



A Connection between Rectangle and Jarasandha Numbers with digits 2,4,5 and 6

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Abstract: We search for rectangles R with dimensions x, y such that $xy + k(x + y) + k^2 = \text{Jarasandha numbers } 81,2025,3025,9801,88209,494209,998001$ in turn. Total number of rectangles satisfying the relation under consideration is also presented.

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I. Introduction

The problems of Heron of Alexandria and Maximus Planude on pairs of rectangles have engaged the attention of Mathematicians since antiquity as can be seen from [1,2]. In [3], a problem dealing with two non-zero integral pairs (x, y) and (u, v) representing the length and breadth respectively of two rectangles which are related such that $u + v = \alpha(x + y)$, $xy = \beta uv$; $\alpha, \beta \in \mathbb{Z}^+ - \{0\}$ has been studied. Some interesting relations among the breadths and lengths of the rectangles have been analyzed. In [4], infinitely many rectangles, where, in each of which, the sum of the area and perimeter is expressed as the difference of squares of the sides, are obtained. In [5], infinitely many rectangles, where, in each of which, the ratio area/perimeter is a perfect square, are obtained. In [6], different pairs of rectangles and Pythagorean triangle, where, each pair satisfies the relation that the sum of the perimeters of a rectangle and a Pythagorean triangle is equal to twice the area of the rectangle added with two, are obtained. In [7], an attempt has been made to search for two non-zero distinct integers such that each of the expressions $na_0, a_0 + a_1$ and $na_0 + a_1, n > 1$ is a perfect square. Also, the process of obtaining the $(n+1)$ tuple $s = \{a_0, a_0, \dots, a_0, a_1\}$, where, the sum of its elements and the sum of any two of its elements is a perfect square, is illustrated. In [8], Patterns of rectangles where, in each of which the area is represented by some polygonal number are given. In [9], the authors have presented pairs of Pythagorean triangles such that, in each pair, the expressions $P_1 - P_2, \frac{2}{3} \left[\frac{A_1}{P_1 - 2z_1} - \frac{A_2}{P_2 - 2z_2} \right]$ represents 6 times the Jarasandha number respectively.

In this communication, we search for rectangles R with dimensions x, y such that $xy + k(x + y) + k^2 = \text{Jarasandha numbers } 81,2025,3025,9801,88209,494209,998001$ in turn. Total number of rectangles satisfying the relation under consideration is also presented.

NOTATIONS:

A=Area of rectangle R,

P= Perimeter of rectangle R,

S= Semi perimeter of rectangle R

DEFINITION:

In our Indian epic Mahabharatha, we come across a demoniac figure 'JARASANDHA'. He had a boon that if he was split into two parts and thrown apart, the parts would region and return to life. In fact, he was given life by the two halves of his body. In the field of Mathematics, we have numbers exhibiting the same property as Jarasandha.

Let N be a positive integer which may be split as two numbers X and C . If these numbers are added and squared, we get the same number XC . ie, $(X + C)^2 = XC$. It is worth to note here that if C is taken to represent a 'n' digit number then we write $(X + C)^2 = 10^n X + C$

II. Method of Analysis:

Let x, y be two non-zero distinct positive integers representing the length and breadth of rectangle R. Let $k \geq 0$ be any given integer.

Let $N_1 = 81$, a 2-digit-Jarasandha number

The problem under consideration is to solve the equation

$$xy + k(x + y) + k^2 = 81 \quad (1)$$

Introduction of the linear transformations $x = u + v, y = u - v; (u > v > 0)$ (2)

in (1) gives $(u + k)^2 - v^2 = 81$

Applying the method of factorization, we have

$$u + k = 41, 15$$

$$v = 40, 12$$

From the above values, the results observed are presented in the table below:

Table for N_1 :

k	Number of rectangles related to 81	Relations
0	2	A=81
1	1	A+S+1=81
2	1	A+P+4=81

A similar observation is made on the Jarasandha numbers given by $N_2 = 2025, N_3 = 3025, N_4 = 9801, N_5 = 88209, N_6 = 494209, N_7 = 998001$

The corresponding results are presented in the following tables:

Table for N_2 & N_4 :

$N_2 = 2025$		$N_4 = 9801$	
k	Number of rectangles related to 2025	k	Number of rectangles related to 9801
0	7	0	7
1,2	6	1,2	6
3,4	5	3-8	5
5-8	4	9,10	4
9-14	3	11-26	3
15-24	2	27-32	2
25,26	1	33-80	1

Table for N_3 & N_6 :

$N_3 = 3025$		$N_6 = 494209$	
k	Number of rectangles related to 3025	k	Number of rectangles related to 494209
0	4	0	4
1-4	3	1-18	3
5-10	2	19-36	2
11-24	1	37-360	1

Table for N_5 & N_7 :

$N_5 = 88209$		$N_7 = 998001$	
k	Number of rectangles related to 88209	k	Number of rectangles related to 998001
0	10	0	10
1,2	9	1,2	9
3-8	8	3-8	8
9,10	7	9-26	7
11-26	6	27-36	6
27-32	5	37-80	5
32-80	4	81-110	4
81-98	3	111-242	3
99-120	2	243-332	2
121-242	1	333-728	1

Note: As in Table for N_1 , one may obtain the relations between A,P,S and the Jarasandha numbers(N_2 to N_7) for each k from each of the above tables.

III. Conclusion

In this paper, we have presented rectangles in connection with 2,4,5 and 6 digit Jarasandha numbers. In particular, we have presented rectangles in connection with their area, perimeter, semi perimeter and the above mentioned Jarasandha numbers. To conclude, one may search for rectangles with other choices of characterizations related to Jarasandha numbers with digits greater than or equal to 4.

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