



Pressure Analysis for the System of Rotating Discs under the Effects of Second Order Rotation

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Abstract: The lubrication of discs can be made kinematically equivalent to gears if they have the same radius at their contact line and rotate at the same angular velocities as the gears. For the system of discs, we will take the origin at the surface of disc of radius R on the line of centers of the two discs. In the present paper, there is analysis about the pressure for the system of lubricating discs in the view of rotatory theory of hydrodynamic lubrication. Some new excellent results have found with the help of geometrical figures, expression for pressure, calculated tables and graphs for the systems of lubricating discs. The analysis of equations for pressure tables and graphs reveal that pressure is not independent of viscosity and increases slightly with viscosity. Also the pressure increases with increasing values of rotation number.

Key words: Film thickness, Reynolds equation, Rotation number, System of discs, Viscosity.

I. Introduction

In theory of hydrodynamic lubrication, two dimensional classical theories [3], [7] were first given by Osborne Reynolds [8]. In 1886, in the wake of the classical experiment by Beauchamp Tower [9], he had given an important differential equation, which was known as: Reynolds Equation [8]. The construction and basic mechanism of the fluid film was observed by that experiment on taking assumptions that fluid film thickness is much small as compare to axial and longitudinal dimensions of the fluid film and if the lubricant layer is to transmit pressure between shaft and bearing, the layer must have varying thickness. Later Osborne Reynolds himself derived the improved version of the Reynolds Equation known as: "Generalized Reynolds Equation" [5], [7], [12], [13], [14] that depends on viscosity, density, film thickness, transverse and surface velocities. The rotation [1] of the fluid film about the axis that lies across film gives some excellent results in the lubrication problems of fluid mechanics. The origin of the rotation can be traced by the certain general theorems based on vorticity in rotating fluid dynamics. The rotation induces the component of vorticity in direction of rotation for fluid film and effects arising from it are predominant, for the large Taylor's Number, it results in streamlines becoming confined to the plane transverse to direction of the rotation of film.

For fully hydrodynamic or "full-film" [5], [8] lubrication, the moving surface of journal is completely separated with the bearing surface by the extremely thin film of lubricant. The applied load creates the centerline of journal to be displaced from centerline of bearing. This eccentricity creates the circular "wedge" in clearance space. The lubricant, by virtue of the viscosity, clings to surface of rotating journal and is drawn into wedge, creating very high pressure that acts to separate the journal from bearing to support applied load.

The bearing eccentricity is written as the centerline displacement divided by radial clearance. The bearing eccentricity increases with the applied load and decreases with the greater journal speed and the viscosity. Thus the hydrodynamic pressure has no relationship at all to engine oil pressure, except that if there is insufficient engine oil pressure to deliver required copious volume of the oil into bearing, the hydrodynamic pressure mechanism will fail and bearing, journal will be destroyed. The pressure distribution in hydrodynamic region of the fluid film bearing increases from quite low in large clearance zone to its maximum at point of the minimum film thickness for incompressible fluid like oil is pulled into converging "wedge" [3], [4] zone of bearing. So, this radial profile does not occur homogeneously across the axial length of bearing. If bearing has the sufficient width then the profile will have nearly flat shape across high pressure region.

If there is adequate supply of the lubricant, the motion of journal starts to drag lubricant into wedge area and the hydrodynamic lubrication starts to occur with the boundary lubrication. If we suppose that viscosity and load remain relatively constant during this initial period, then the revolution per minute increases, hydrodynamic operation strengthens until it is completely developed and it moves journal into the steady state orientation. The direction of eccentricity and minimum film thickness, do not exist in the line with load vector and are angularly

displaced from load. There are also some other position of fluid film lubrication that includes the squeeze film lubrication [7] i.e., piston engine etc. The figure-1.1 shows hydrodynamic journal bearing and journal is rotating. The rotation of journal creates pumping of lubricant that flows around bearing in direction of rotation. If there is no applied force to journal then its position remains unchanged and concentric to bearing position. The loaded journal rotates from the concentric position and originates the converging gap between the bearing and journal surfaces. Now the movement of the journal forced lubricant to squeeze through gap that generates the pressure. The pressure falls to cavitations pressure [7] i.e., closer to atmospheric pressure in gap in that the cavitations form.

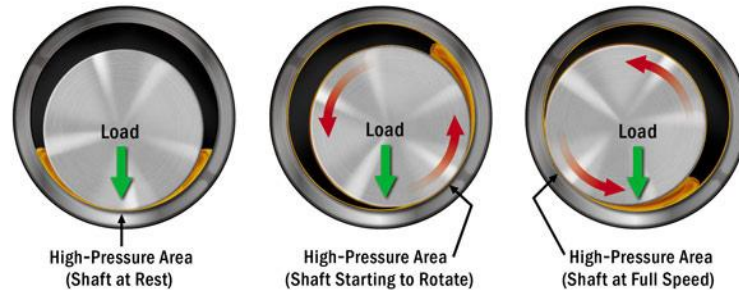


Figure-1.1 (Journal Bearing Rotation [10])

The new extended version of “Generalized Reynolds Equation” [5], [7] is said to be “Extended Generalized Reynolds Equation” [1], [2], which takes into account of the effects of the uniform rotation about the axis, which lies across fluid film and also depends on rotation number M [1], i.e. the square root of Taylor’s Number. The general form of the old theory of hydrodynamic lubrication is known as the “Rotatory Theory of Hydrodynamic Lubrication” [1], [2]. The “First Order Rotatory Theory of Hydrodynamic Lubrication” and the “Second Order Rotatory Theory of Hydrodynamic Lubrication” [2], [6] had given by maintaining the terms in the equation upto M and M^2 respectively by neglecting the terms of higher powers of M .

The lubrication of discs can be made kinematically equivalent to gears if they have the same radius at their contact line and rotate at the same angular velocities as the gears. For the system of discs, we will suppose the origin at surface of the disc of radius R on the line of centers of the two discs. The geometry of the system of discs is given by the figure (1.2) and figure (1.3).

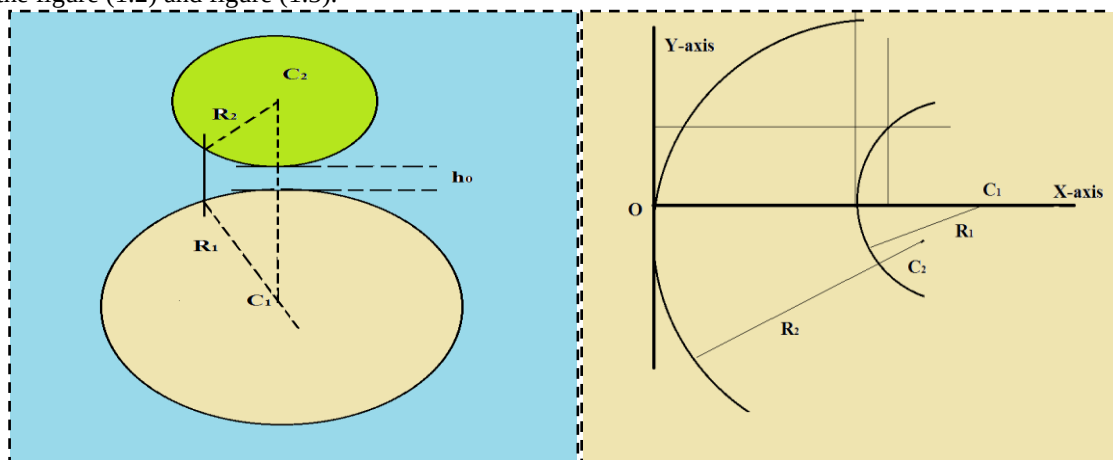


Figure-1.2 (Geometry of system of discs)

Figure-1.3 (Film thickness of system of discs)

The film thickness ‘ h ’ is given by

$$h = h_0 \left[1 + \frac{y^2}{2h_0} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \right], \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{1}{R} \quad (1.1)$$

$$\tan \theta = \frac{y}{\sqrt{2Rh_0}}, h = h_0 \sec^2 \theta \quad (1.2)$$

Let us suppose that the disc is stationary at the lower surface transverse to the fluid film where sliding is absent and $U=+U$ (constant). Suppose the variation of pressure in x -direction is much low as compared to variation of pressure in y -direction. Then the terms containing pressure gradient $\partial p / \partial x$ can be neglected as compared to the terms containing $\partial p / \partial y$ in the differential equation for pressure, so that P may be taken as function of y only.

II. Governing Equations and Boundary Conditions

In the second order rotatory theory of hydrodynamic lubrication the “Extended Generalized Reynolds Equation” [7, 11] with the above assumption reduces to equation (2.1).

$$\begin{aligned} \frac{\partial}{\partial y} \left[-\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \rho \frac{\partial P}{\partial y} \right] + \frac{\partial}{\partial x} \left[-\frac{M\rho^2h^5}{120\mu^2} \left(1 - \frac{31M^2\rho^2h^4}{3024\mu^2} \right) \frac{\partial P}{\partial y} \right] \\ = -\frac{\partial}{\partial x} \left[\frac{\rho U}{2} \left\{ h - \frac{M^2\rho^2h^5}{120\mu^2} \left(1 - \frac{31M^2\rho^2h^4}{3024\mu^2} \right) \right\} \right] - \frac{\partial}{\partial y} \left[\frac{M\rho^2U}{2} \left\{ -\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \right\} \right] \end{aligned} \quad (2.1)$$

Taking $h=h(y)$, $U=+U$, $P=P(y)$

$$\frac{d}{dy} \left[-\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \rho \frac{dP}{dy} \right] = -\frac{\partial}{\partial y} \left[\frac{M\rho^2U}{2} \left\{ -\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \right\} \right] \quad (2.2)$$

For the determination of pressure distribution in the positive regions, the boundary conditions are as follows:

(i) $P=0$ at $h=h_0$ or $P=0$ at $y=0$ or $P=0$ at $\theta=0$ (ii) $P=dP/d\theta=0$ at $y=y_1$ or $\theta=\gamma$ (say) (2.4)

Where is determined by putting $\theta=\gamma$ and $P=0$ in the equation of pressure.

III. Determination of Pressure

The solution of the differential equation under the boundary conditions imposed is given by

$$P = -\sqrt{\frac{Rh_0}{2}} M\rho U \left[\frac{17M^2\rho^2h_0^4}{1680\mu^2} F(\theta) - \tan \theta F(\gamma) \right] \quad (3.1)$$

Where $F(\theta)$ is given by

$$F(\theta) = \tan \theta \left[\frac{1}{9} \sec^8 \theta + \frac{8}{63} \sec^6 \theta + \frac{48}{315} \sec^4 \theta + \frac{192}{945} \sec^2 \theta + \frac{384}{945} \right] \quad (3.2)$$

IV. Calculation tables and graphs

By taking the values of different mathematical terms in **C.G.S.** system the calculated tables and graphical representations are as follows:

$U = 80$, $\rho = 1.0$, $R = 3.35$, $h_0 = 0.0167$, $\mu = 0.0002$, $\theta = 30^\circ$, $\gamma = 60^\circ$

M	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
P	93.22554 478	186.4498 789	279.6717 919	6145.652 604	466.1035 115	559.310 897	652.5110 187	745.7026 661	838.8846 285	932.0556 954

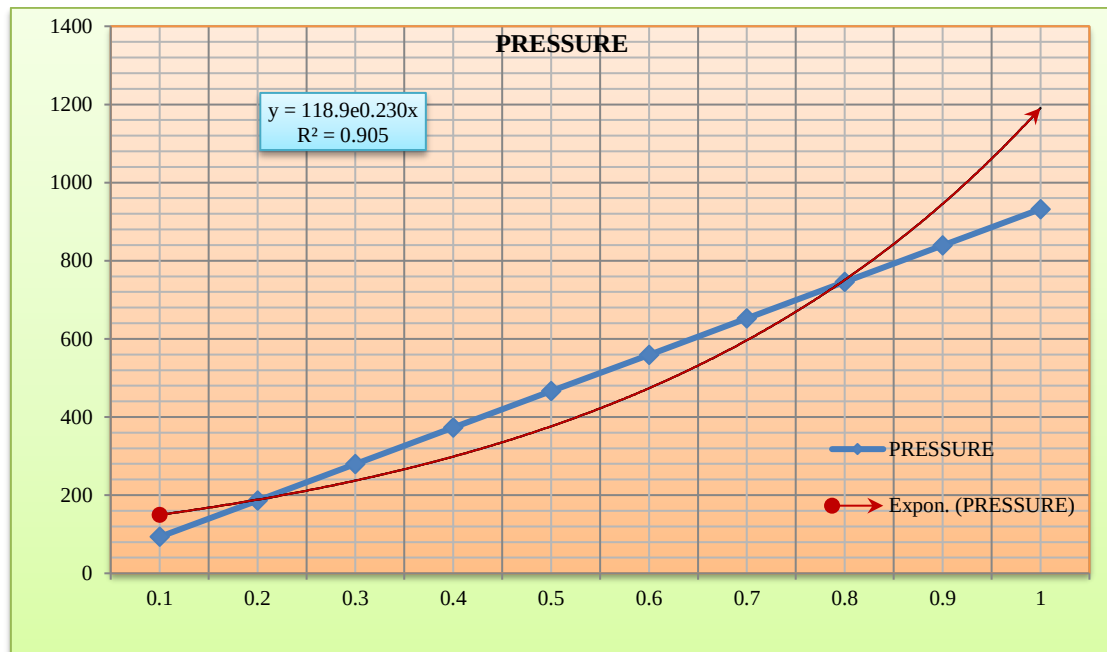


Figure-4.1 (Variation of pressure with respect to rotation number M with exponential trendline)

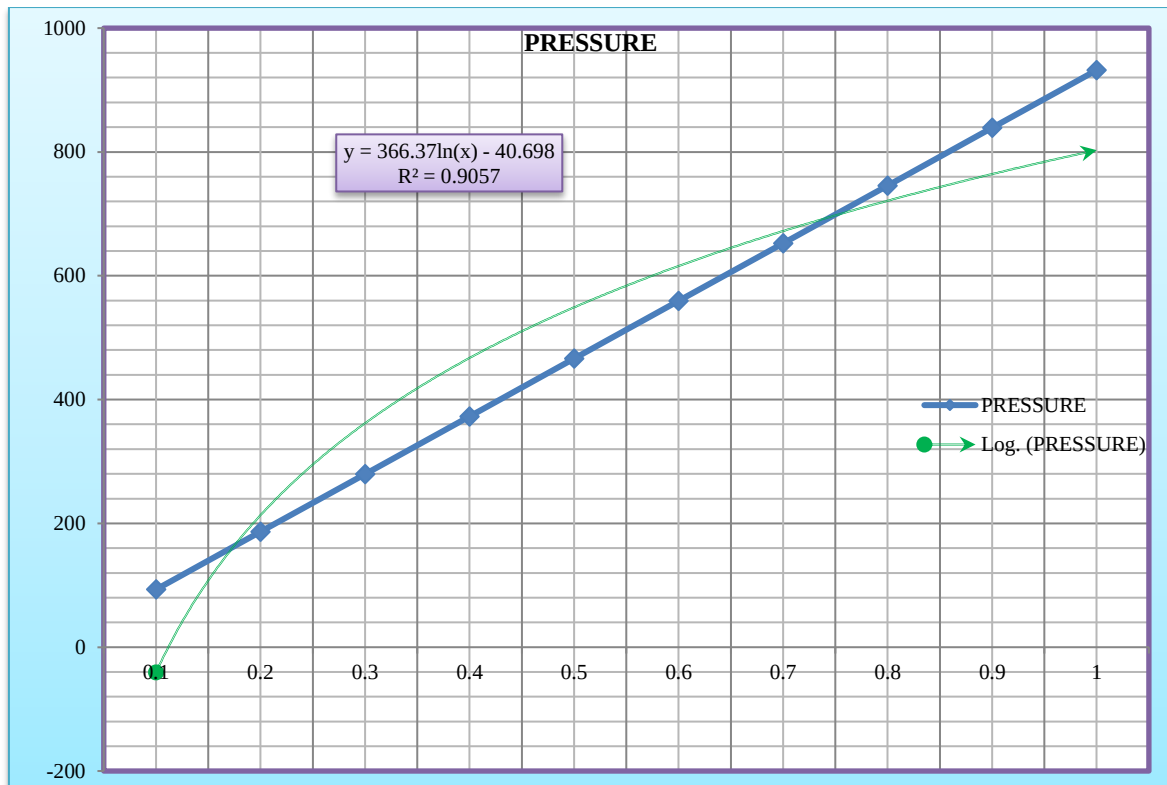


Figure-4.2 (Variation of pressure with respect to rotation number M with logarithmic trendline)

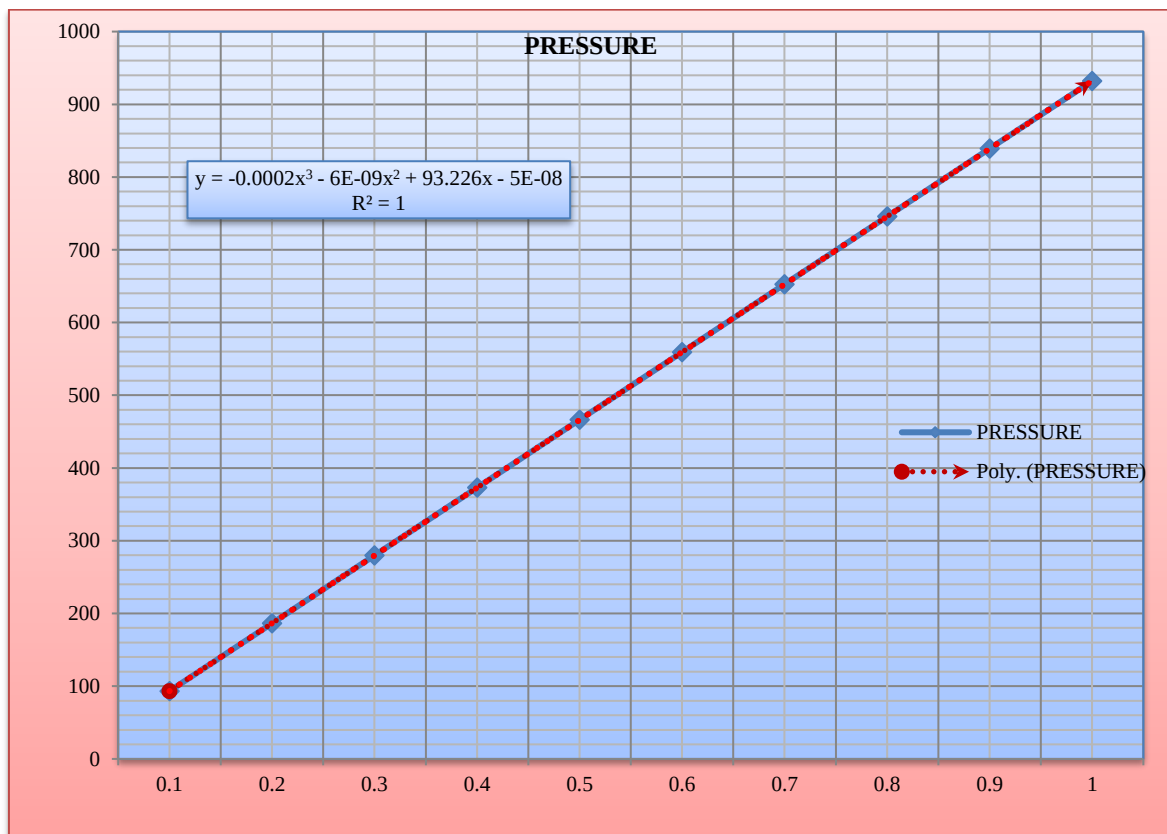


Figure-4.3 (Variation of pressure with respect to rotation number M with third degree polynomial trendline)

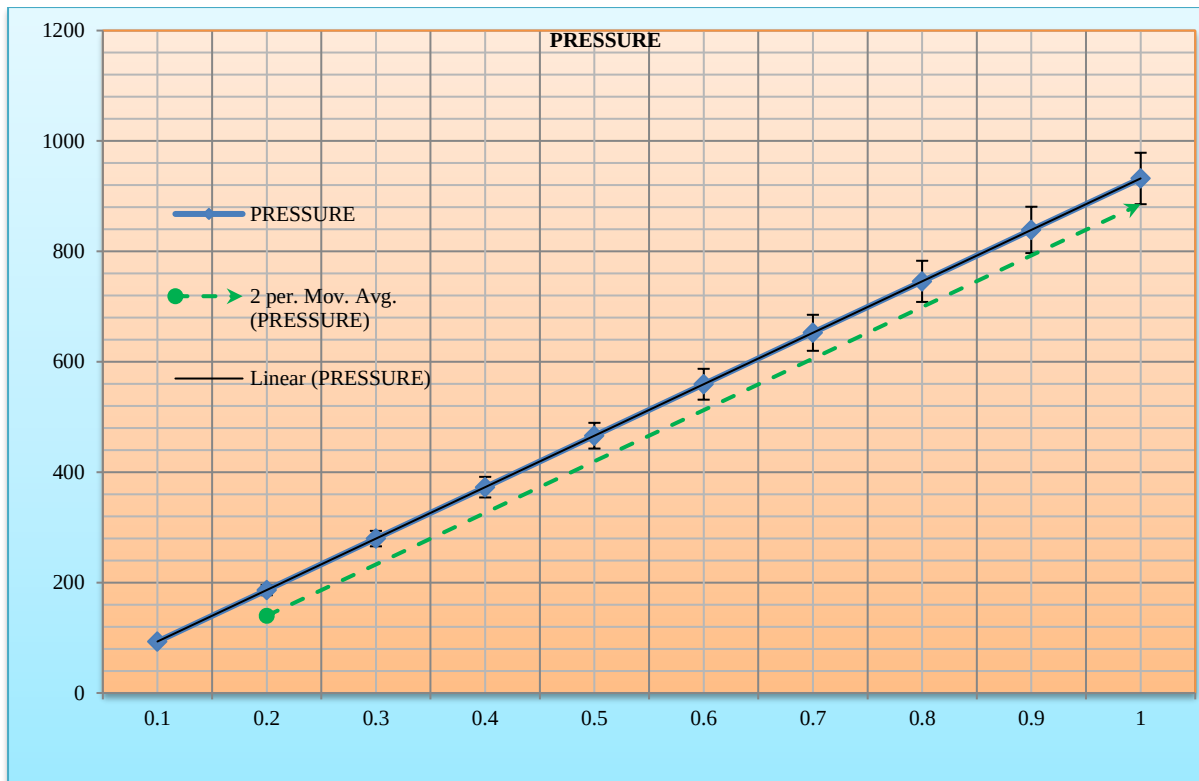


Figure-4.4 (Variation of pressure with respect to rotation number M with moving average with 200% error bars)

V. Results and Discussion

The variation of pressure with respect to rotation number M is shown by the table and graphs. The figure-4.1 shows the exponential trendline by $y = 118.9 e^{0.230x}$ with $R^2 = 0.905$. The figure-4.2 shows the logarithmic trendline by $y = 366.3 \log_e x - 40.69$ with $R^2 = 0.905$. The figure-4.3 shows the third degree polynomial trendline by $y = -0.000x^3 - 6E-09x^2 + 93.22x - 5E-08$ with $R^2 = 1$. The figure-4.4 shows the variation of pressure with respect to rotation number M with moving average with 200% error bars.

VI. Conclusion

The derived equation of pressure is given by equation (3.1). The calculated values of the pressure against rotation number M is shown in the table and graphical representation for the variation of pressures is also shown by figure-4.1 to figure-4.4. The comparisons of the pressures have been done with the help of geometrical figures, expressions, calculated tables and graphs for the lubricating discs in the second order rotatory theory of hydrodynamic lubrication. The analysis of equations for pressures, tables and graphs show that pressures pressure is not independent of viscosity and increase with increasing values of rotation number.

VII. References

- [1]. Banerjee, M.B., Gupta, R.S. and Dwivedi, A.P. (1981) The Effects of Rotation in Lubrication Problems, *WEAR*, 69, 205.
- [2]. Banerjee, M.B., Dube, G.S., Banerjee, K. (1982) The Effects of Rotation in Lubrication Problems: A New Fundamental Solutions, *WEAR*, 79, pp. 311-323.
- [3]. Cameron, A. (1981) *Basic Lubrication Theory*, Ellis Harwood Limited, Coll. House, Watergate, Chichester, p. 45-162.
- [4]. Cameron, A. (1958) The Viscous Wedge Trans., *ASME*, 1, 248.
- [5]. Dowson, D. (1962) A Generalized Reynolds Equations for Fluid Film Lubrication, *Int. J. Mech. Sci.*, 4, 159.
- [6]. Dube, G.S. and Chatterjee, A. (1988) *Proc. Nat. Acad. Sci. India*, 58(58), I: 79.
- [7]. Hori, Y. (2005) *Hydrodynamic Lubrication*, Springer Science & Business Media p.23-31.
- [8]. Pinkus, O. and Sternlicht, B. (1961) *Theory of Hydrodynamic Lubrication*, Mc. Graw Hill Book Company, Inc. New York, 5-64.
- [9]. Reynolds, O. 1886. *Phil. Trans. Roy. Soc. London*, Part I, 177.
- [10]. <http://www.machinerylubrication.com/Read/29654/journal-bearing-oil>
- [11]. Miyan, M., (2016) Load Capacity for Fitted Bearings of Hydrodynamic Lubrication under Low and High Rotation Number, *International Journal of Pure and Applied Researches*; 2016 Vol. 1(2); pp. 109-118. <http://ijopar.com/files/CurrentIssue/14A16107.pdf>
- [12]. Miyan, M., (2016) Differential Acoustic Resonance Spectroscopy Analysis of Fluids in Porous Media, *International Journal of Pure and Applied Researches*; 2016 Vol. 2(1); pp. 22-30. <http://ijopar.com/files/CurrentIssue/A16209.PDF>
- [13]. Miyan, M., (2016) The Fluid Flow Motion through Porous Media for Slightly Compressible Materials, *International Journal of Pure and Applied Researches*; 2016 Vol. 3(1); pp. 28-35. <http://ijopar.com/files/CurrentIssue/A16306.PDF>
- [14]. Miyan, M., (2016) Lattice Boltzmann Method for PAKKA-Model Structures in Fluid Flow Motion, *International Journal of Pure and Applied Researches*; 2016 Vol. 3(2); pp. 146-152. <http://ijopar.com/files/CurrentIssue/A16308.pdf>