non-dominated solutions. Thomas Saaty [28], developed the AHP, it becomes a useful tool for estimating judgment elements by quantifying subjective decisions. It is used to derive relative weights of decision elements and then synthesize them to obtain the corresponding weights for alternatives and criteria. It is a building block for decision making.

IV. Method for generating non-dominated solutions

A preferred (best) solution is a non-dominated solution which is chosen by the DM his self that is lies in the region of acceptance of all DMs. Non-dominated solution is to design the best alternative by considering the various interactions within the design constraints that best satisfy the DM by way of obtaining some acceptable levels of quantifiable objective functions. This method be distinguished with; a set of quantifiable objective functions, a set of well-defined constraints and a process of obtaining some trade-off information, between the stated quantifiable objective functions. The most common strategy for finding non-dominated solutions of MLP problems is to convert it into a SOP. DMs provide their preference and converting MLP problem into a SOP by finding vector of weights for objectives.

A non-dominated solution is one in which no one objective function can be improved without a simultaneous detriment to at least one of the other objectives of the vector maximum problem (VMP). A given multiple objective mathematical problem which contains only maximization type objective functions is called the VMP. A feasible solution $\bar{x}^* \in X$ (decision space) is a non-dominated solution to the VMP iff there does not exist any other feasible solution $\bar{x} \in X$ such that $z_p(\bar{x}^*) \leq z_p(\bar{x})$, $p = 1, 2, \dots, P$ and $z_p(\bar{x}^*) < z_p(\bar{x})$, for at least one p

A non-dominated solution is one for which there is no other solution giving equal or greater values of each and every objective function. But in even the smallest problem, the number of non-dominated solutions generated may be infinite. This is because all points on the line joining two non-dominated and extreme points are themselves non-dominated. A preferred solution is a non-dominated solution which is chosen by the DM, through some additional criteria, as the final decision. As such it lies in the region of acceptance of all the criteria values for the problem. A preferred solution is also known as the ‘best’ solution.

The most common strategy for finding non-dominated solutions of a multi-level problem is to convert it into a scalar optimization problem (SOP). This class of method does not require any assumption or information regarding the DMs utility function.

V. Weighting method for multi-level linear fractional programming problems

In the weighting problem $\bar{p}(w)$ in the absence explicit preference structure, the strategy is to generate all or representative subsets of non-dominated solutions from which a DM can select the suitable solution.

Let a linear fractional MLPP be represented as:

$$\begin{aligned} \max_{x_1} \quad & z_1(\bar{x}) \\ \max_{x_2} \quad & z_2(\bar{x}) \\ & \cdot \\ & \cdot \\ \max_{x_p} \quad & z_p(\bar{x}) \end{aligned} \tag{3}$$

subject to

$$x \in S = \{A_1 x_1 + A_2 x_2 + \dots + A_p x_p (\geq, =, \leq) B, x \geq 0\}$$

$$\bar{x}_1 \geq 0, \bar{x}_2 \geq 0, \dots, x_p(\bar{x}).$$

Where, $z_1(\bar{x}), z_2(\bar{x}), \dots, z_p(\bar{x})$, and $A_i x_i (\geq, =, \leq) B$ are linear fractional objective functions and linear or non linear constraints respectively. $\bar{x}_1 \geq 0, \bar{x}_2 \geq 0, \dots, x_p(\bar{x})$ are decision vectors under the control of the first level, second and so on p-th level DM.

$$\text{Let } X = \text{set of feasible solutions} = \{ \bar{x} : \bar{x} \in R^n, A_i x_i (\geq, =, \leq) B \},$$

$$\bar{x} = \text{decision vector in n-dimensional Euclidean space} = x = x_1 \cup x_2 \cup \dots \cup x_p.$$

Solving the SOP involves finding $\bar{x}^* \in X$ such that $z(\bar{x}^*) \geq z(\bar{x}) \quad \forall \quad \bar{x} \in X$. The point $\bar{x}^*$ is said to be global optimum. If strict inequality holds for the objective functions, then $\bar{x}^*$ is the unique global optimum. If the inequality holds for some neighbourhood of $\bar{x}^*$, then $\bar{x}^*$ is a local or relative optimum while it is strict local optimum if strict inequality holds in a neighbourhood of $\bar{x}^*$.

By using the AHP pairwise comparison process, weights or priorities are derived from a set of judgments. While it is difficult to justify weights that are arbitrarily assigned, it is relatively easy to justify judgments and the basis (hard data, knowledge, experience) for the judgments. Suppose already the relative weights of p-objective functions are known, for p-level hierarchical objective functions a complete pairwise comparison matrix A can be expressed as; $A = [a_{ij}]$, $i, j = 1, 2, \dots, p$ is a matrix of size $p \times p$ with the following properties;

$a_{ij} > 0, a_{ii} = 1$, and $a_{ij} = 1/a_{ji}, i, j = 1, 2, \dots, p$ where a_{ij} is the numerical answer given by the each DM for the question "How many times objective i is more important than objective j?"

$$A = \begin{matrix} & \begin{matrix} z_1 & z_2 & \dots & \dots & \dots & z_p \end{matrix} \\ \begin{matrix} z_1 \\ z_2 \\ \cdot \\ \cdot \\ \cdot \\ z_p \end{matrix} & \begin{bmatrix} 1 & a_{12} & a_{13} & \dots & a_{1p} \\ a_{21} & 1 & a_{23} & \dots & a_{2p} \\ a_{31} & a_{32} & 1 & \dots & a_{3p} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ a_{p1} & a_{p2} & a_{p3} & \dots & 1 \end{bmatrix} \end{matrix}$$

After the normalized matrix, N of pairwise comparison matrix A for a hierarchical p-level structure is designed, the normalized principal eigen vector (priority vector) can be obtained by some ways such as averaging across the rows where, it shows the relative weights for objectives. The weighting problem is to find the p-dimensional weight vector $W = (w_1, w_2, \dots, w_p)$ such that the appropriate ratios of the components of W reflect or, at least, approximate all the a_{ij} values ($i, j = 1, 2, \dots, p$) given by DMs.

Then, the weighting problem for multi-level (p-level) linear fractional programming problem is formulated as follows:

$$P(\bar{w}) = \max_{\bar{x} \in X} \sum_{n=1}^p \bar{w}_n z_n(\bar{x})$$

subject to

$$g_i(x) \leq b_i, \quad i = 1, 2, \dots, m$$

$$\bar{L}_1 \leq \bar{x}_1 \leq \bar{U}_1$$

$$\bar{L}_2 \leq \bar{x}_2 \leq \bar{U}_2 \quad (4)$$

$$\bar{L}_p \leq \bar{x}_p \leq \bar{U}_p$$

Where, $\bar{w} \in W = \{ \bar{w} : \bar{w} \in R^p, w_n \geq 0, n = 1, 2, \dots, p. \text{ and } \sum_{n=1}^p \bar{w}_n = 1 \}$; and

$$z_n(x_1, x_2, \dots, x_p) = \frac{a_n x + r_n}{b_n x + s_n} = \frac{a_{n1}x_1 + a_{n2}x_2 + \dots + a_{np}x_p + r_n}{b_{n1}x_1 + b_{n2}x_2 + \dots + b_{np}x_p + s_n}$$

$\bar{U}_p$ and $\bar{L}_p$ are the upper and lower bounds of decision vector provided by the respective DM. Finally the linear fractional programming problem (4), with a single objective function is solved. Here the weighting coefficients convey the importance attached to an objective function. Suppose that the relative importance of the both objective functions is known and is constant. Then the preferred solution is obtained by solving $p(\bar{w})$ where

$\bar{w}_p, s \geq 0$ are the weighting coefficients. The $\bar{w}_p$'s are normalized since, $\sum_{n=1}^p \bar{w}_n = 1$.

This method can be used to generate non-dominated solutions by utilizing various values of $\bar{w}$. In such a case the weighting coefficients $\bar{w}$ do not reflect the relative importance of the objective functions in the proportional sense, but are only parameters varied to locate the non-dominated points.

VI. Discussion

MLP problems are characterized that a DM at a certain level of the hierarchy may have his objective function and decision space determined partially by other levels where each DM controls over some decision variables. So, the followers can take part of the system decision which be concerned by their control variables because they always try to optimize their objective functions but they must take the goal or preference of the leader into consideration. DM1 defines his objective function and decision variables, this information then constrains the DM2's feasible space and so on. So, the preference information is delivered from the upper-levels to the lower-levels sequentially. This is in accordance with a hierarchical system where the upper level DM is the main decision maker. Each unit independently maximizes or minimizes its own benefits, but is affected by the action of other units through externalities. Compared with the traditional methods, the method has the following merits:

1. Proposed approach does not require any assumption or information regarding the DMs utility function. The method has no special requirement for the characters of the function and overcome the difficulty discussing the conditions and the algorithms of the optimal solution with the definition of the differentiability of the function.
2. This method can be used to generate non-dominated solutions by utilizing various values of $\bar{w}$. AHP gives the relative weights to form a super objective function.
3. This paper demonstrates the merit of this technique in deciding optimal solution of multi-level linear fractional decision-making problem taking into consideration the various constraints and complexities representing the real situation.
4. The main advantage of the proposed weighting approach is that the possibility of rejecting the solution again and again by the fuzzy approach and re-evaluation of the problem repeatedly, by redefining the elicited membership functions, needed to reach to the satisfactory decision does not arise.

VII. The Numerical Results

In this section we present numerical example to demonstrate the solution procedures by proposed weighting method to solve MLLFPP. The following example considered by Dey and Pramanik et al.[14] is again used to demonstrate the solution procedures and clarify the effectiveness of the proposed approach. Consider the following Tri-Level Linear Fractional Programming Problem (TLLFPP)[14]:

$$\begin{aligned} \max_{x_1} z_1(x) &= \frac{2x_1 + x_2 + 3x_3 + 3}{x_1 + x_2 + x_3} && \text{(First upper level)} \\ \max_{x_2} z_2(x) &= \frac{x_1 + 4x_2 - 2x_3 + 1}{2x_1 + 2x_2 + x_3 + 3} && \text{(Second upper level)} \\ \max_{x_3} z_3(x) &= \frac{3x_1 + x_2 - x_3 + 2}{4x_1 + 3x_2 - x_3} && \text{(Loewr level)} \end{aligned}$$

subject to

$$2x_1 + x_2 + x_3 \leq 7; \quad x_1 - 2x_2 + 3x_3 \leq 4; \quad -x_1 + 2x_2 + 2x_3 \geq 1; \quad x_1 + 2x_2 \geq 3; \quad x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0$$

Solution: Let the bounds provided by the respective DMs be $0 \leq x_1 \leq 35$, $0 \leq x_2 \leq 20$, and $0 \leq x_3 \leq 30$. The pair wise comparison matrix $\tilde{A}$ of order 3 and its normalized matrix $\tilde{N}$ [24,35] for the hierarchical objective functions are given as:

$$\tilde{A} = \begin{bmatrix} z_1 & z_2 & z_3 \\ 1/2 & 1 & 3 \\ 1/5 & 1/3 & 1 \end{bmatrix}; \quad \tilde{N} = \begin{bmatrix} 1/1.7 & 2/3.33 & 5/9 \\ 0.5/1.7 & 1/3.33 & 3/9 \\ 0.2/1.7 & 0.333/3.33 & 1/9 \end{bmatrix} = \begin{bmatrix} 0.588 & 0.6 & 0.556 \\ 0.294 & 0.3 & 0.333 \\ 0.118 & 0.1 & 0.111 \end{bmatrix}$$

Thus the normalized weights are

$$\bar{w}_1 = \frac{(0.588 + 0.6 + 0.556)}{3} = 0.581,$$

$$\bar{w}_2 = 0.309 \quad \text{and} \quad \bar{w}_3 = 0.110 \text{ as:}$$

$$\tilde{A}\bar{w} = \begin{bmatrix} 1 & 2 & 5 \\ 1/5 & 1 & 3 \\ 1/5 & 1/3 & 1 \end{bmatrix} \begin{bmatrix} 0.581 \\ 0.309 \\ 0.110 \end{bmatrix} = \begin{bmatrix} 1.749 \\ 0.930 \\ 0.329 \end{bmatrix} \quad P_{\max} = 1.749 + 0.930 + 0.329 = 3.008 \quad \text{and} \quad P = 0.3$$

Thus, Consistency Index (CI) = $(3.008 - 3)/3 - 1 = 0.008/2 = 0.004$;

Random Index (RI) = $1.98(3-2)/3 = 0.66$ which gives

$$\text{Consistency Ratio (CR)} = \text{CI/RI} = 0.004/0.66 = 0.006.$$

The weighting problem is therefore formulated as:

$$P(\bar{w}) = \max_x \left[0.581 \frac{2x_1 + x_2 + 3x_3 + 3}{x_1 + x_2 + x_3} + 0.309 \frac{x_1 + 4x_2 - 2x_3 + 1}{2x_1 + 2x_2 + x_3 + 3} + 0.110 \frac{3x_1 + x_2 - x_3 + 2}{4x_1 + 3x_2 - x_3} \right]$$

$$\text{subject to } 2x_1 + x_2 + x_3 \leq 7; \quad x_1 - 2x_2 + 3x_3 \leq 4; \quad -x_1 + 2x_2 + 2x_3 \geq 1; \quad x_1 + 2x_2 \geq 3; \quad 0 \leq x_1 \leq 35, \\ 1 \leq x_2 \leq 20, \quad 0 \leq x_3 \leq 30, \quad x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$$

The non-dominated solution set is generated by parametrically varying the weights and is tabulated below:

$\bar{w}_1, \bar{w}_2, \bar{w}_3$	x_1, x_2, x_3	$P(\bar{w})$
0.1,0.4,0.5	2.60,1.23,0.57	1.4
0.1,0.5,0.4	2.60,1.23,0.57	1.4
0.2,0.3,0.5	2.60,1.23,0.57	1.4
0.3,0.4,0.3	2.60,1.23,0.57	1.4
0.4,0.4,0.2	2.60,1.23,0.57	1.4
0.5,0.3,0.2	2.60,1.23,0.57	1.4
0.581,0.309,0.110	2.60,1.23,0.57	1.4
0.6,0.3,0.1	2.60,1.23,0.57	1.4
0.7,0.2,0.1	2.60,1.23,0.57	1.4
0.9,0,0.1	2.60,1.23,0.57	1.4
0.9,0.1,0	2.60,1.23,0.57	1.4

Solution of the above problem obtained Dey and Pramanik et al.[14] using fuzzy mathematical programming is:

Approach	Decision variables			Objective values		
FGP model(1)	$x_1 = 0$	$x_2 = 2$	$x_3 = 0$	$z_1 = 2.5$	$z_2 = 1.286$	$z_3 = 0.667$
FGP model(2)	$x_1 = 0$	$x_2 = 2$	$x_3 = 0$	$z_1 = 2.5$	$z_2 = 1.286$	$z_3 = 0.667$

The proposed weighting approach gives the solution as:

Approach	Decision variables			Objective values		
Non-dominated Weighted Solution Approach	$x_1 = 2.60$	$x_2 = 1.23$	$x_3 = 0.57$	$z_1 = 2.53$	$z_2 = 0.78$	$z_3 = 0.77$

Which is very close or improved to the results obtained by the existing methods Dey and Pramanik et al[14]. The solution by this approach is more reasonable and modified as each control variable has some non-zero value.

VIII. Conclusion

Solving MLLFP problems is, generally, a very difficult goal. In these particular optimization problems, the objectives often conflict across a high-dimensional problem space and may also require extensive computational resources. In general, MLLFP is a non-convex optimization problem; therefore, there is no general algorithm to solve it. This problem can be non-convex even when all functions and constraints are bounded and continuous. In this paper we consider the solution of a multi-level linear fractional programming problems by Weighting method. The Weighting method determines a subset of the complete set of non-dominated solutions. From this subset the DM chooses the most satisfying solution, making implicit trade-offs between objective functions based on some previously un-indicated or non-quantifiable criteria. We also observe that even though we vary the weight vector, the solution remains more or less the same. Thus the non-dominated solution set reduces to a point (almost). The numerical result shows the proposed algorithm is feasible and efficient, can find optimal solutions with less computational burden.

References

- [1] M. A. Abo-Sinna and I.A.Baky, Fuzzy goal programming procedure to bilevel multiobjective linear fraction programming problems, *International Journal of Mathematics and Mathematical Sciences*, (2010), 01-15, ID 148975 (2010) 01-15 doi:10.1155/2010/48975.
- [2] M. Ahlatcioglu and F. Tiryaki, Interactive fuzzy programming for decentralized two-level linear programming (DTLLFP) problems, *Omega*, 35 (4) (2007), 432-450.
- [3] G. Alemayehu and S.R Arora, On the bilevel integer linear fractional programming problem, *Journal of the Operational Research Society of India (OPSEARCH)*, 38 (5) (2001), 508-519.
- [4] G. Anandalingam, A mathematical programming model of decentralized multi-level systems, *Journal of the Operational Research Society*, 39 (11) (1988), 1021-1033.
- [5] S.R. Arora, N. Malhotra and D. Thirwani, An algorithm for bilevel fractional program when the follower controls few variables, *Indian Journal of Pure and Applied Mathematics*, 33 (2) (2002), 227-235.
- [6] M.A.K. Azad M. Sakawa, K. Kato and H. Katagiri, interactive fuzzy programming for multi-level nonlinear integer programming problems through genetic algorithms, *Scientiae Mathematicae Japonicae Online*, e-2006, 223-236.
- [7] F.W Bialas and M. H. Karwan, Two-Level Linear Programming, *Management Science*, vol. 30, no. 8, pp. 1004-1020, 1984.
- [8] H.I Calvete and C. Galé., Optimality conditions for the linear fractional/quadratic bilevel problem, *Monografias del Seminario Matematico Garcia de Galdeano*, 31 (2004), 285-294.
- [9] H. I. Calvete and C. Galé, Solving linear fractional bilevel programs, *Operations Research Letters*, 32 (2) (2004), 143-151.
- [10] H. I. Calvete and C. Galé, The bi-level linear/linear fractional programming problem, *European Journal of Operational Research*, 114 (1) (1999), 188-197.
- [11] H. I. Calvete and C. Galé, On the quasiconcave bilevel programming problem, *Journal of Optimization Theory and Applications*, 98 (3) (1998), 613-622.
- [12] M. Chakraborty and S. Gupta, Fuzzy mathematical programming for multi objective linear fractional programming problem, *Fuzzy Sets and Systems*, 125 (3) (2002), 335-342.
- [13] A. Charnes and W.W. Cooper, Programming with linear fractional functions, *Naval Research Logistics Quarterly*, 9 (3-4) (1962), 181-186.
- [14] Partha Pratim Dey and Surapati Pramanik and C. Bibhas, Multi-Level Linear Fractional Programming Problem Based on Fuzzy Goal Programming Approach, *International Journal of Innovative Research in Technology and Science(IJIRTS)*,4(2),2014, 17-26.
- [15] N. Guzel and M. Sivri, Taylor series solution of multiobjective linear fractional programming problem, *Trakya University Journal Science*, 6 (2) (2005), 80-87.
- [16] R. Jan. and M. Chern, Nonlinear Integer Bilevel Programming, *European Journal of Operational Research*, vol. 72, pp. 574-587, 1994.
- [17] Y.J. Lai, Hierarchical optimization. A satisfactory solution, *Fuzzy Sets Syst.* 77 (1996) 321-335.
- [18] N. Malhotra. and S.R. Arora, An algorithm to solve linear fractional bilevel programming problem via goal programming, *Journal of the Operational Research Society of India(OPSEARCH)*, 37 (1) (2000), 01-13.
- [19] K. Mathur and M.C. Puri, On bilevel fractional programming, *Optimization*, 35 (3) (1995), 215-226.
- [20] S. Mishra, Weighting method for bi-level linear fractional programming problems, *European Journal of Operational Research*, 183 (1) (2007), 296-302.
- [21] S. Mishra and A. Ghosh, Interactive fuzzy programming approach to bi-level quadratic fractional programming problems, *Annals of Operational Research*, 143 (1) (2006), 249-261.
- [22] I. Nishizaki, M. Sakawa and K. Kato, Computational methods through Genetic Algorithms for obtaining Stackelberg solutions to Two-level Zero-One Programming Problems, *Proceedings of 2000 IEEE International Conference on Industrial Electronics, Control and Instrumentation*, pp. 2750-2755, 2000.
- [23] I. Nishizaki and M. Sakawa., Computational Methods through Genetic Algorithms for obtaining Stackelberg solutions to Two-Level Mixed Zero-One Programming Problems, *Cybernetics and Systems: An International Journal*, Vol. 31, pp. 203-221, 2000.
- [24] M.S. Osman et al, A Compromise Weighted Solution For Multilevel Lineart Programming Problems, *Int. Journal of Engineering Research and Applications*, ISSN : 2248-9622, Vol. 3, Issue 6, Nov-Dec 2013, pp.927-936.
- [25] B.B. Pal, B.N. Moitra and U. Maulik, A goal programming procedure for fuzzy multiobjective linear fractional programming, *Fuzzy Sets and Systems*, 139 (2) (2003), 395-405.

- [26] M. Sakawa and I. Nishizaki, Interactive fuzzy programming for two-level linear fractional programming problem, *Fuzzy Sets and Systems*, 119 (1) (2001), 31-40.
- [27] M. Sakawa and I. Nishizaki, Interactive fuzzy programming for cooperative two-level linear fractional programming problems with multiple decision makers, *International Journal of Fuzzy Systems*, 1 (1) (1999), 48-59.
- [28] T.L. Satty, The Analytical Hierarchy Process, Plenum Press, New York, 1980.
- [29] H.S. Shih, Y.J. Lai, E.S. Lee, Fuzzy approach for multi-level programming problems, *Comput. Operat. Res.* 23 (1) (1996) 73–91.
- [30] D. Thirwani and S.R. Arora, An algorithm for the integer linear fractional bilevel programming problem, *optimization*, 39 (1) (1997) 53-67.
- [31] D. Thirwani and S.R. Arora, Bi-level linear fractional programming problem, *Cahiers Du Cero*, 35 (1-2) (1993), 135-149.
- [32] M.D. Toksarı, Taylor series approach for bi-level linear fractional programming problem, *Selçuk Journal of Applied Mathematics*, 11 (1) (2010), 63-69.
- [33] M.D. Toksarı, Taylor series approach to fuzzy multiobjective linear fractional programming, *Information Sciences*, 178 (4) (2008), 1189-1204.
- [34] L.N. Vicente and P.H. Calamai, Bilevel and multilevel programming: a bibliography review, *Journal of Global Optimization*, 5 (3) (2004), 291-306.
- [35] Mishra Savita, Verma Arun Bihari, American International Journal of Research in Science, Technology, Engineering & Mathematics, 11(2) (2015), 125-131.