



Special Pythagorean Triangles in Connection with the Narcissistic Numbers of Order 3 and 4

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Abstract: We aim to find special pythagorean triangles in connections with the Narcissistic number of order 3 and 4. Also we present the number of primitive and non-primitive pythagorean triangles.

Keywords: Narcissistic number, primitive and non-primitive, pythagorean triangles

I. Introduction

Number theory is about properties of natural numbers, integers or rational numbers. The main goal of number theory is to discover interesting and unexpected relationships between different sorts of numbers and to prove that these relationships are true.

For various problems and ideas one may refer [1-3]. Always there is some one to one correspondence between pythagorean triangles and special numbers. From [4] to [11] one can understand the relations between pythagorean triangles and special polygonal numbers.

In [12], [13] special relations between pythagorean triangles with Jarasandha numbers and Harshad numbers are studied.

Recently in [14] special pairs of pythagorean triangles with Narcissistic numbers are obtained.

In this paper we aim to find special pythagorean triangles in connection with the Narcissistic numbers of order 3 and 4. We have presented pythagorean triangles each with a leg, hypotenuse, generators represented by 3rd and 4th order narcissistic numbers 153, 370, 371 and 8208 respectively and some expressions representing the considered narcissistic numbers.

Also we present the number of primitive and non-primitive pythagorean triangles.

II. Basic Definitions

Definition 2.1: Narcissistic Numbers:

An n-digit number which is the sum of nth power of its digits is called an n-narcissistic number. It is also known as Armstrong number.

Definition 2.2:

The ternary quadratic Diophantine equation given by $x^2 + y^2 = z^2$ is known as Pythagorean equation where x, y, z are natural numbers. The above equation is also referred to as Pythagorean triangle.

Definition 2.3:

Most cited solution of the Pythagorean equation is $x = m^2 - n^2$, $y = 2mn$, $z = m^2 + n^2$ where m, n are generators ($m > n > 0$). This solution is called primitive, if m, n are of opposite parity and $\gcd(m, n) = 1$.

III. Method of Analysis

Case 1:

1. $x = 153$ (3rd order narcissistic number), i.e., $m^2 - n^2 = \text{narcissistic number}$

m	n	x	y	z	x^2	y^2	$x^2 + y^2 = z^2$
13	4	153	104	185	23409	10816	34225
27	24	153	1296	1305	23409	1679616	1703025
77	76	153	11704	11705	23409	136983616	137007025

Out of these 3 pythagorean triangles 2 are primitive and one is non-primitive.

2. $x = 370$ is not possible, since 370 cannot be written as difference of two square numbers.

3. $x = 371$ (3rd order narcissistic number), i.e., $m^2 - n^2 = \text{narcissistic number}$

m	n	x	y	z	x^2	y^2	$x^2+y^2= z^2$
30	23	371	1380	1429	137641	1904400	2042041
186	185	371	68820	68821	137641	4736192400	4736330041

In this case both the triangles are primitive.

4. $x = 8208$ (4th order narcissistic number) ie, $m^2-n^2 =$ narcissistic number

m	n	x	y	z	x^2	y^2	$x^2+y^2= z^2$
92	16	8208	2944	8720	67371264	8667136	76038400
93	21	8208	3906	9090	67371264	15256836	82628100
103	49	8208	10094	13010	67371264	101888836	169260100
127	89	8208	22606	24050	67371264	511031236	578402500
132	96	8208	25344	26640	67371264	642318336	709689600
183	159	8208	58194	58770	67371264	3386541636	3453912900
237	219	8208	103806	104130	67371264	10775685636	10843056900
348	336	8208	233856	234000	67371264	54688628736	54756000000
517	509	8208	526306	526370	67371264	276998005636	277065376900
687	681	8208	935694	935730	67371264	875523261636	875590632900
1028	1024	8208	2105344	2105360	67371264	4432473358336	4432540729600
2053	2051	8208	8421406	8421410	67371264	70920079016836	70920146388100

In this case all the pythagorean triangles are non-primitive

Case 2:

In this case the numbers 370 and 8208 are only considered, since 153,371 cannot be written in the form of $2mn$

1. $y = 370$ (3rd order narcissistic number) ie, $2mn =$ narcissistic number

m	n	x	y	z	x^2	y^2	$x^2+y^2= z^2$
37	5	1344	370	1394	1806336	136900	1943236

In this case the pythagorean triangle is non-primitive

2. $y = 8208$ (4th order narcissistic number) ie., $2mn =$ narcissistic number

m	n	x	y	z	x^2	y^2	$x^2+y^2= z^2$
72	57	1935	8208	8433	3744225	67371264	71115489
76	54	2860	8208	8692	8179600	67371264	75550864
108	38	10220	8208	13108	104448400	67371264	171819664
152	27	22375	8208	23833	500640625	67371264	568011889
171	24	28665	8208	29817	821682225	67371264	889053489
216	19	46295	8208	47017	2143227025	67371264	2210598289
456	9	207855	8208	208017	43203701025	67371264	43271072289
513	8	263105	8208	263233	69224241025	67371264	69291612289
1026	4	1052660	8208	1052692	1108093075600	67371264	1108160446864
1368	3	1871415	8208	1871433	3502194102225	67371264	3502261473489
2052	2	4210700	8208	4210708	17729994490000	67371264	17730061861264

Out of these 11 pythagorean triangles 3 are primitive and the remaining are non-primitive.

Case 3: $z = 153,370$ (3rd order narcissistic number) ie., $m^2 + n^2 =$ narcissistic number

m	n	x	y	z	x^2	y^2	$x^2+y^2= z^2$
12	3	135	72	153	18225	5184	23409
17	9	208	306	370	43264	93636	136900
19	3	352	114	370	123904	12996	136900

In this case all the pythagorean triangles are non-primitive

Note that since 371 and 8208 cannot be written as sum of two squares, $z \neq 371, 8208$

Case 4: The generators m, n are narcissistic numbers.

m	n	x	y	z	$x^2+y^2=z^2$
370	153	113491	113220	160309	25698975481
371	153	114232	113526	161050	25937102500
8208	153	67347855	2511648	67394673	4542041948776930
8208	370	67234364	6073920	67508164	4557352206650900
8208	371	67233623	6090336	67508905	4557452254299020

Out of these 5 pythagorean triangles 2 triangles are primitive and the remaining are non-primitive

Case 5: The expressions a). $\frac{4A}{P} - y + z, x$ b). $x - \frac{2A}{P}, \frac{1}{2}[z + x - y]$ c). $\frac{2A}{P}, \frac{1}{2}[y + x - z]$ represents the narcissistic number 153 which is presented below in the Table (a), Table (b), Table (c) respectively

Table (a)

m	n	x	y	z
13	4	153	104	185
27	24	153	1296	1305
77	76	153	11704	11705

Out of these 3 pythagorean triangles 2 triangles are primitive and one is non-primitive

Table (b)

m	n	x	y	z
54	3	2907	324	2925
26	9	595	468	757
26	17	387	884	965
54	51	315	5508	5517
154	153	307	47124	47125
154	1	23715	308	23717

Out of these 6 pythagorean triangles, 4 triangles are primitive and 2 are non-primitive

Table (c)

m	n	x	y	z
17	8	225	272	353
51	48	297	4896	4905

Out of these 2 pythagorean triangles one triangle is primitive and the other is non-primitive.

For simplicity, in Table (d), we exhibit the connections between special pythagorean triangles and the other narcissistic numbers 370,371and 8208 respectively.

Table (d)

Narcissistic Number	Expressions	No. of Triangles	Remark
370	$x - \frac{2A}{P}, \frac{1}{2}[z + x - y]$	4	2- primitive 2-non-primitive
	$\frac{2A}{P}, \frac{1}{2}[y + x - z]$	8	4 - primitive 4-non-primitive
371	$\frac{4A}{P} - y + z, x$	2	2 - primitive
	$x - \frac{2A}{P}, \frac{1}{2}[z + x - y]$	2	2 - primitive
	$\frac{2A}{P}, \frac{1}{2}[y + x - z]$	4	4 - primitive
8208	$\frac{4A}{P} - y + z, x$	12	12-non-primitive
	$x - \frac{2A}{P}, \frac{1}{2}[z + x - y]$	20	3 - primitive 17-non-primitive
	$\frac{2A}{P}, \frac{1}{2}[y + x - z]$	40	5 - primitive 35-non-primitive

IV. Conclusion

One can also search for connections between the Pythagorean triangles and other Narcissistic number of higher orders.

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