



Finite Element Analysis and Exact Solution of Some Discontinuity in Structure

Dr. Lattif Shekher Jabur

Mechanical Technics Department, Southern Technical University,
Technical Institute of Nasiriyah, Nasiriyah, Postal Code: 64001, Iraq

Abstract: Among other, Numerical methods can be used to determine the maximum stress concentration for different and complicated geometries without considering time and cost limitations. In this paper, stress concentration factor (SCF, K_t) of some discontinuity structure such as; U-shaped notched in shaft, plates and fillets shaft are considered under tension and bending. The maximum stress in the fillets and notches are calculated using finite element carried out on ABAQUS software. Then, the results compared with theoretical calculation results getting by the solutions of equations available in literatures. The static analysis shows a good agreement between the analytical and numerical results which it means, there are no significant differences between the theoretical results and those obtained through simulation. Also, it has been found, for all investigated cases, there is inverse relationship between the stress concentration factor (K_t) and the (r/d) ratios' value, while it is positive relationship between (K_t) and the values of (D/d) ratios.

Keywords: stress concentration, fillet, discontinuity, finite element, notched plates

I. Introduction

It is very important for the engineers to be aware of the effects of stress raiser such as holes, notches and sharp corner in their design. Stress concentration is one of the main problems which designers have to worry about when they are designing any type of machinery or equipment, because it can cause fracture to various mechanical parts and the machine or equipment leading to lost their operating conditions. Therefore, designers should try to eliminate as far as possible the stress concentration on mechanical parts. It should pay particular attention to analyze in the best way possible stress concentrators for designs that are made to work properly and be as reliable as possible [1].

Dimensional changes and discontinuity of a member in a loaded structure lead to variations of stress and high stresses concentrate near these dimensional changes. As a result, stress distribution is not uniform through out the cross section, leading to stress concentration regions, then failure occurs at these points [2]. This situation of high stresses near dimensional changes and discontinuities of a member such (holes, sharp corner, cracks .. etc) is called *stress concentration*, SC, and the ratio of peak stress near stress raiser to a average stress over the member is called *stress concentration factor*, SCF, which it characterizes as a function of ; a geometry or shape of the part but not its size or material, type of loading applied to the part (axial, bending and torsional), specific geometric stress raiser in the part (fillet radius, notch or hole). And always defined with respect to a particular nominal stress and assumes a linear elastic, homogenous, isotropic material [3]. Also, it is commonly determined by experimental or analytical methods and presented in graphical form for ease interpretation. The stress distribution along the surface is substantially uniform until reaching the vicinity of the holes or notches, where efforts suddenly increase. The maximum value of stress at points is found by multiplying the average effort by a (K_t) factor of stress concentration [4].

The stress concentration factor of notched plates, a round bar with a circumferential groove and shoulder fillet shaft are mainly used in practice for the design of structure and machine elements. Exact stress concentration factors are desirable for the research, the authors proposed accurate engineering formulas with minimum error percentage useful for any dimensions of notch in a round bar and plate under tension and bending [5].

Jasmin Kisija et. al [6] investigated the problem of circular cross section bar with u – shape grooved subjected to tension and bending load by using FEA performed in ABAQUS. They described a procedure for determination of SCF, then they calculated its value over a range of geometrical variations for both tension and bending. Josip Kacmarick et. al [7] have been used new mathematical model of stress concentration factor of rectangular bar with opposite edge u – notches subjected to tensile load, and their model parameters' determination implemented by combining MATLAB software and numerical solution using FEMAP software. Nao – Aki and Yasushi [5] considered a round bar with a circular – arc or V – shaped notch under tension, bending and

torsion to calculate stress concentration factors by using body force method, then they applied the least squares method for calculations too. Francisco Javier Ortega Herrera et. al [1] analyze the SCF in a flat plate with two fillets and two notches under an axial load. They used the least squares method to solved a polynomial equations which used to determine SCF at the fillets, whereas they calculated the maximum stress in the fillets and the notches by using ANSYS software. Also, A. M. wahl and R. Beeuwkes [8] described photoelastic tests and strain measurements to determine the problem of SC effects produced by holes and notches in bar under tension. K. S. Babulal et. al [2] studied the maximum stress developed at semi – circular, u – shaped and v – shaped notches using FEM of solidworks simulation software. A. Santos [4] determined SCF in flat plate with central hole subjected to axial load by using ANSYS software, meanwhile he studied the effect of location of three holes located transversely and longitudinally on SCF. Besides that, Shubhrata Nagpal et. al [9] analyzed and determined the optimum size and position of hole in axially loaded thin plate by using FEA, then they proposed a generalized analytical form for mitigation curve which can be used for similar type of discontinuity. By using FEM of ABAQUS software, Mohammed Aryaral and Mohamood Heshmati [10] evaluated and analyzed the SCF reduction techniques on perforated plates under tensile loading.

Generally, and to solve the stress distribution heterogeneity problems, several methodologies can be followed, which can be classified in three general categories. These are experimental, analytical and numerical method. Though experimental methods give the most reliable results but it is very costly, as it requires special equipments, testing facilities ... etc. In other side, analytical solution of every problem is almost impossible because of complex boundary conditions and shapes. For these reasons the numerical methods become the ultimate choice by the researchers in the last few decades [9]. In addition to, the invention and rapid improvement of the computing machines and programs played an important role for the increasing popularity of the numerical methods.

II. Analytical calculation

The first mathematical study on stress concentration was published shortly after 1990, In recent years they have started using computer simulations based on finite elements [4].

The theoretical stress concentration factor (K_t) can be defined as the ratio of the peak stress in the body to nominal stress that would exist in the member if the distribution of stress remained uniform [6, 11, 12, 13], i.e.

$$K_t = \frac{\sigma_{\max.}}{\sigma_{nom.}} \quad , \quad (1)$$

Where:

Nominal stress is found using basic strength – of – materials formulas, and the calculations can be based on the properties of the net cross section at the stress raiser, so it can be calculated as following [6,14]:

$$\sigma_{nominal} = \frac{4P}{\pi d^2} \quad , \quad \text{for circular cross section bar with U-shaped or shoulder fillet subjected to axial tension.} \quad (2a)$$

$$\sigma_{nominal} = \frac{P}{td} \quad , \quad \text{for opposite single U-shaped notches in finite width plate subjected to axial tension.} \quad (2b)$$

$$\sigma_{nominal} = \frac{6M}{td^2} \quad , \quad \text{for opposite single U-shaped notches in finite width plate subjected to in-plane bending} \quad (2a)$$

$d = (D - 2h)$ = diameter of net cross section of circular bar; and d is equal to the width of the plate,
 h = depth of notch, t = thickness of the plate.

To calculate the theoretical stress concentration factor (K_t) for the cases study including U – shaped circumferential grooved circular shaft and shoulder fillet shaft which loaded by tension stress and U-shaped notched plate that loading by tension and bending stresses, the following equation was selected with two different values of constants (C_1 , C_2 , C_3 and C_4), and to compare their results with results have obtained by *Numerical Method*. The selected equation is:

$$K_t = C_1 + C_2 \left(\frac{2h}{D} \right) + C_3 \left(\frac{2h}{D} \right)^2 + C_4 \left(\frac{2h}{D} \right)^3 \quad (3) \quad [3,6,11,12,13]$$

Where : the equations which used to calculate the values of constants C_1 , C_2 , C_3 and C_4 are available in references [12 and 13] ; -

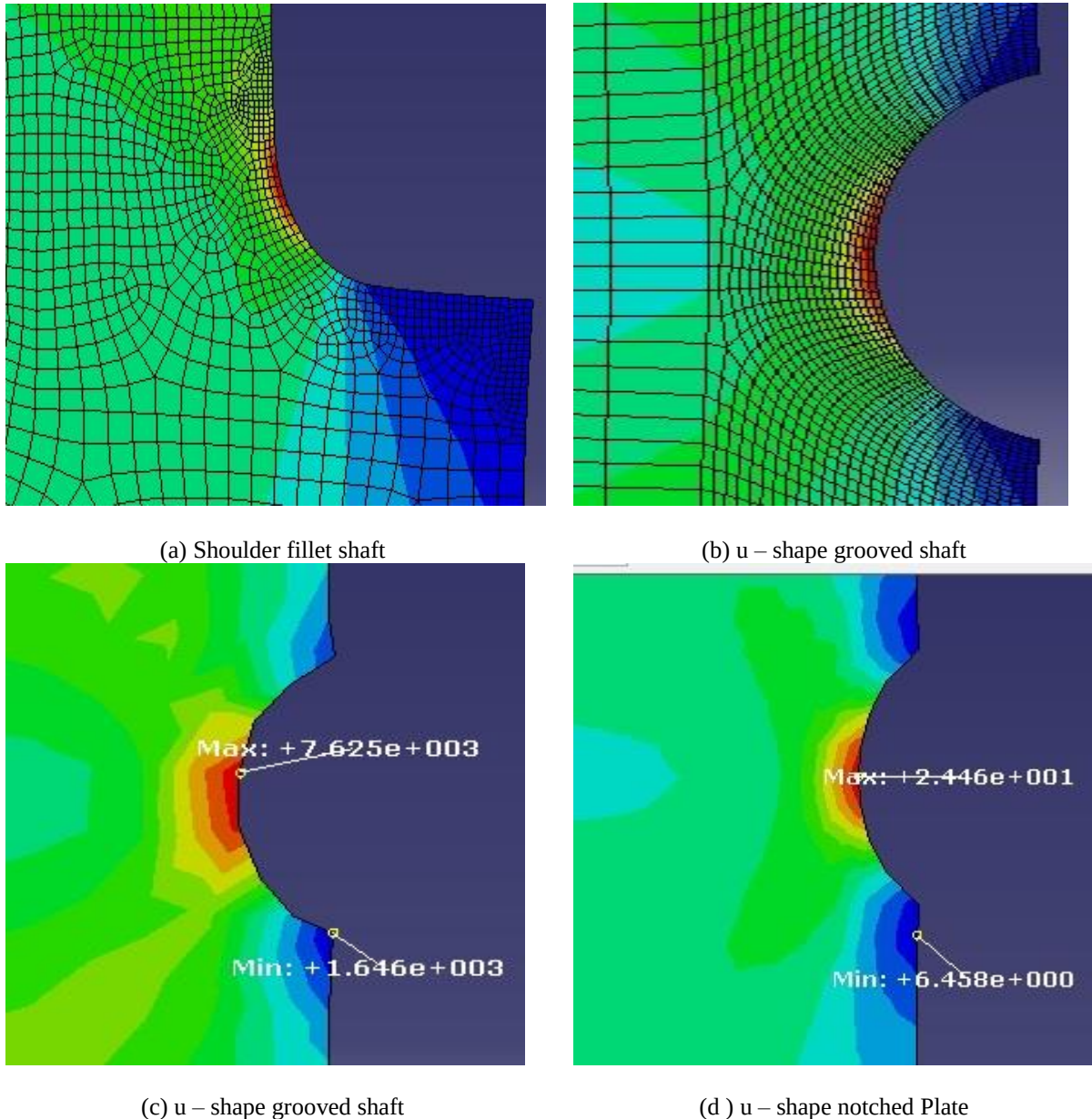
III. Finite Element Analysis

Numerical simulation is carried out on ABAQUS software ver.6.9. The investigated stress concentration cases are analysis as axisymmetric problem for u – shape circumferential groove shaft and shoulder fillet shaft subjected to axial tension , whereas for u – shaped plate loaded by axial tension and in – plane moment is considered as 2D problem.

Because of the differences in geometry parameters of study's cases such as (D/d and r/d) ratios, and to get more accurate results, more than two element sizes were used and the smallest one was at the region of maximum

stress concentration as shown in figure (1a and 1b). Quadratic (CAX8 - an 8-nodes) shape elements are implemented. Example of a mesh and location of maximum and minimum stresses of FEM analysis results for some cases are illustrated in Figure (1c & 1d).

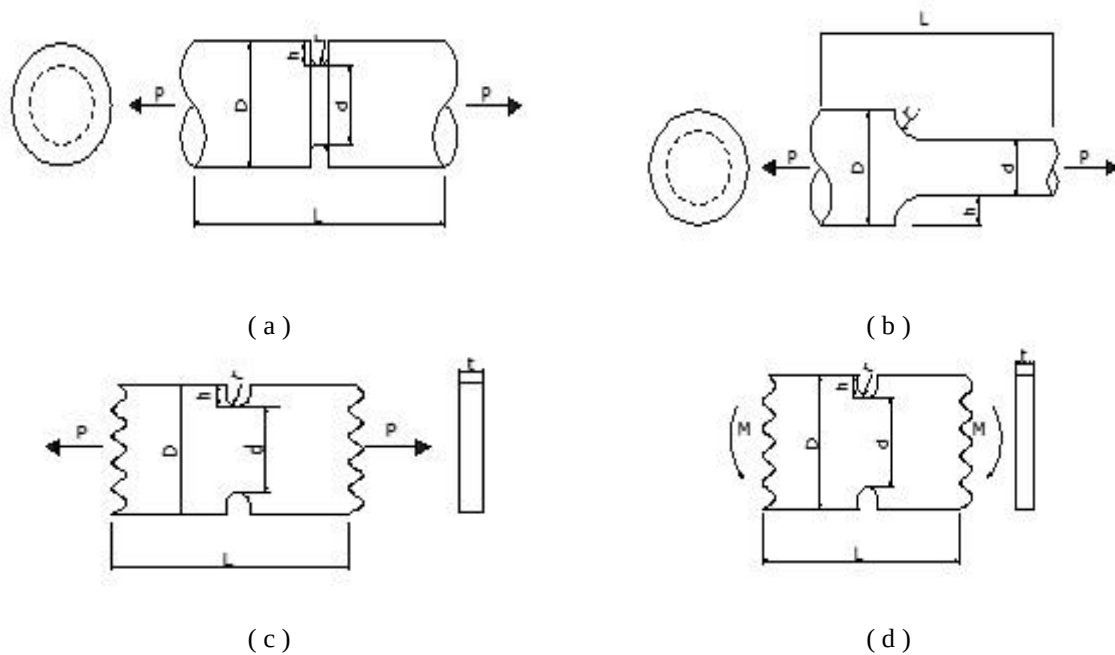
Figure 1 : Finite element analysis; a & b) Von misses mesh, b & c) location of maximum and minimum stresses



IV. Materials of Numerical Simulation Models

The present work is to determine the maximum stress developed at U – shaped circumferential grooved circular shaft and shoulder fillet shaft which loaded by tension stress and U-shaped notched plate that loading by tension and bending stresses. Figure (2) illustrates the geometry of investigated grooved shaft, shoulder fillet shaft and notched plate which made up of carbon steel with young's modulus of (208 GPa), poisson's ratio of (0.28) and density of (7820 Kg / m³) [15]. Considering homogeneous and isotropic material, the diameter (d) and length (L) of the shaft which used in finite element modeling were 40 mm, 150 mm respectively, while the dimension of the tested notch plate was; the width ; d = 40 mm, the length; L = 150 mm; and it's thickness, t = 10 mm. The ratios of (D / d) used in this investigations were (1.1 to 1.5 , by 0.05 step), and the ratios of (r / d) were (0.05 to 0.3, by 0.025 step). The axial and moment loads were (5 KN and 10 N.M) respectively.

Figure 2 : Geometry of investigated specimen; a : u – shape circumferential grooved circular cross section shaft, b : shoulder fillet shaft, c & d ; u – shaped notched plate

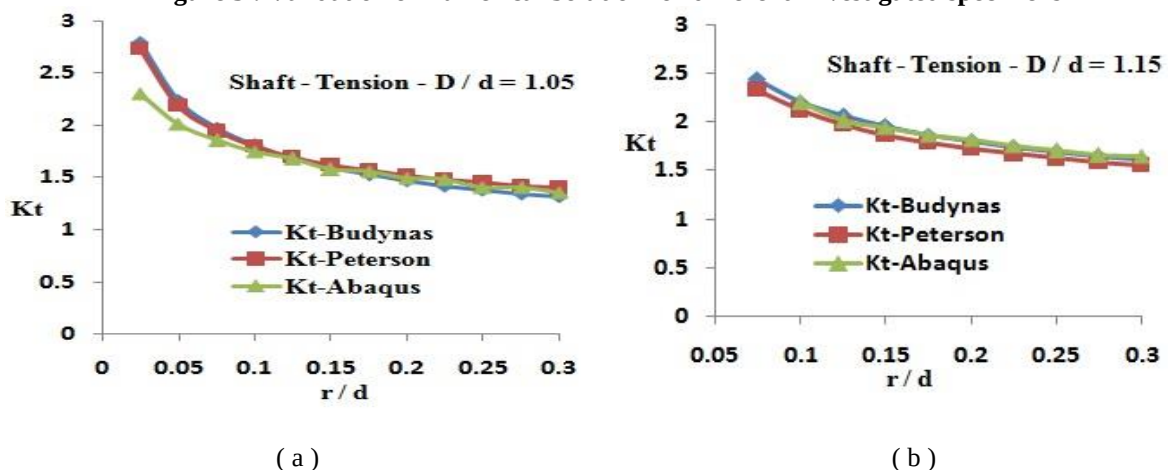


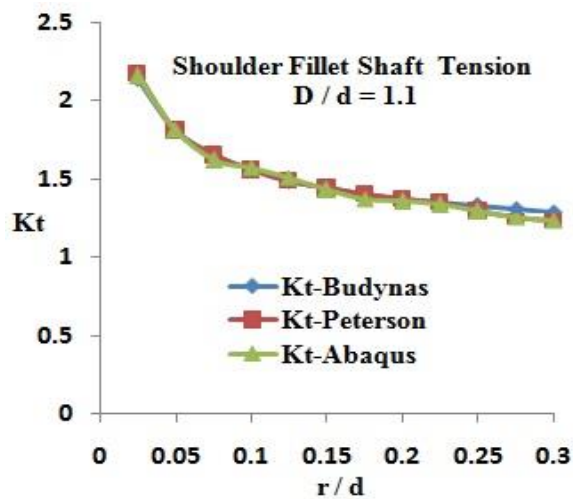
V. Validity of Numerical Simulation

Validation of the finite element approach with results available in the literatures, experimental results or theoretical calculation results which could be obtained by solutions of equations are the most important for acceptance of the FEM model used in the computation [16]. The analytical results have been compared with results of finite element method by using ABAQUS simulation software ver. 6.9.

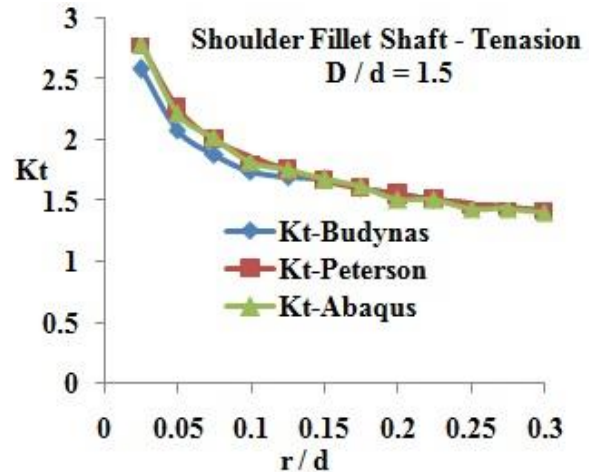
In the present investigations, the software validation was done by comparing the data obtained from simulation performed on cases study with results of the same conditions of stress concentration by solving equation (3), then the results of the maximum stresses obtained from them were used to determine the SCF, K_t . From Figures (3), it is clear that the theoretical calculations of stress stress concentration factor are very close to results of analytical solutions at different parameters such as (D/d) and (r/d) for all study cases. A good agreement is found between the results, and the static analysis shows that the percentage of error between analytical and finite element solution has been found to be less than 3 %. This percentage error affirm that there are no significant differences between the theoretical results and those obtained through simulation. This means, FE modeling using ABAQUS software can be used to determine the SCF for grooved shaft, shoulder fillet shaft and notched plate configurations.

Figure 3 : Validation of Numerical Solution for different investigated specimens

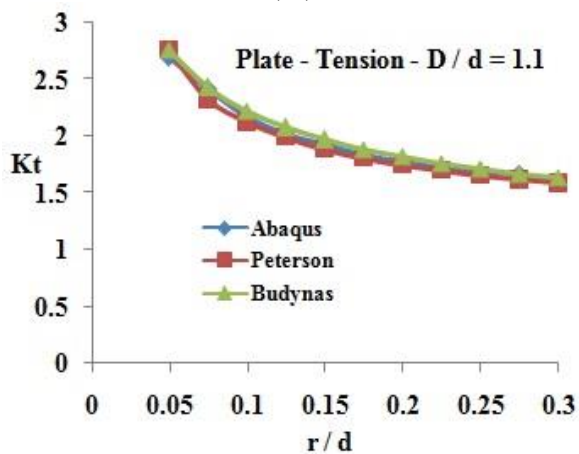




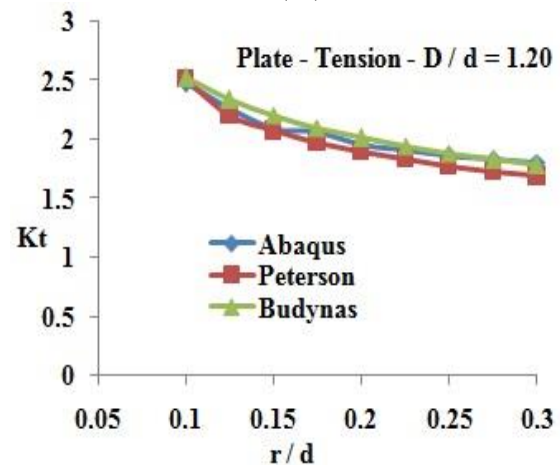
(c)



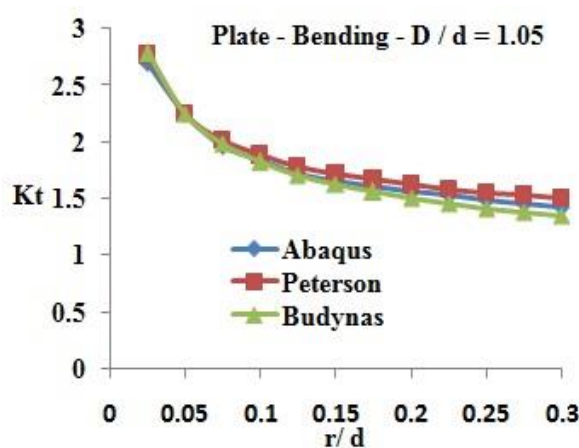
(d)



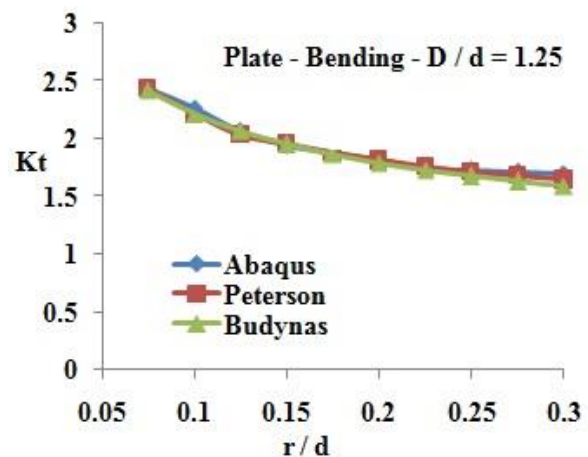
(e)



(f)



(g)



(h)

VI. Results and Discussion

6.1. U – shaped circumferential groove shaft subjected to axial tension

Figure (4) , shows the variation of stress concentration factor (K_t) value with different values of (r/d) ratios. It is clear to notice that, K_t value is more than 2 at low value of (r/d) ratio then decreases with increasing (r/d)

ratio, which it means, the maximum stress generated in the loaded u – shaped grooved shaft decreases with increasing (r/d) ratio. Also, it can be seen, the lower of (D/d) ratio the lower value of (K_t) is. As shown in Figure (5), the location of maximum and minimum stresses is not effecting by varying of both (r/d) and (D/d) ratios.

Figure 4 : Variation of stress concentration factor's values (K_t) with different value of (r/d)

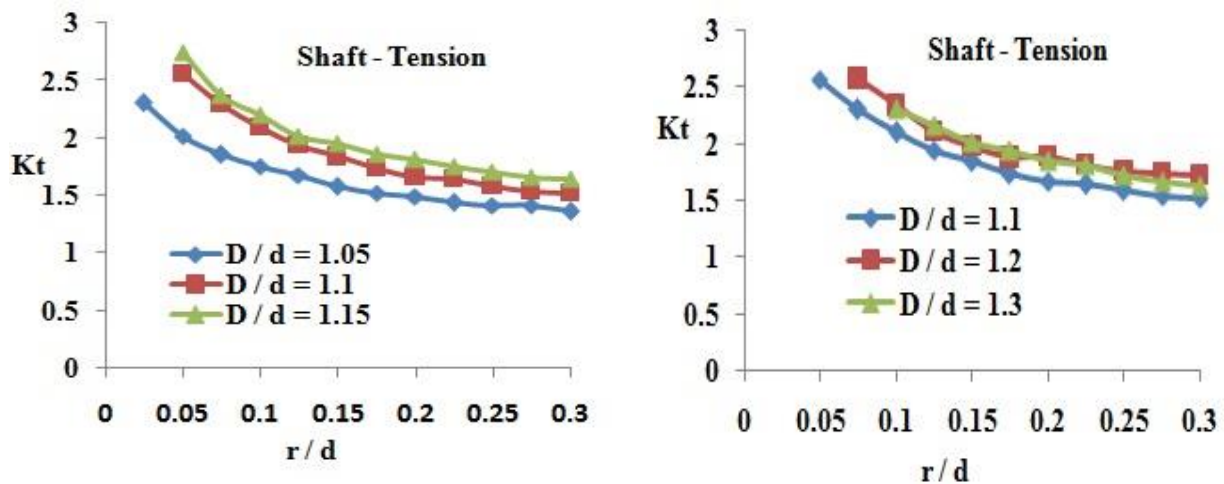
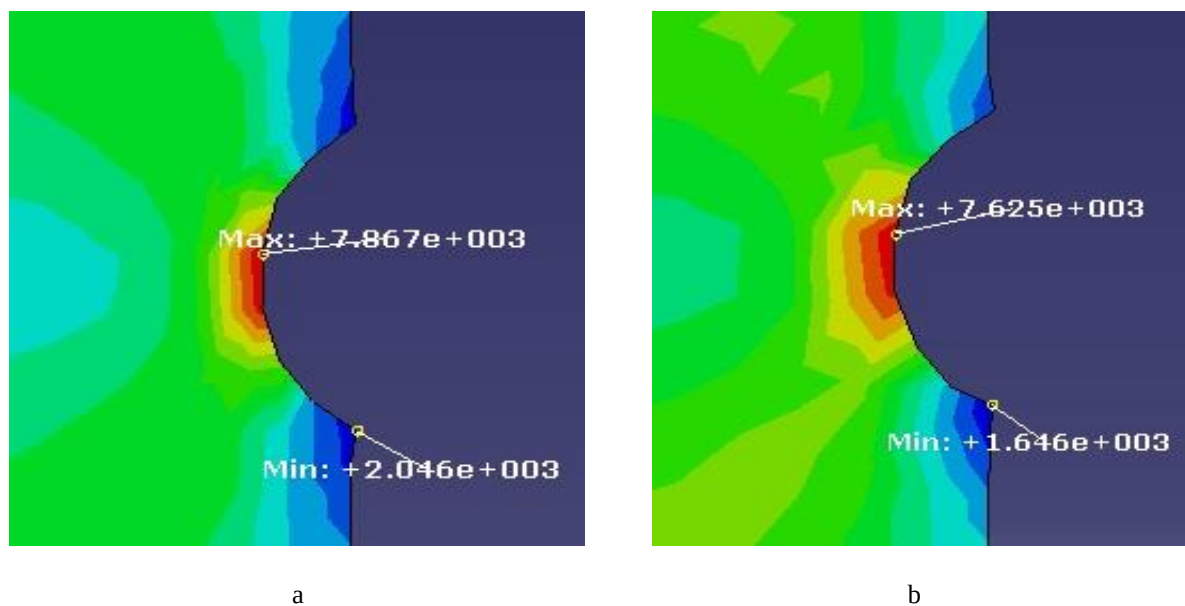


Figure 5 : location of maximum stress concentration of u – shape grooved shaft subjected to axial tension at different (D/d) & (r/d) ratios; a) $D/d = 1.2$ & $r/d = 0.15$; b) $D/d = 1.3$ & $r/d = 0.2$



6.2. Shoulder Fillet of Circular shaft

By comparing the values of (K_t) at different values of (D/d) ratios as shown in Figure (6), there is no considerable differences in values as that have been found in investigated cases of u – shape grooved shaft (see figure 4), where despite of the different values of (D/d) but the K_t 's curves are closely to each other. It means, the depth of fillet (h) has less effect on maximum stress generated due to axial tensile load in shoulder fillet than that have been found in case of u – shape grooved shaft. Also, it clearly appears, the maximum stress arises in shoulder fillet is less than that found in u – shape grooved shaft for all values of (D/d) and (r/d) ratios (see figure 7).

Figure 6: Variation of (K_t) verse (r/d) ratios of Shoulder Fillet Shaft subjected to axial Tension

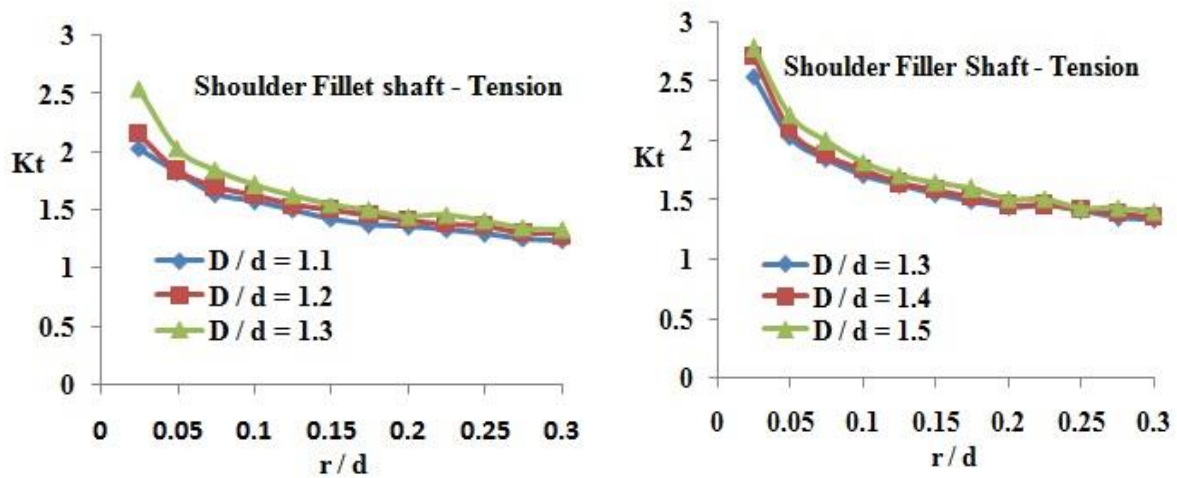
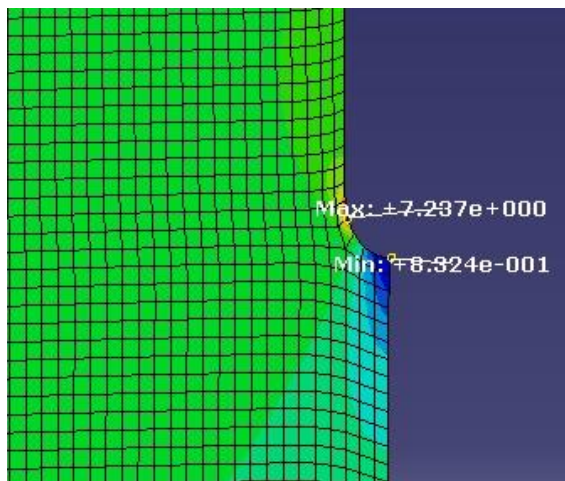
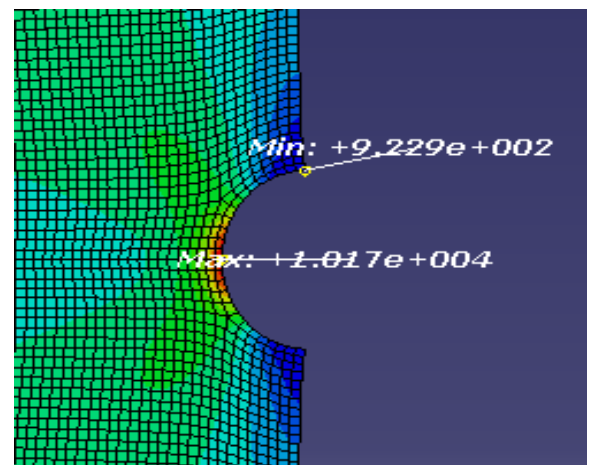


Figure 7 : location & values of maximum stresses arises in shoulder fillet and u – shaped groove shaft subjected to axial tension ; $D/d = 1.1$, $r/d = 0.05$ for both



(a) shoulder fillet shaft



(b) u – shape grooved shaft

6.3. U – Shaped Notched Plate Subjected to axial Tension

The same inverse relationship between (K_t) values and values of (r/d) ratios is found as clearly shown in figure (8), while the relationship between (K_t) value and (D/d) is positive relationship and the bigger value of (D/d), the bigger value of (K_t) is too.

Figure 8 : The relationship between (K_t) values and values' of (r/d) of u – shape notched plate subjected to axial tension.

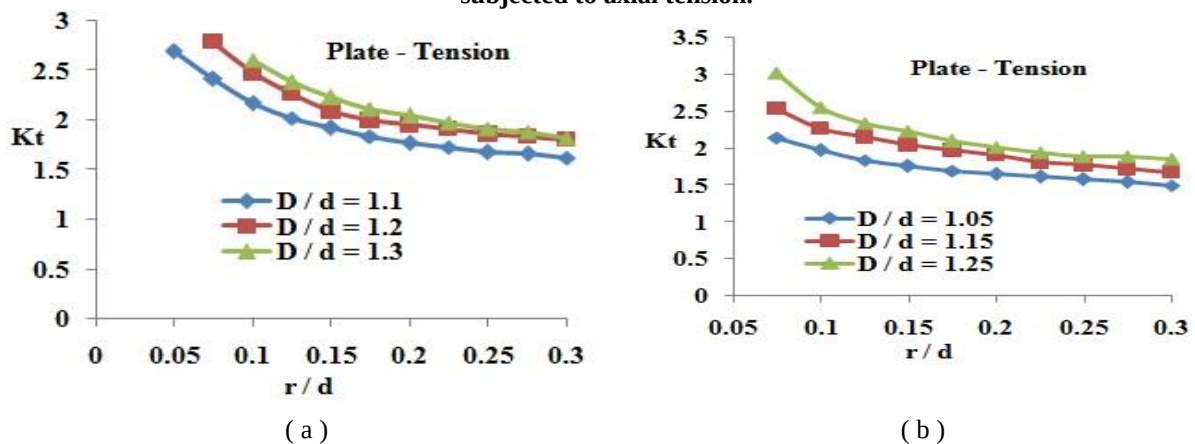
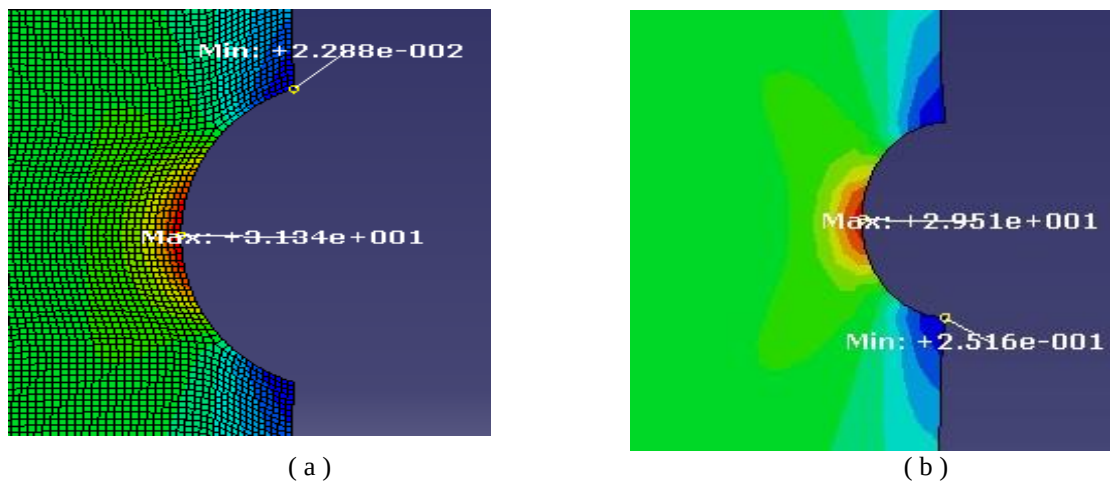


Figure 9 : Location of maximum and minimum stresses of u – shape notched plate subjected to axial tension; a) $D/d = 1.3$ & $r/d = 0.1$, b) $D/d = 1.3$ & $r/d = 0.15$



6.4. U – Shaped Notched Plate Subjected to in – plane Moment

From figure (10), it can be seen, the value of (K_t) decreases with increasing the (r/d) ratio's value for all investigated values of (D/d) ratio. Also, figure (11) shows the maximum and minimum stresses arise in the U – shape notched plate loaded by in – plane bending. There is no significant change in location of stress concentration between an axial tensile load and in – plane bending.

By comparing the values of (K_t) of U – shape notched plate loading by tension with that loaded by in – plane moment, it has been found, the (K_t) value of bending less than that of tension until reaches to ($r/d = 0.25$) then approximately stabilized at constant specific value. This means, the maximum stress arises by moment load is less than that arises by tensile load as can be seen in figures (9 & 11).

Figure 10 : variation of (K_t) values with (r/d) ratios of U – shape notched plate subjected to moment

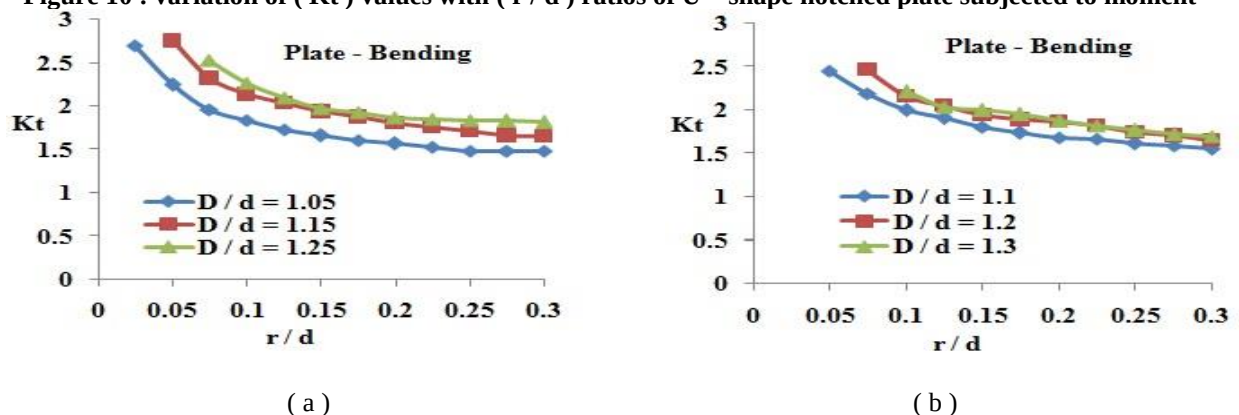
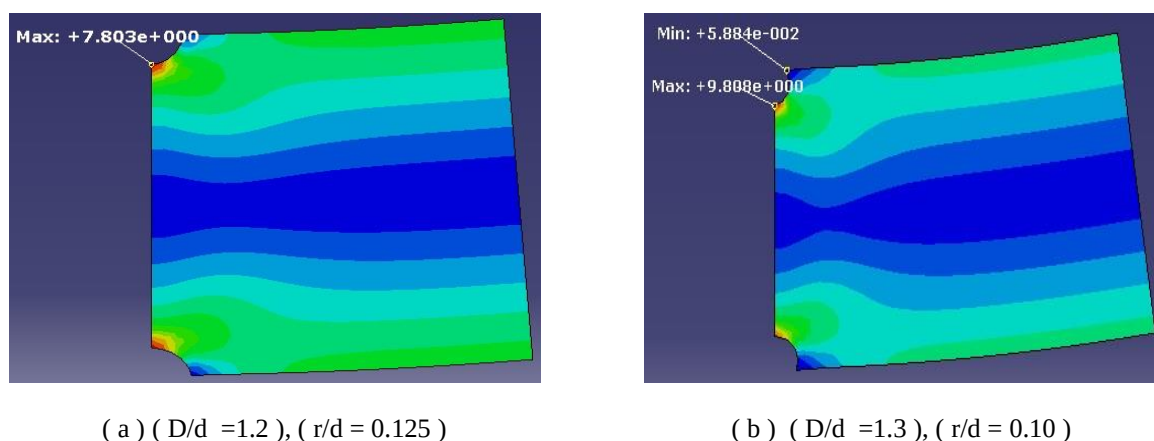


Figure 11 : Location of maximum stress concentration of u – shape notched plate subjected to moment at different (D/d) & (r/d)



VII. Conclusion

The results of present investigation can be summarized as:-

1. Stress concentration factor's (K_t) values have been obtained by Numerical Simulation carried out on ABAQUS software show a good agreement with values have obtained by solving theoretical equations available in literatures.
2. There are inverse relationships between (K_t) values and (r/d) ratios' values, whereas the relationship between values of (K_t) and (D/d) are positive.
3. The depth of groove or fillet (h) has no considerable effect on the location of maximum stress concentration. On other hand, its effect on (K_t) values is bigger in case of u – shape groove shaft than that of shoulder fillet shaft (K_t values for u - shape grooved shaft are bigger than K_t values of fillet shaft for all tested value of (h)).
4. Among other, the ($r/d = 0.2$ until 0.3) gives stable and the less (K_t) value for all investigated cases, so it is the best one.

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