



Mathematical Modeling to express the Amount of Heat transfer in a Human-Clothing-Atmosphere System

Dr. Biswapati Chatterjee¹, Mr. Anirban Dutta², Md.Samsu Alam³

^{1,2,3}Department of Textile Technology, Govt. College of Engg & Textile Technology Serampore,
Serampore, West Bengal, 712201, INDIA.

Abstract: The Clothing Comfort consists of 4 elements: Thermal Comfort, Sensorial or Tactile Comfort, Physiological Comfort and Fitting Comfort (ease comfort). Thermal comfort of Clothing is achieved by maintaining the thermal balance between the skin and the inner surface of the porous clothing system. Most of the research work is related to characterization of fabric in terms of their comfort parameters measured in the laboratory. In the present work the evaluation of transmission of heat of garment as a combination of fabric properties, and garment parameters under a given atmosphere for various levels of garment fit is carried out by developing a suitable algorithm and programme. Human body is idealized by three simple shaped geometrical volume of circular in cross section comprising 1) Neck to Shoulder, 2) Bust to the Waist and 3) Shoulder to Bust region respectively. Four types of fit for garment namely close, normal, loose, very loose are considered. Total volume of the human body and Garment surface area is calculated assuming the same geometry of garment as the bare body but with added ease allowance for different fit. The heat transfer equation through air-fabric composite system including fit is derived from the model by taking the human body temperature is 37° C. The porosity of the fabric is also considered. A suitable computer program is developed to calculate conductive and radiation heat transfer including entrapped air at four fitting levels, six standard body sizes, and ambient temperatures. The heat transfer rates are calculated for two typical fabrics 3/1 Twill and Plain weave commonly used for suitings and shirtings. The nature of heat transfers at various levels of fit through the composite human-fabric system is discussed from the calculated data.

Keywords: Heat Transfer, Thermal Comfort, Clothing, Garment, Fabric, Comfort

NOMENCLATURE

- r_1 = the radius at neck girth
- r_2 = radius at bust girth
- r_3 = radius at waist girth
- n = shoulder length
- i = front waist level height
- g = nape to waist height
- k = conductivity of material
- A = Surface area
- ΔT = Temperature difference between both the sides
- D = Fabric thickness
- f = fit ratio of the garment
- A = human body surface area (m^2)
- A_1 = Garment surface area (m^2)
- K_f = Thermal conductivity of fabric ($Wm^{-1}K^{-1}$).
- d = fabric thickness (m)
- R = Thermal resistance of the Garment (Km^2W^{-1})
- θ_1 = Body temperature (Degree Celsius)
- θ_2 = Temperature of the air trapped between the Garment and the Body
- θ_3 = Ambient Temperature (Degree Celsius)

I. Introduction

The criteria for selection of garment in the modern consumer world is not only limited in the look and aesthetics of the garments, but also depends upon the wear comfort. And the apparel market is also highly competitive, to meet and even exceed consumer's need.

Clothing comfort [1],[2] is a pleasant state arising out of physiological, psychological and physical harmony between a human being and the environment. The Physiological comfort depends on the aesthetic properties of fabric, i.e. Drape, lusture, colour, crease, pilling, staining etc. On the other hand the fitting comfort is related with the size and fit of clothing [6].

The thermal exchange between body and environment takes place through heat conduction, convection and radiation. In additionally the heat transfer the exchange of the body moisture takes place through perspiration and sweating [3]. In a steady state situation, the heat produced by the human body is balanced by the heat loss to the environment. The human body tries to maintain a constant temperature of about 37°C . The actual value varies slightly from person to person. Thermal comfort of Clothing is achieved by maintaining the thermal balance between the skin and the inner surface of the clothing system [4],[5].

Most of the research work [5] is related to characterisation of fabric in terms of their comfort parameters measured in the laboratory. But the actual comfort properties depends on not-only the fabric, but also a garment size and fineness which is again governed by the dimension of human body.

The present work thus aims at the evaluation of comfort of garment as a combination of fabric properties, environmental temperature and also studies the effect of garment fitness on comfort. A suitable mathematical expression is developed based on simple heat transfer process to evaluate comfort of a garment under a given atmosphere for various garment fitness and hence developing an algorithm to predict comfort of a worn garment.

The present work thus aims at the evaluation of comfort of garment as a combination of fabric properties, environmental factors and garment parameters and also optimizes the effect of garment fitness on comfort. The specific objective is to establish a suitable mathematical expression to evaluate comfort of a garment under a given atmosphere for various garment fitness and hence developing an algorithm to predict comfort of a garment under given condition.

II. Theoretical Analysis

Assumptions:

1. Human body is idealized by three simple geometric volumes each of circular in cross section comprising a) Neck to Shoulder by truncated conic, b) Shoulder to Bust by cylinder and c) Bust to the Waist by truncated conic region. (figure 1)
2. Only girth ease is allowed to the body garment
3. Girth in shoulder tip region is equal to the girth in bust region
 - The latent height is the average of front waist level and nape to waist height.
4. The allowance at Bust, Waist and Neck girth are in 3:2:1 ratio.

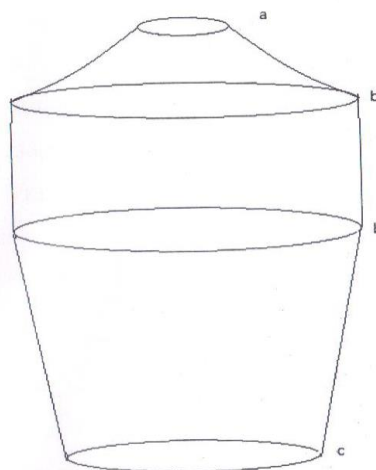


Figure 1: Geometrical Representation of Human Body shape.

Four types of fit ratio (in %) for garment are considered in this work, which are

1. Close fit (Allowance of 0-5%)
2. Normal fit (Allowance of 7-10%)
3. Loose Fit (Allowance of 12-15%)

4. Very loose fit (Allowance of 16-20%)

A mid value of fit allowance in all the cases is taken for the analysis. Accordingly, the allowances for the close fit, normal fit, loose fit and very loose fit are 3%, 8.5%, 13.5% and 18%.

Calculation of Volume and Surface area for different Segments of human body and garment:

Total volume of the human body (V_1) is calculated geometrically by the following formula after stepwise derivation

$$V_1 = 1/3 \{ \pi (r_1^2 + r_2^2 + r_1 \cdot r_2 (n^2 - (r_2 - r_1)^2)^{0.5}) + \{ \pi (r_2^2 + r_3^2 + r_2 \cdot r_3 ((p-h)^2 - (r_2 - r_3)^2)^{0.5}) \} + \{ \pi \cdot r_2^2 (h-n) \} \} \quad (1)$$

After considering the ease allowances, the r_1 is changed into r_{11} and similarly r_2 is changed into r_{22} and r_3 is changed into r_{33} . Hence the total volume of human body and trapped air under the garment is

$$V_{11} = 1/3 \{ \pi (r_{11}^2 + r_{22}^2 + r_{11} \cdot r_{22} (n^2 - (r_{22} - r_{11})^2)^{0.5}) + \{ \pi (r_{22}^2 + r_{33}^2 + r_{22} \cdot r_{33} ((p-h)^2 - (r_{22} - r_{33})^2)^{0.5}) \} + \{ \pi \cdot r_{22}^2 (h-n) \} \} \quad (2)$$

So the net volume of air entrapped between the garment and body is calculated as $V_{11} - V_1$.

The body surface area (A) is calculated as

$$A = \pi \{ (r_1 + r_2)n + (r_2 + r_3)(p-h) + 2 \cdot r_2 \cdot (h-n) \} \quad (3)$$

And the Garment Surface area (A') is calculated as

$$A' = \pi \{ (r_{11} + r_{22})n + (r_{22} + r_{33})(p-h) + 2 \cdot r_{22} \cdot (h-n) \} \quad (4)$$

In order to calculate the volume and surface area for a different fitness and body sizes a suitable computer programme is developed by using C-language.

The theoretical calculation is of heat transfer through garment:

The theoretical calculation is of heat transfer through garment is also made in this study. The heat transfer is generally done by conduction, convection and radiation process. But in case of fabric the heat flow through a fabric is due to a combination of conduction and radiation. The convection heat loss through a fabric being negligible [8].

Calculation of Conductive Heat Transfer:

The Conduction heat transfer through fabric is done according to the law of conductive heat transfer which is $Q = K \cdot A \cdot \Delta T / d$ (5)

For garment the relation of conductive heat transfer is expressed as

$$Q/t = (K(\theta_1 - \theta_2) \cdot \sqrt{\pi \cdot A}) / f = \{ K f (\theta_1 - \theta_2) \cdot A_1 \} / d \quad (5)$$

Since the thermal conductivity of the air (K) is in the range of 0.024 to 0.0262 ($\text{Wm}^{-1}\text{K}^{-1}$)

So, we take $K = 0.025$

And the human body temperature (θ_1) is 37°C

The below figure is furnished in support of the above mentioned equation.

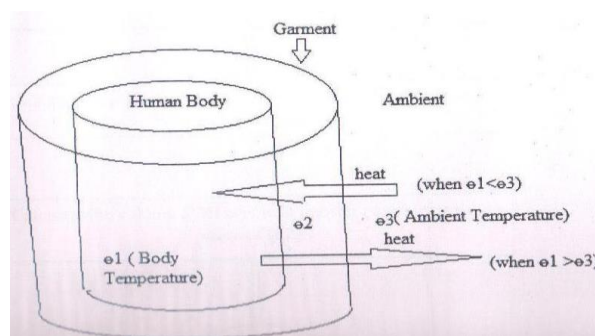


Figure 2: Schematic Diagram of thermal interaction in human body –garment- atmosphere system

Now by taking the value of ambient temperature, human body size , fit ratio , thermal resistance of garment as inputs , the values of A , A₁ , θ_2 , Q/t can be calculated by using the equations mentioned above . In case the ambient temperature is lower than the body temperature , then the heat flow from body to the atmosphere and vice versa.

In this case also , in order to calculate the conductive transfer heat for a different fittings levels and body sizes at different ambient temperatures a suitable computer program is developed by using C-Language

Calculation of heat transfer by radiation through garment:

The radiation heat loss through garment cloth can be calculated by Stefan-Boltzman Law of radiation heat transfer ,, which is given below :-

$$q = \epsilon \cdot \sigma \cdot (T_h^4 - T_c^4) \cdot A \quad (7)$$

q = the amount of transfer of heat (Watt)

ϵ = emissivity of the object

σ = Stefan-Boltzman constant

T_h = hot body absolute temperature (°K)

T_c = Cold Body absolute temperature (°K)

A = area of the object (in square meter)

The emissivity (ϵ) of cotton cloth material is 0.77 [9] .

As the fabric is a porous material so the porosity of the fabric will be considered. So the heat transfer equation is considered as below:-

$$q = \epsilon \cdot \sigma \cdot (T_b^4 - T_a^4) \cdot A \cdot P \quad (8)$$

T_b = Body absolute temperature (°K)

T_a = Ambient absolute temperature (°K)

A = Garment surface area (in square meter)

P = porosity of the fabric.

III. Case Study

In the second segment of the present work , In order to calculate the heat transfer through the Human Body-Garment system the typical values of the fabric are required. For this case study, some data from the published literature are taken, A detailed fabric sample properties are taken from the standard published paper of V.K.Kothari and D.Bhattacharjee [9] , where a numbers of fabric were tested by a standard instrument ALAMBETA tester.

Two types of fabrics one is 3/1 Twill and another one is Plain weave are used for the calculation, since these two types of weaves are commonly used for Suitngs & Shirts . (Table No. 1)

Using these values the Heat transfer rates are calculated according to the equations developed in this work (Equation Number 6 & 8) at different body sizes and at different fit percentages.

Table-2 : Properties of the fabrics used in the Case study [ref.no.5]

Fabric ref. No	Weave	Warp (Ne)	Weft (Ne)	Ends/m	Picks/m	Thickness (mm)	GSM	Porosity	Thermal Resistance (Km ² W ⁻¹)
37	3/1 Twill	41	36	5920	4160	0.44	151	0.680	0.0097
25	Plain	38	38	5600	3200	0.28	151	0.619	0.0060

A detailed result of Heat transfer through 3/1 Twill fabric at ambient temperature 20°C , 28°C , 42°C , where Human body temeprature is 37°C , Garment emissivity is taken as 0.77 (for 100% Cotton garment) , fabric porosity as 0.68 , Thermal resistance of fabric as 0.0097 Km²W⁻¹ is represented graphically , as described under :-

- 1) Conductive Heat transfer through fabric Vs Fit % at ambient temperature 20°C at seven different body sizes.
- 2) Radiative Heat transfer through fabric Vs Fit % at ambient temperature 20°C at seven different body sizes.
- 3) Combined Heat transfer through fabric Vs Fit % at ambient temperature 20°C at seven different body sizes.
- 4) Conductive Heat transfer through fabric Vs Fit % at ambient temperature 28°C at seven different body sizes.
- 5) Radiative Heat transfer through fabric Vs Fit % at ambient temperature 28°C at seven different body sizes.
- 6) Combined Heat transfer through fabric Vs Fit % at ambient temperature 28°C at seven different body sizes.
- 7) Conductive Heat transfer through fabric Vs Fit % at ambient temperature 42°C at seven different body sizes.
- 8) Radiative Heat transfer through fabric Vs Fit % at ambient temperature 42°C at seven different body sizes.
- 9) Combined Heat transfer through fabric Vs Fit % at ambient temperature 42°C at seven different body sizes.

The Graphs are as shown below:-

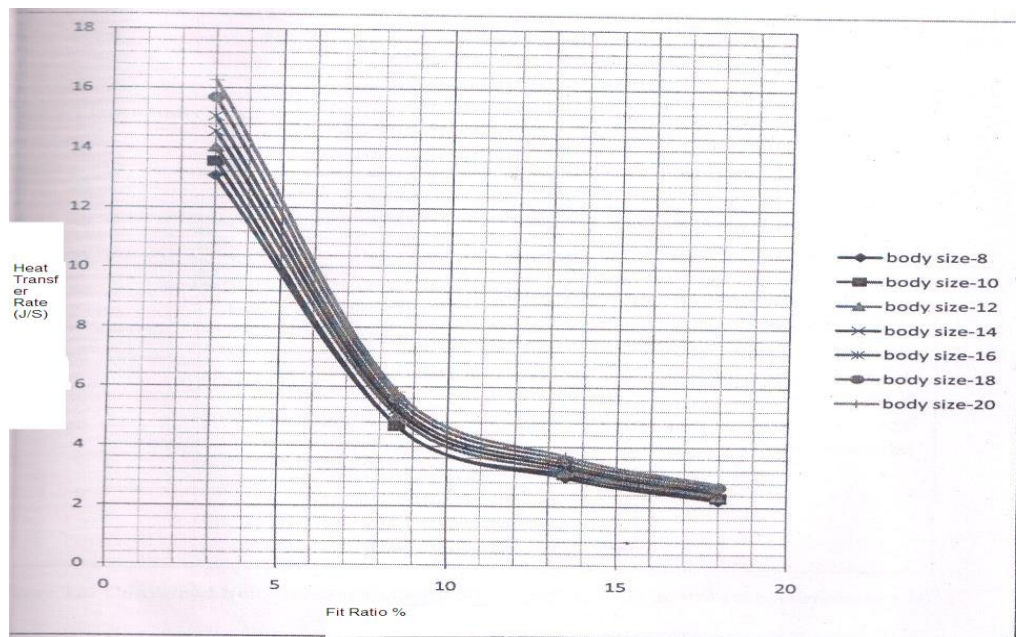


Figure 5: Conductive Heat Transfer through fabric (ref No.37) at ambient temperature 20°C

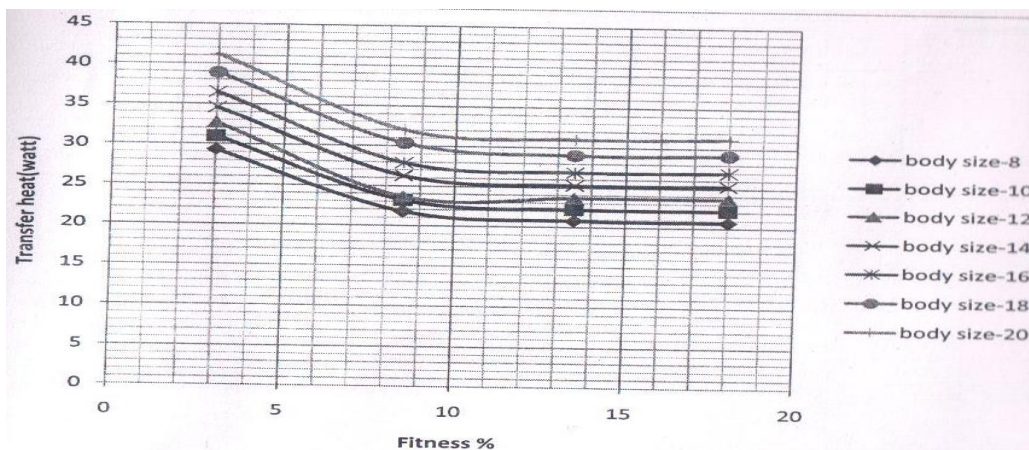


Figure 6: Radiative heat transfer through fabric (Ref No. 37) at ambient temperature 20°C

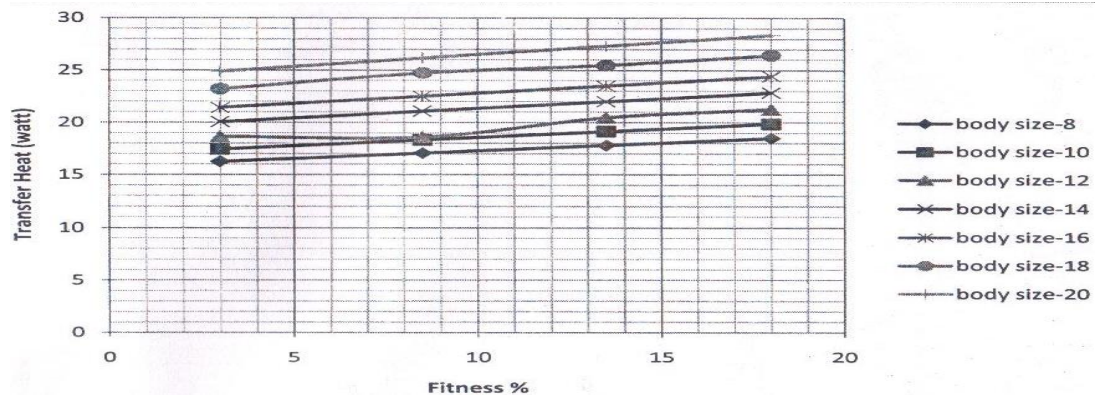


Figure 7: Combined heat transfer through fabric (fab ref. No. 37) at ambient temperature 20°C

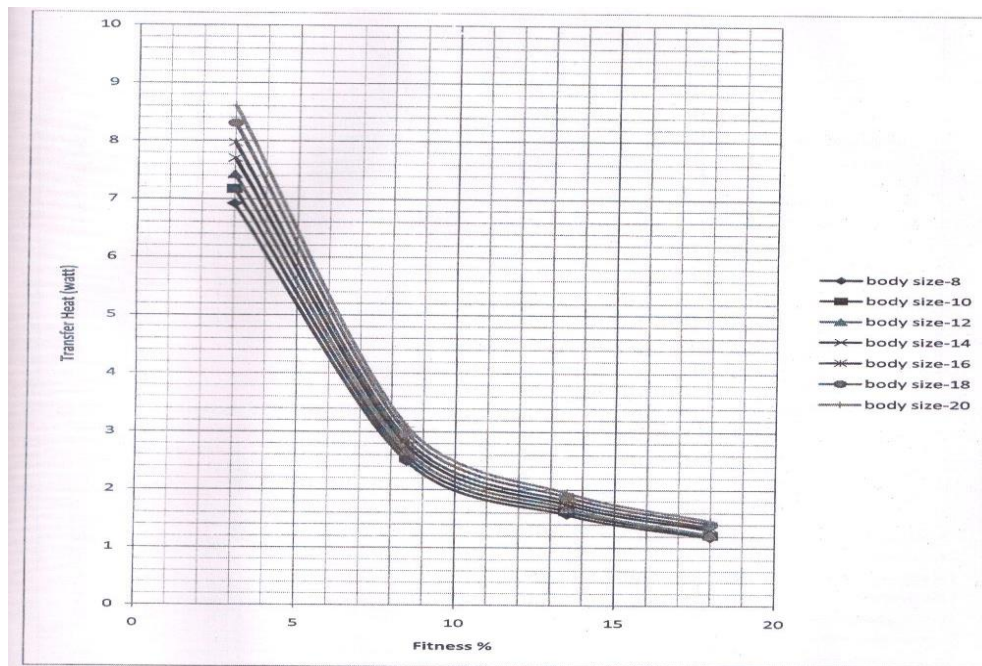


Figure 8: Conductive heat transfer through fabric (ref. No.37) at ambient temperature 28°C .

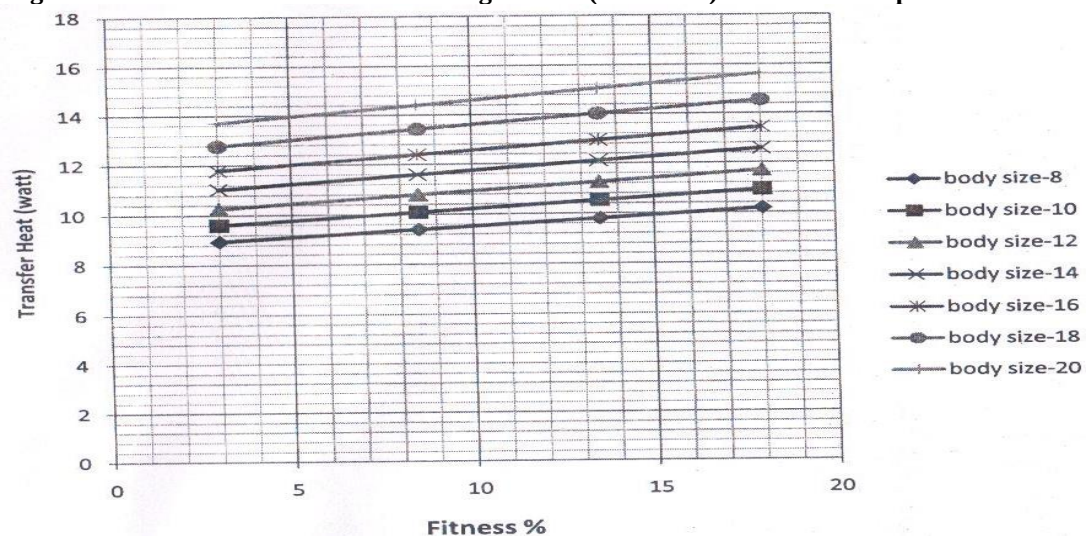


Figure 9: Radiative heat transfer through fabric (ref.No.37) at ambient temperature 28°C .

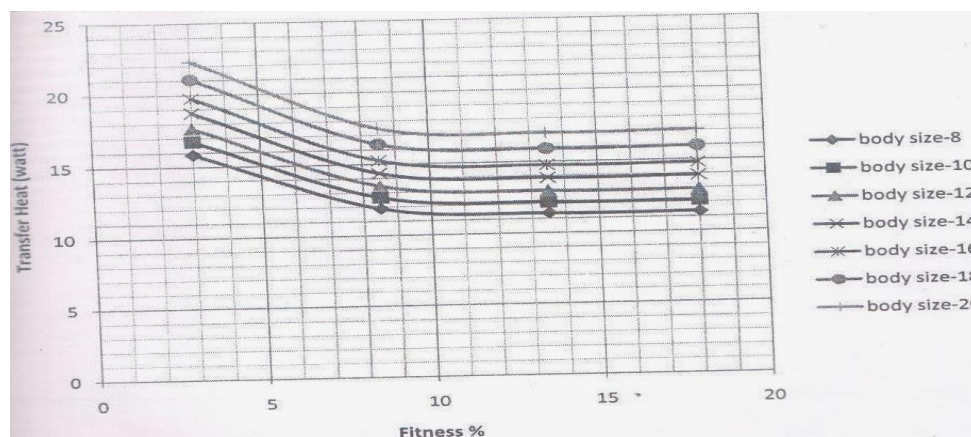


Figure 10: Combined heat transfer through fabric (ref No.37) at ambient temperature 28°C.

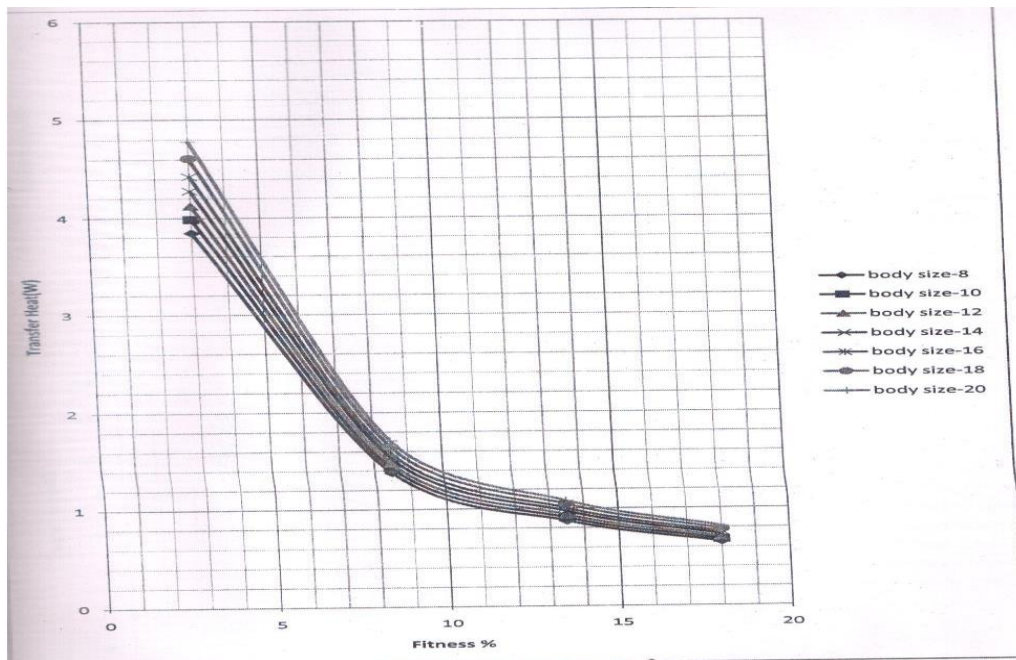


Figure 11: Conductive heat transfer through fabric (ref No.37) at ambient temperature 42°C .

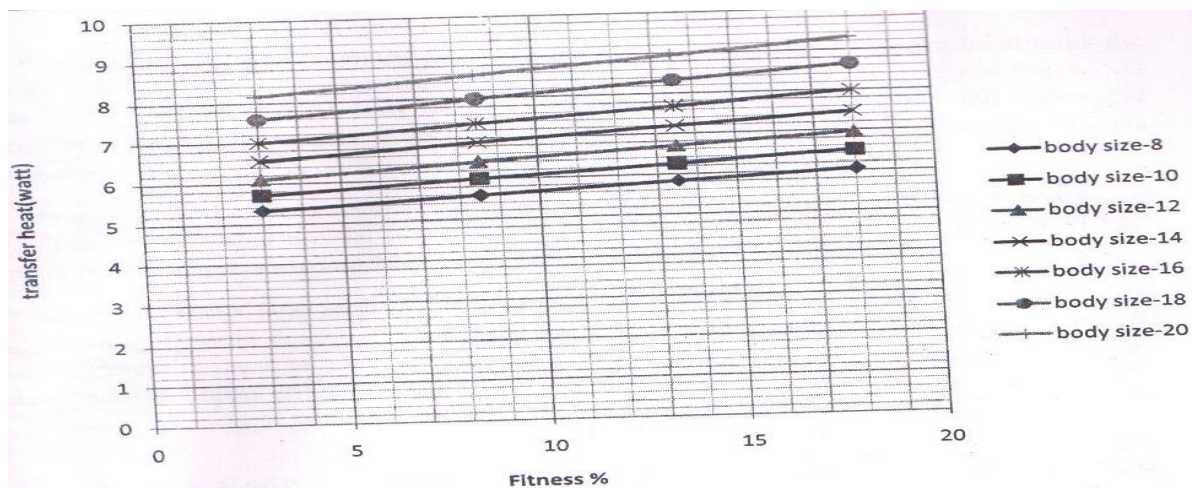


Figure 12: Radiative heat transfer through fabric (ref no.37) at ambient temperature 42°C .

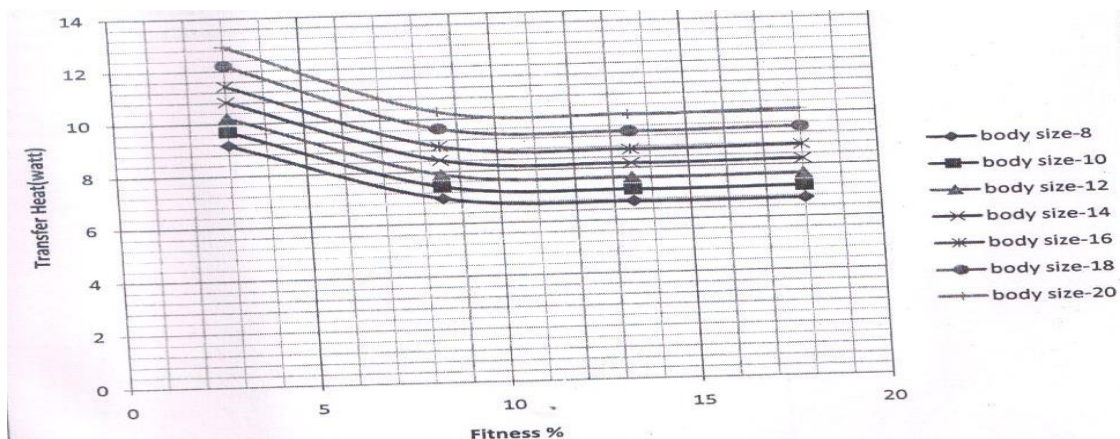


Figure 13: Combined heat transfer through fabric (ref.No.37) at ambient temperature 42°C .

From the detailed graphical analysis, it is found that at all three ambient conditions the rate of conductive heat transfer near about inversely proportional to the garment fit percentage. At close fit the rate of transfer heat is too high and at normal fit level it falls down rapidly and thereafter at large and very loose fir levels the rate falls slightly . But as the fit ratio increases the rate of radiative heat transfer also increases significantly.

Similarly , A detailed result of Heat transfer through Plain woven fabric at ambient temperature 20°C , 28°C , 42°C , where Human body temperature is 37°C , Garment emissivity is taken as 0.57 (for 100% Cotton garment) , fabric porosity as 0.68 , Thermal resistance of fabric as 0.0049 Km²W⁻¹ is represented graphically , as described under :-

- 1) Conductive Heat transfer through fabric Vs Fit % at ambient temperature 20°C at seven different body sizes.
- 2) Radiative Heat transfer through fabric Vs Fit % at ambient temperature 20°C at seven different body sizes.
- 3) Combined Heat transfer through fabric Vs Fit % at ambient temperature 20°C at seven different body sizes.
- 4) Conductive Heat transfer through fabric Vs Fit % at ambient temperature 28°C at seven different body sizes.
- 5) Radiative Heat transfer through fabric Vs Fit % at ambient temperature 28°C at seven different body sizes.
- 6) Combined Heat transfer through fabric Vs Fit % at ambient temperature 28°C at seven different body sizes.
- 7) Conductive Heat transfer through fabric Vs Fit % at ambient temperature 42°C at seven different body sizes.
- 8) Radiative Heat transfer through fabric Vs Fit % at ambient temperature 42°C at seven different body sizes.
- 9) Combined Heat transfer through fabric Vs Fit % at ambient temperature 42°C at seven different body sizes.

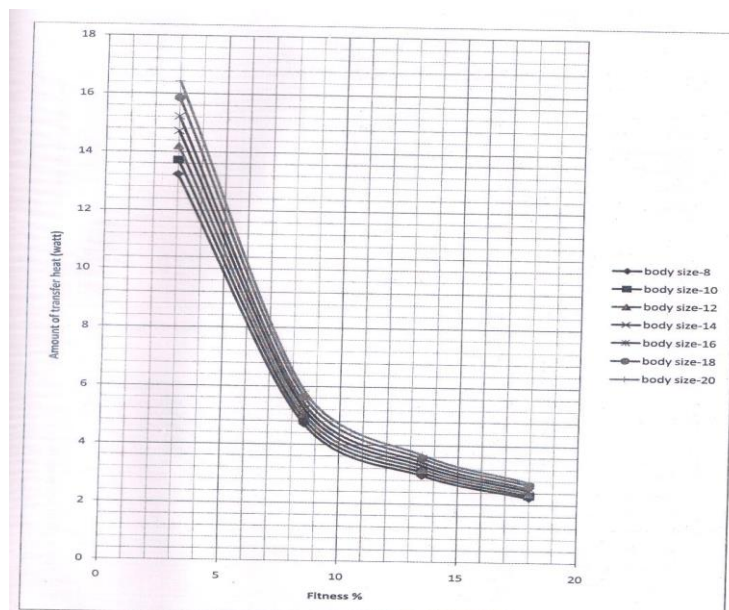


Figure 14: Conductive heat transfer through fabric (ref no. 26) at ambient temperature 20°C .

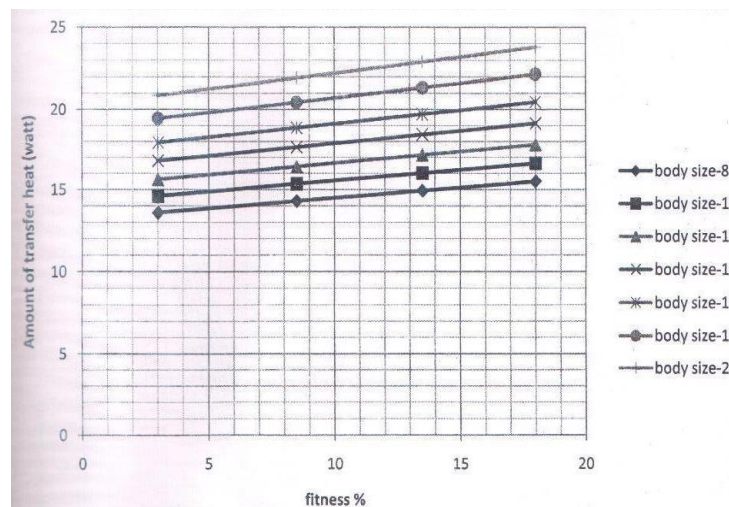


Figure 15: Radiative heat transfer through fabric (ref.no. 26) at ambient temperature 20°C

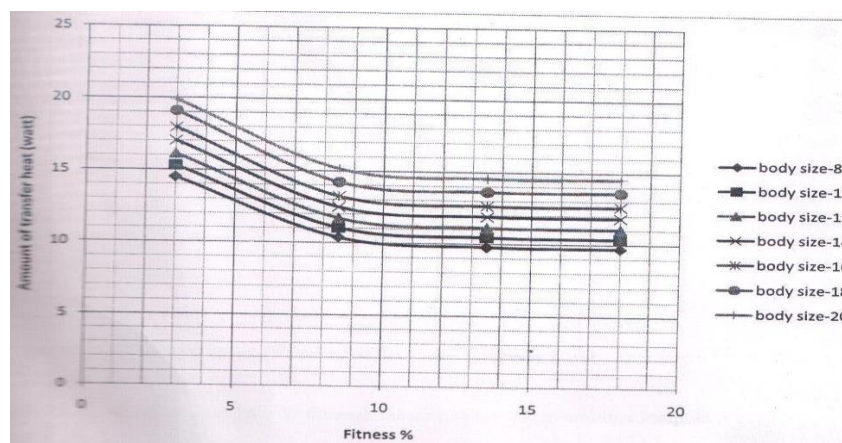


Figure 16: Combined heat transfer through fabric (ref. no. 26) at ambient temperature 20°C .

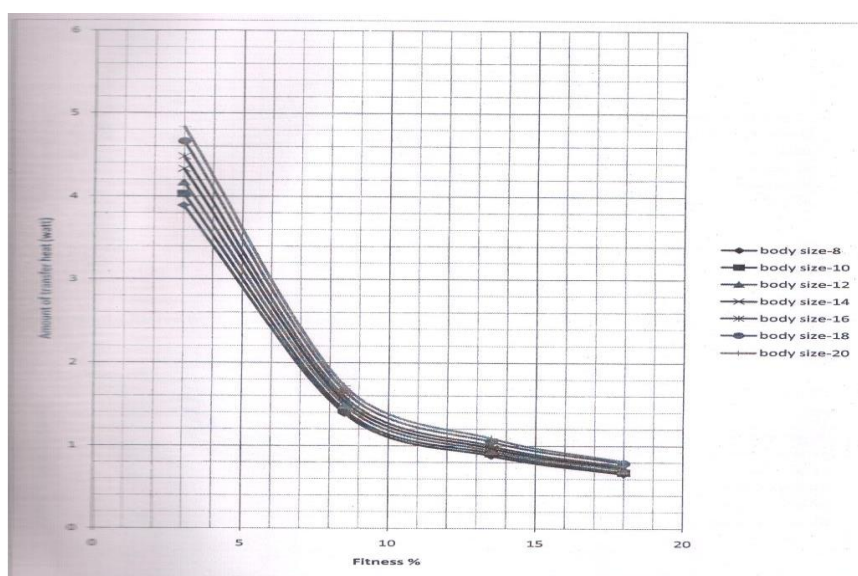


Figure 17: Conductive heat transfer through fabric (ref no.26) at ambient temperature 28°C .

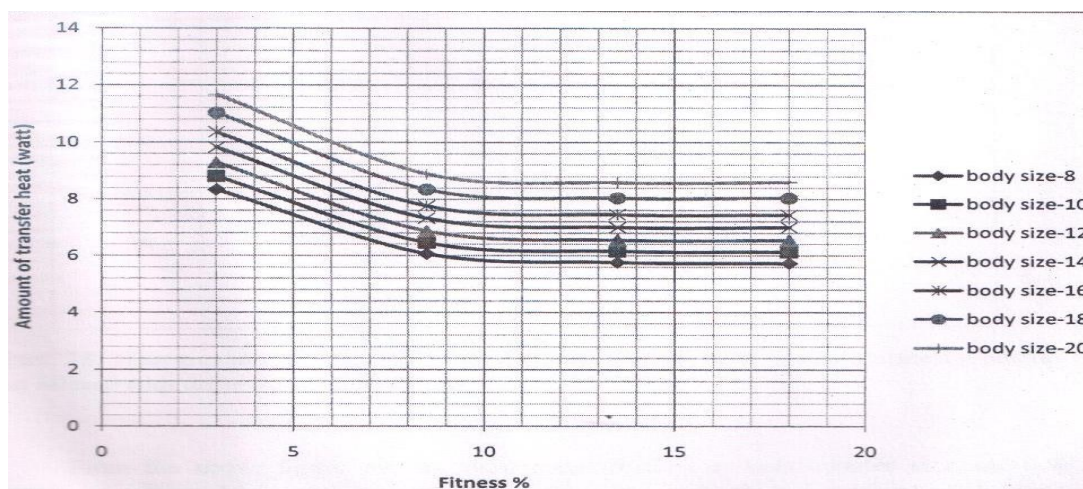


Figure 18: Combined heat transfer through fabric (ref no. 26) at ambient temperature 28°C .

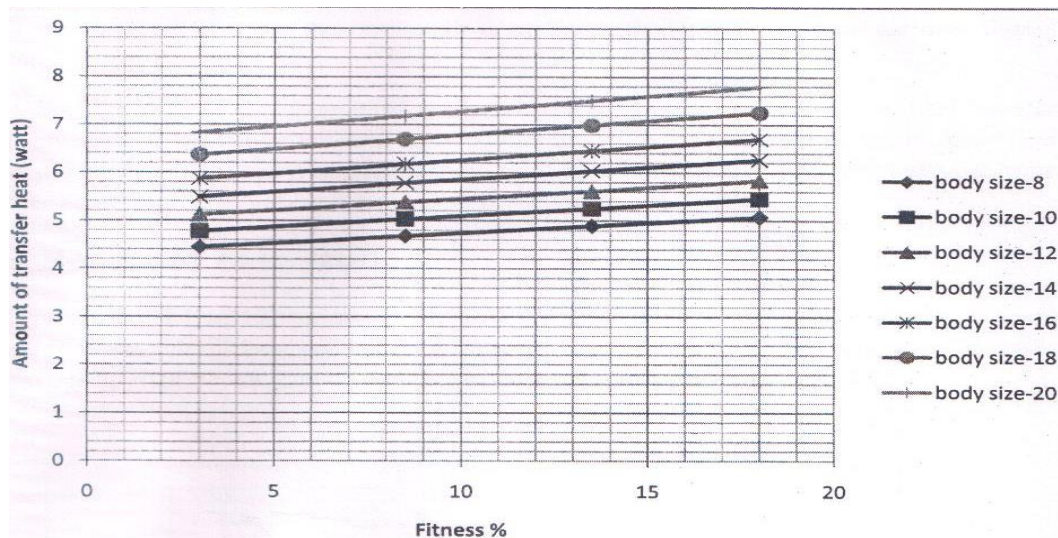


Figure 19: Radiative heat transfer through fabric (ref no. 26) at ambient temperature 28°C .

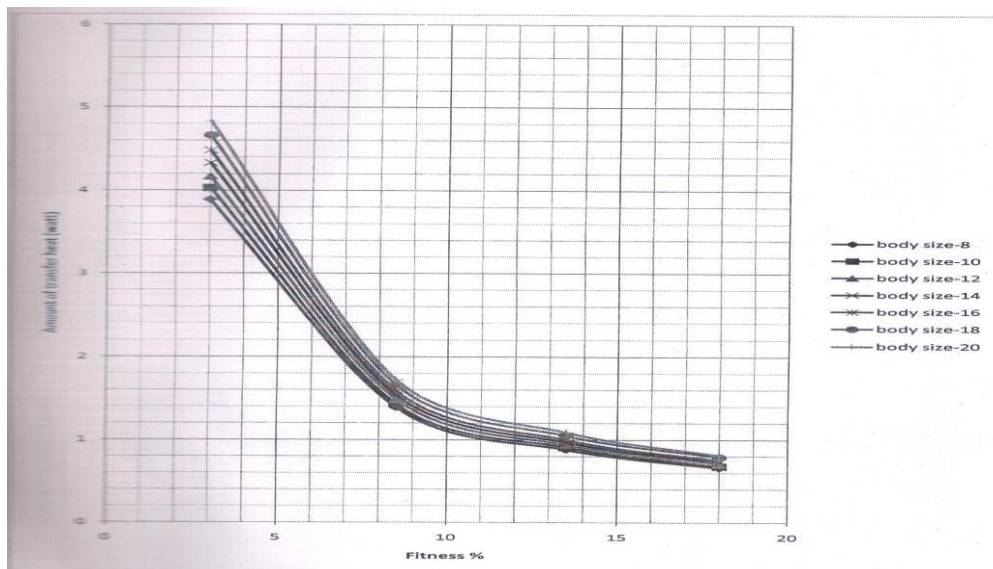


Figure 20: Conductive heat transfer through fabric (ref no. 26) at ambient temperature 42°C .

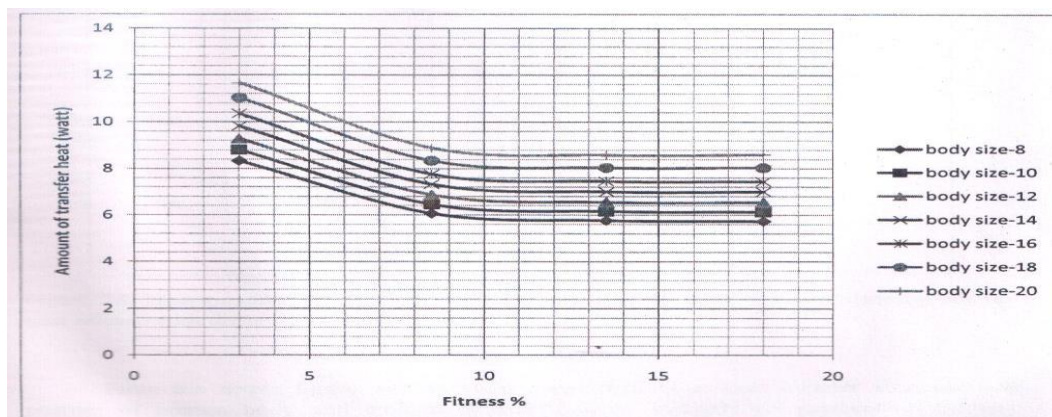


Figure 21: Combined heat transfer through fabric (ref. no. 26) at ambient temperature 42°C .

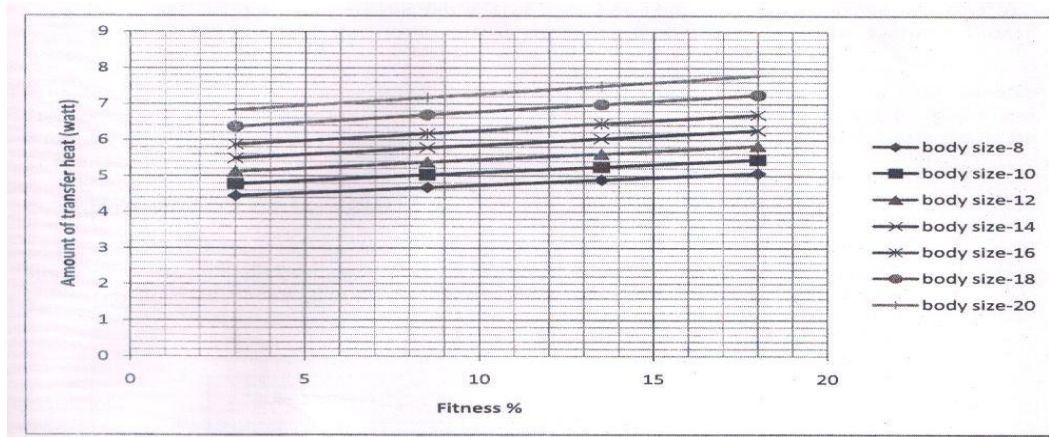


Figure 22: Radiative heat transfer through fabric (ref no.26) at ambient temperature 42°C .

The result of the plain weave fabric is similar to that found in case of 3/1 Twill fabric.

IV Conclusion

It is found that for both varieties of fabrics, at all three ambient conditions the rate of conductive heat transfer is inversely proportional to the garment fitness. At close fit the rate of transfer heat is too high and at normal fit level it falls down rapidly and thereafter at large and very loose fit levels the rate falls slightly. But as the fit ratio increases the rate of radiative heat transfer also increases significantly.

From these numerical models thus developed and through the case studies, it is found that the normal fit of a garment is the most favorable than other fit levels in respect of the Heat Transfer aspect.

References

1. Slater K 1977 , 'Comfort Properties of Textile' , Textile Progress 9(4), 1-42
2. Li Y 2001 , 'The science of Clothing Comfort' , Textile Progress 31(1-2) , 54
3. Goldman R.F 2005 , 'The 4 'F's of Clothing comfort' , Elsevier Ergonomics Book series : Health and performance in the thermal environment 3 , 315-319
4. Savillie B.P 2005 , 'Physical Textile of Textiles' , Woodhead Publication
5. Kothari V K and Bhattacharjee D 2009, Heat Transfer through woven Textiles', International journal of Heat and Mass Transfer , 52 , 2105 - 2160