



Solution of Bi-level Quadratic Fractional Programming Problem using Fuzzy Linear Membership Function

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Abstract: In this paper we propose a Fuzzy programming approach for obtaining a satisfactory solution to a "Bi-level quadratic fractional programming problem". Fuzzy programming approach to this problem is a supervised search procedure which will generate a satisfactory solution for a "Bi-level quadratic fractional programming problem". Linear membership function is being used to find the compromise solution. Finally, an illustrative numerical example is provided to clarify the proposed approach.

Key words: Bi-level programming problem, Fuzzy programming, Linear membership function, Satisfactory solution, Optimal solution.

I. INTRODUCTION

Bi-level programming is characterized by two optimization problems located at different levels, in which the constraint region of the upper level problem is implicitly determined by the lower level problem. Bi-level programming is a powerful technique for solving hierarchical decision-making problems. In this paper we deal with the bi-level quadratic fractional programming problem (BLQFPP) with the essentially co-operative decision makers (DMs). Fuzzy programming approach to this problem is a supervised search procedure which will generate a satisfactory solution for a "bi-level quadratic fractional programming problem". Linear membership function is being used to find the compromise solution. A bi-level programming problem (BLPP) consists of two levels, namely, the first level and the second level. The first level decision maker (DM) is called the centre. The second level DM called follower, executes its policies after the decision of higher level DM called leader (centre) and then the leader optimizes its objective independently but may be affected by the reaction of the follower i.e. BLPP is a sequence of two optimization problems in which the constraints region of one is determined by the solution of second. Bi-level programming structure is used for central economic planning at the regional or national level to create model problems concerning organizational design, facility location, signal optimization, traffic assignment etc. In a decentralized firm, top management or an executive of headquarters makes a decision such as budget of the firm, and then each division determines production plan in the full knowledge of the budget.

A bi-level organization has following common features:

- Interactive decision making units within a predominantly hierarchical structure.
- Execution of decision is sequential, from upper level to lower level.
- Each unit independently maximizes or minimizes its own benefits, but is affected by the action of other units through externalities.
- External effect on decision maker's problems can be reflected in both the objective function and the set of feasible decision space

A bi-level programming problem (BLPP) is a special case of multi-level programming problem (MLPP). Multi-level optimization plays an important role in engineering design, management, and decision making in general. There are many areas in which the techniques of BLQPPs can be used. In general in the area of non-linear programming, the fractional programming has great significance on account of economic development that has the necessity of optimizing the productivity per unit of inputs invested. Practical instances where the use of MLPPs (or BLPPs) can be observed are agriculture, government organization, autonomous institutes, military, maintenance set-up, managerial decision making, schools, hospitals, banks, economic systems, finance control, warfare, transportation, network designs, and is especially suitable for conflict resolution in which all these are directly or otherwise involved to achieve a goal. Fractional programming

gains significant stature since many of the real world problems represented as fractional function are often encountered in the following situation: return on investment, current ratio, risk-assets to capital, actual capital to required capital, foreign loans to total loans, residential mortgages to total mortgages, for finance or corporate planning, debt-to-equity ratio etc. for production planning, inventory to sales, actual cost to standard cost, output per employee, and so forth.

The concept of BLPP was first introduced by Candler and Townsley [7], as well as Fortuny –Amat and McCarl [10] presented the formal formulation of BLPP. The formulation and different version of BLPP are given by Bard [2], Candler [7], Herminia and Carmen [11] and Bialas and Karwan [4]. Bialas and Karwan [3] are the pioneers for linear BLPP who presented vertex enumeration method, called Kth- best solution. These were solved by simplex method. Anandalingam [1] proposed Stakelberg solution concept to MLPP as well as Bi-level decentralized programming problem. Simple type of MLPP were studied by Bellmann [5], Charnes and Cooper [8], Craven and Mond [9], Anandalingam and Apprey [1]. In 1978 Zimmermann [12] first applied Fuzzy set theory in decision making problems with several conflicting objectives. In 1996 Lai [14] first incorporated the concept of tolerance membership function of Fuzzy set theory to MLPP for obtaining satisfactory solution. Shin et al [25], Shin and Lee [24] extended Lai's satisfactory solution concept and theory proposed a supervised search approach by using non-compensatory max-min aggregation operator for MLPP. In the present era number of work has done in Fuzzy set theory and Fuzzy approach. Linguistic variables and triangular fuzzy numbers were used to quantify the vagueness inherent in decision parameters. Again Mishra and Ghosh [15] presented fuzzy programming approach to solve bi-level linear fractional programming problems. Luhandjula [18] has given an approach On Fuzzy Random-Valued Optimization. The main idea behind this approach consists of taking advantage of interplays between fuzzy random variables and random sets in a way to get an equivalent stochastic program. Omar and Hafez [22] has given an algorithm for Solving Bi-Level Integer Linear Fractional Programming Problem (BLIFPP) based on Fuzzy Approach. At the first phase the Charnes & Cooper transformation is used to convert the BLIFPP to an equivalent bi-level linear programming problem (BLPP). At the second phase, a membership function is constructed to develop a fuzzy model for obtaining the optimal solution of the BLPP. Sinna [20] proposed an approach to solve Stability Of Multi-Objective Bi-Level Linear Programming Problems under fuzziness. Saraj and Safaei [19] has used, Taylor series and the Kuhn-Tucker conditions approach to solve Fuzzy Linear Fractional Bi-Level Multi-Objective Programming Problems (FLFBL-MOP). Sultan and Emam et al [27] has introduced A Fuzzy Approach for Solving a Three-Level Large Scale Linear Programming Problem. This problem can be thought as a static version of the Stackelberg strategy, it uses the concept of tolerance membership function to develop a Tchebycheff problem for generating Pareto optimal solution for this problem. This proposed approach was used to obtain non-dominated solutions. Emam, Bakr et al [27] has introduced A Fuzzy Approach for Solving Bi-Level Programming Problem of Banking Crisis Management, here the concept of tolerance membership function is used to develop a fuzzy Max-Min decision model for generating Pareto optimal solution for this problem. Fuzzy programming approach for portfolio selection problems with fuzzy coefficients was introduced by Ramadan and Eldin [13]. Neha Gupta and Abdul Bari [21] has proposed an approach to determine the optimum compromise solution of multi objective capacitated transportation problem (MOCTP) with mixed constraints and a Fuzzy multi objective programming approach has been applied in which they used three different forms of membership functions viz. linear, exponential and hyperbolic. Haseen and Bari [26] has given a fuzzy approach to bi-objective bi-level programming problem. Here using ranking method the fuzzy bi-objective bi-level programming problem (FBOBPP) is converted into a crisp model. The transformed problem was further solved by adopting a two level Stackelberg game theory and fuzzy decision model of Sakawa. Mishra Savita and Verma [16] has introduced analytic hierarchy process for Solving bi-level quadratic fractional programming problems. Mishra and Verma [17] presents an algorithm based on the fitness function for solving bi-level linear fractional programming problems.

In this paper we propose a Fuzzy programming approach for obtaining a satisfactory solution to a "Bi-level quadratic fractional programming problem". Fuzzy programming approach to this problem is a supervised search procedure which will generate a satisfactory solution for a "bi-level quadratic fractional programming problem". Linear membership function is being used to find the compromise solution.

II. BI-LEVEL QUADRATIC FRACTIONAL PROGRAMMING PROBLEMS

A bi-level quadratic fractional programming problem (BLQFPP) consists of two levels, namely, the first level and the second level and each has quadratic fractional objective function. A multi-level programming problem (MLPP) can be defined as a p -person, nonzero sum game with perfect information in which each player moves sequentially from top to bottom. This problem is a nested hierarchical structure. When $p = 2$, we call the system a bi-level programming problem.

For instance, by adopting a criterion with respect to finance or corporate planning as an objective function at the upper level and employing a criterion regarding production planning as an objective function at the lower level, a bi-level linear or non-linear fractional programming problem can be formulated for hierarchical decision problems in firms.

III. FORMULATION OF BI-LEVEL QUADRATIC FRACTIONAL PROGRAMMING PROBLEMS

A bi-level quadratic fractional programming problem is mathematically formulated as:

$$\begin{aligned} & \underset{x_1}{\text{Minimize}} && z_1(x_1, x_2) && \text{Where } x_2 \text{ solves} \\ & \underset{x_2}{\text{Minimize}} && z_2(x_1, x_2) && \text{subject to } A_1 x_1 + A_2 x_2 \leq b, \quad x_1 \geq 0, x_2 \geq 0. \end{aligned} \quad (1)$$

Where objective functions $z_i(x_1, x_2), (i = 1, 2)$ are represented by a quadratic fractional function

$$z_i(x_1, x_2) = \frac{p_i(x_1, x_2)}{q_i(x_1, x_2)} = \frac{x_1' Q_{i1} x_1 + x_2' Q_{i2} x_2 + c_{i1} x_1 + c_{i2} x_2 + c_{i3}}{x_1' R_{i1} x_1 + x_2' R_{i2} x_2 + d_{i1} x_1 + d_{i2} x_2 + d_{i3}} \quad (2)$$

$x_i, (i = 1, 2)$ is an n_i -dimensional decision variables;

Q_{i1} and $R_{i1}, (i = 1, 2)$ are $n_1 \times n_1$ positive definite matrix.

Q_{i2} and $R_{i2}, (i = 1, 2)$ are $n_2 \times n_2$ positive definite matrix;

c_{i1} and $d_{i1}, (i = 1, 2)$ are n_1 —dimensional row vectors.

c_{i2} and $d_{i2}, (i = 1, 2)$ are n_2 —dimensional row vectors;

c_{i3} and $d_{i3}, (i = 1, 2)$ are constants.

b is an m -dimensional constant column vector; $A_i, (i = 1, 2)$ is an $m \times n_i$ constant matrix; and it is assumed that the denominators are positive i.e. $q_i(x_1, x_2) > 0, (i = 1, 2)$. For the sake of simplicity, we use the following notations:-

$x = (x_1, x_2) \in \mathbb{R}^{n_1+n_2}, c_i = (Q_{i1}, Q_{i2}, c_{i1}, c_{i2}, c_{i3}), d_i = (R_{i1}, R_{i2}, d_{i1}, d_{i2}, d_{i3}), (i = 1, 2)$.

Also let DM1 denote the DM at the upper level and DM2 denote the DM at the lower level. In the bi-level quadratic fractional programming problem (1), $z_1(x_1, x_2)$ and $z_2(x_1, x_2)$ respectively represent objective functions of DM1 and DM2, and x_1 and x_2 represent decision variables of DM1 and DM2 respectively.

In contrast to formulation (1), in proposed dissertation, we employ the following representation in order to deal with a bi-level quadratic fractional problem with cooperative DMs:

$$\begin{aligned} & \underset{\text{upperlevel}}{\text{Max}} \quad z_1(x) = \frac{Q_{11}x_1^2 + Q_{12}x_2^2 + c_{11}x_1 + c_{12}x_2 + c_{13}}{R_{11}x_1^2 + R_{12}x_2^2 + d_{11}x_1 + d_{12}x_2 + d_{13}} \\ & \underset{\text{lowerlevel}}{\text{Max}} \quad z_2(x) = \frac{Q_{21}x_1^2 + Q_{22}x_2^2 + c_{21}x_1 + c_{22}x_2 + c_{23}}{R_{21}x_1^2 + R_{22}x_2^2 + d_{21}x_1 + d_{22}x_2 + d_{23}} \end{aligned} \quad (3)$$

$$\text{subject to } A_1 x_1 + A_2 x_2 \leq b, \quad x_1 \geq 0, \quad x_2 \geq 0.$$

IV. FUZZY PROGRAMMING APPROACH TO BI-LEVEL QUADRATIC FRACTIONAL PROGRAMMING PROBLEM.

The decision maker first solve the upper level problem then the Lower level problem

$$\underset{x_1}{\text{Max}} \quad z_1(x) = \frac{Q_{11}x_1^2 + Q_{12}x_2^2 + c_{11}x_1 + c_{12}x_2 + c_{13}}{R_{11}x_1^2 + R_{12}x_2^2 + d_{11}x_1 + d_{12}x_2 + d_{13}} \quad (4)$$

$$\text{subject to } A_1 x_1 + A_2 x_2 \leq b, \quad x_1 \geq 0, \quad x_2 \geq 0, \quad (5)$$

$$\underset{x_2}{\text{Max}} \quad z_2(x) = \frac{Q_{21}x_1^2 + Q_{22}x_2^2 + c_{21}x_1 + c_{22}x_2 + c_{23}}{R_{21}x_1^2 + R_{22}x_2^2 + d_{21}x_1 + d_{22}x_2 + d_{23}} \quad (6)$$

$$\text{subject to } A_1 x_1 + A_2 x_2 \leq b, \quad x_1 \geq 0, \quad x_2 \geq 0. \quad (7)$$

Let (x_1^u, x_2^u, z_1^u) be the solution for the upper level problem where $z_1^u = z_1(x_1^u, x_2^u)$ and (x_1^L, x_2^L, z_2^L) be the solution for the lower level problem where $z_2^L = z_2(x_1^L, x_2^L)$. Then the decision maker at upper level gives some tolerance to the decision vector x_1^u and the objective function z_1 . Tolerance on the decision vector x_1^u enables the

decision maker at lower level to search for his optimum in a wider feasible range. By giving some tolerance to his objective function the decision maker at upper level directs the decision maker at lower level to search for his solutions in the right direction. Let $\pm p_1$ be the tolerance on x_1^u .

Let the membership function [Zimmermann (1991)][29] for decision vector x_1 be defined as

$$\mu_{x_1}(x_1) = [x_1 - (x_1^u - p_1)] / p_1, (x_1^u - p_1) \leq x_1 \leq x_1^u$$

$$[(x_1^u + p_1) - x_1] / p_1, x_1^u < x_1 \leq x_1^u + p_1$$

$$0, \text{ otherwise}$$
(8)

Let the membership function [Zimmermann (1991)] for the objective function z_1 be defined as:

$$1, z_1(x) \geq z_1^u$$

$$\mu_{z_1}[z_1(x)] = [z_1(x) - z_1^L] / [z_1^u - z_1^L], z_1^L < z_1(x) \leq z_1^u$$

$$0, z_1(x) < z_1^L$$
(9)

The above information which is suitably represented mathematically is sent to the decision maker at lower level as his additional constraints. Thus the problem for the decision maker at lower level is

$$\underset{x_2}{\text{Max}} \quad z_2(x) = \frac{Q_{21}x_1^2 + Q_{22}x_2^2 + c_{21}x_1 + c_{22}x_2 + c_{23}}{R_{21}x_1^2 + R_{22}x_2^2 + d_{21}x_1 + d_{22}x_2 + d_{23}}$$

$$\text{subject to} \quad A_1x_1 + A_2x_2 \leq b,$$
(10)

$$\mu_{x_1}(x_1) \geq \alpha I; \mu_{z_1}[z_1(x)] \geq \beta;$$

$$x_1, x_2 \geq 0; \alpha \in [0,1]; \beta \in [0,1]$$

Where, α is the minimum acceptable degree of satisfaction for decision vector x_1 and β is the minimum acceptable degree of satisfaction of objective $z_1(x)$. I is a column vector whose elements are all '1' and is of the same dimension as x_1 . To rate the satisfaction of each of his potential solution, the decision maker at lower level builds the membership function (Unlu,1987[28]) for the objective function z_2 as:

$$1, \quad z_2(x) \geq z_2^u$$

$$\mu_{z_2}[z_2(x)] = [z_2(x) - z_2^L] / [z_2^u - z_2^L], z_2^L < z_2(x) \leq z_2^u$$

$$0, \quad z_2(x) < z_2^L$$
(11)

The auxiliary problem for the decision for the decision maker at lower level is

$$\text{Max} : \delta = \mu_{z_2}[z_2(x)] = [z_2(x) - z_2^L] / [z_2^u - z_2^L]$$
(12)

$$\text{Subject to} \quad A_1x_1 + A_2x_2 \leq b;$$

$$\mu_{x_1}(x_1) \geq \alpha I; \mu_{z_1}[z_1(x)] \geq \beta;$$

$$x_1, x_2 \geq 0; \alpha \in [0,1]; \beta, \delta \in [0,1];$$
(13)

Where, δ is the minimum acceptable degree of satisfaction of objective function $z_2(x)$. To resolve the conflict between the two decision makers and to avoid rejection of solution by the decision maker at upper level, the decision maker at the lower level should try $\max \alpha, \beta$ and δ simultaneously. Let $\lambda = \min\{\delta, \alpha, \beta\}$, Then the lower level problem (10) is formulated as:

$$\text{Max: } \lambda \quad (14)$$

Subject to

$$\begin{aligned} A_1 x_1 + A_2 x_2 &\leq b; \\ [x_1 - (x_1^u - p_1)] / p_1 &\geq \lambda I; \\ [(x_1^u + p_1) - x_1] / p_1 &\geq \lambda I; \\ \mu_{z_1}[z_1(x)] &= [z_1(x) - z_1^L] / [z_1^u - z_1^L] \geq \lambda; \\ \mu_{z_2}[z_2(x)] &= [z_2(x) - z_2^L] / [z_2^u - z_2^L] \geq \lambda; \end{aligned} \quad (15)$$

$$x_1, x_2 \geq 0; \lambda \in [0,1]$$

The problem stated in equation (14) and (15) is a fuzzy programming problem (FPP). It is solved by the decision maker at lower level.

V. NUMERICAL EXAMPLE

In this section we present numerical example to demonstrate the solution procedures by proposed approach to solve bi-level quadratic fractional programming problem (BLQFPP). The following example considered by Mishra [18] is again used to demonstrate the solution procedures and clarify the effectiveness of the proposed approach:

Consider the following BLFPP

$$\begin{aligned} \max_{x_1} z_1 &= \frac{10x_1^2 + 15x_2^2 + 5}{x_1^2 + 2x_2^2 + 1} \quad \text{where } x_2 \text{ solves} \\ \max_{x_2} z_2 &= \frac{25x_1^2 + 9x_2^2}{2x_1^2 + x_2^2 + 1} \end{aligned}$$

subject to

$$\begin{aligned} 4x_1 - 5x_2 &\leq 15; \quad 3x_1 - x_2 \leq 21; \quad 2x_1 + x_2 \leq 27; \quad 3x_1 + 4x_2 \leq 45; \quad x_1 + 3x_2 \leq 30; \\ x_1 &\geq 0, \quad x_2 \geq 0. \end{aligned}$$

Solution by proposed approach:

The satisfactory solution is as follows:

x_1	x_2	z_1	z_2
5.34	1.27	9.6	12.20

Realized degree of satisfaction is:

$$\mu_{x_1} = [x_1 - (x_1^u - p_1)] / p_1 = 0.96;$$

$$\mu_{x_2} = [(x_1^u + p_1) - x_1] / p_1 = 1.4 > 1;$$

$$\mu_{z_1} = (z_1 - z_1^L) / (z_1^u - z_1^L) = 0.7$$

$$\mu_{z_2} = (z_2 - z_2^L) / (z_2^u - z_2^L) = 0.7;$$

The solution to the problem is feasible. Hence, (5.34,1.27) is the optimal solution of the given BLQFPP.

VI. CONCLUSION

Fuzzy programming approach is a supervised search procedure under the control of the upper level DM. This is in accordance with a hierarchical system where the upper level DM is the main decision maker. This approach gives a satisfactory solution of the system, which is obtained by considering all the points enclosed, by the solution

space. This satisfactory solution is more reasonable as it is obtained by considering the decision variables and the objective functions. Also this approach is interactive since the tolerance limit can only be decided by the DMs. A membership function should be chosen to best represent the objective function. The membership function and tolerance limit are always decided by the experience of the DMs. We also observe that the positive tolerance on decision variables x_1 and x_2 has a greater effect on the value of λ than the negative tolerance. We see that, for higher value of positive tolerance of decision variables x_1 the value of λ is also high which implies that overall satisfactory solution of the system has improved.

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