



Mathematical Modeling and Simulation of Chemically Reacting Double Diffusive Flow in Low Heat Resistance Sheet under Radiation and Viscous Heating Effect

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Abstract: The objective of this paper is to analyze the radiation and mass transfer effects on low-heat-resistance sheet that moves downwards in a ,electrically conducting chemically reacting fluid, along a vertical moving semi-infinite permeable plate with suction, embedded in a uniform porous medium, in the presence of transverse magnetic field , by taking into account the effects of viscous dissipation. The equations of continuity, linear momentum, energy and diffusion, which govern the flow field, are solved by using a regular perturbation method .The Spectral collocation method is adopted to find the numerical solution of the nonlinear coupled differential Equations. The solution is done computationally using FORTRAN 77 programming Language . The behavior of the velocity, temperature, concentration, skin-friction, Nusselt number and Sherwood number has been discussed for variations in the governing parameters.

I. Introduction

The process of developing a mathematical model is termed **mathematical modeling**. Mathematical models are used in the natural sciences (such as physics, biology, earth science, meteorology) and engineering disciplines (such as computer science, artificial intelligence), as well as in the social sciences (such as economics, psychology, sociology, political science). A model may help to explain a system and to study the effects of different components, and to make predictions about behavior

With the development of computer technology, numerical simulation has been more and more widely used in many fields of our society. Simulation techniques not only play very important roles in scientific study, but also occupy very important places in education, military, entertainment and almost any fields.

It is well known that most fluids which are encountered in chemical and allied processing applications do not satisfy the classical Newton's law and are accordingly known as non-Newtonian fluids. Due to the important applications of non-Newtonian fluids in biology, physiology, technology, and industry, considerable efforts have been directed toward the analysis and understanding of such fluids Bejan and Khair [6] studied the buoyancy induced heat and mass transfer from a vertical plate embedded in a saturated porous medium.

At high operating temperatures, radiation effect can be quite significant. Many processes in engineering areas occur at high temperatures and knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles are example of such engineering areas. Takhar et al. [7] studied the radiation effects on MHD free convection flow for nongray-gas past semi-infinite vertical plate. Ghaly and Elbarbary [9] reported the effect of radiation on free convection flow on MHD along a stretching surface with uniform free stream. Anjali Devi and Kayalvizhi [10] presented analytical solution of MHD flow with radiation over a stretching sheet embedded in a porous medium.

In all the studies mentioned above the heat due to viscous dissipation is neglected. Gebharat [11] has shown the importance of viscous dissipative heat in free convection flow in the case of isothermal and constant heat flux at the plate. Israel-Cookey et al [13] investigated the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Suneetha et al [14] investigated radiation and mass transfer effects on MHD free convection flow past an impulsively started isothermal vertical plate with viscous dissipation. . Mohammed Ibrahim and Bhaskar Reddy [15] studied the radiation and mass transfer effects on MHD free convection flow along a stretching surface with viscous dissipation and heat generation. Recently S.Rawat and S. Kapoor [16],[17], [18] have

worked on MHD Flow Heat and Mass Transfer of Micropolar Fluid over a Nonlinear Stretching Sheet with Variable Micro Inertia Density.

However the interaction of chemical reaction and radiation effects of an electrically conducting and diffusing fluid past a stretching surface has received little attention. Hence an attempt is made to investigate the radiation effects on a steady free convection flow near an isothermal vertical stretching sheet in the presence of a magnetic field, heat generation/absorption, viscous dissipation and chemical reaction.

II. MATHEMATICAL MODEL

Let us consider the problem of cooling of a low-heat-resistance sheet that moves downwards in a viscous fluid when the velocity of the fluid far away from the plate is equal to zero. The variation of surface temperature are linear. The flow configuration and coordinate system is shown in Fig.5.A. All the fluid properties are assumed to be constant except for the density variations in the buoyancy force term of linear momentum. The magnetic Reynolds number is assumed to be small, so that the induced magnetic field is neglected. Electric field is assumed to exist and both viscous and magnetic dissipation are neglected. The Hall Effect, viscous dissipation and the joule heating term are neglected. Under these assumption along with the Bousineque approximation, the boundary layer equation for the problem.

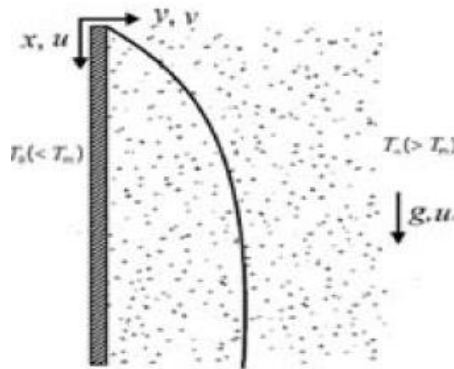


Fig. 1- Physical Model of the Problem

III. GOVERNING EQUATION

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \left(\frac{\partial^2 u}{\partial y^2} \right) - \frac{\sigma_0 B_0^2}{\rho} u + g\beta(T - T_\infty) + g\beta(S - S_\infty) \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + Q_0 (T - T_\infty) + \frac{\mu}{\rho c_p} \left(\frac{\partial u}{\partial y} \right)^2 \quad (3)$$

$$u \frac{\partial S}{\partial x} + v \frac{\partial S}{\partial y} = \sigma \frac{\partial^2 S}{\partial y^2} + \Gamma S \quad (4)$$

The fluid is assumed to be slightly conducting, and hence the magnetic field is negligible in comparison with the applied magnetic field. It is further assumed that there is no applied voltage, so that electric field is absent. The fluid is considered to be a gray, absorbing emitting radiation but non-scattering medium and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. It is also assumed that all the fluid properties are constant except that of the influence of the density variation with temperature and concentration in the body force term (Boussinesq's approximation).

Then, under the above assumptions, the governing boundary layer equations are Where J is Current density, Γ is the chemical reaction rate parameter.

Neglecting the displacement current, the Maxwell equation and Ohm's law becomes

Where σ is electrical conductivity

By using the Rosseland approximation, we have

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \quad (5)$$

i.e equation reduce to

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad (6)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \left(\frac{\partial^2 u}{\partial y^2} \right) - \frac{\sigma_0 B_0^2}{\rho} u + g\beta(T - T_\infty) + g\beta(S - S_\infty) \quad (7)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{4\sigma^*}{3k^* \rho c_p} \frac{\partial T^4}{\partial y^2} + Q_0 (T - T_\infty) + \frac{\mu}{\rho c_p} \left(\frac{\partial u}{\partial y} \right)^2 \quad (8)$$

$$u \frac{\partial S}{\partial x} + v \frac{\partial S}{\partial y} = \sigma \frac{\partial^2 S}{\partial y^2} + \Gamma S \quad (9)$$

Subject to the boundary conditions

$$u = 0, \quad v = 0, \quad T = T_0, S = S_0 \text{ at } y = 0 \quad (10)$$

$$u \rightarrow 0, \quad T \rightarrow \infty, S \rightarrow \infty \text{ as } y \rightarrow \infty, \quad (11)$$

$$\psi = [g\beta(T - T_\infty)v^2 x_0^3]^{\frac{1}{4}} f(\eta), \quad (12)$$

$$T = T_\infty + (T - T_\infty) \left[\frac{x_0}{x_0 - x} \right]^3 \theta(\eta), \quad (13)$$

$$S = S_\infty + (S - S_\infty) \left[\frac{x_0}{x_0 - x} \right]^3 \phi(\zeta) \quad (14)$$

$$\eta = \left[\frac{g\beta(T - T_\infty)x_0^3}{v^2} \right]^{\frac{1}{4}} \frac{y}{(x_0 - x)}, \quad (15)$$

$$\zeta = \left[\frac{g\beta(S - S_\infty)x_0^3}{v^2} \right]^{\frac{1}{4}} \frac{y}{(x_0 - x)} \quad (16)$$

$$\frac{d^3 f}{d\eta^3} - \left(\frac{df}{d\eta} + M \right) \frac{df}{d\eta} + Gr\theta - Gc\phi = 0 \quad (17)$$

$$\frac{1}{Pr} \frac{d^2 \theta}{d\eta^2} + 3 \frac{df}{d\eta} \theta + 4(1 + r\theta)^3 \frac{d^2 \theta}{d\eta^2} + 12r(1 + r\theta)^2 \left(\frac{d\theta}{d\eta} \right)^2 + 3fQ\theta + 3Fec \left(\frac{d^2 f}{d\eta^2} \right)^2 = 0 \quad (18)$$

$$\frac{1}{Sc} \frac{d^2 \phi}{d\eta^2} + 3 \frac{df}{d\eta} \phi - Kr\phi = 0 \quad (19)$$

η, ζ are the dimensionless y-coordinate

Kr is the local dimensionless chemical reaction rate parameter. Q is the internal heat, q_r is the radiation parameter, $r = \frac{T_w - T_\infty}{T_\infty} Pr = \frac{\theta}{\alpha}$ is the Prandtl Number, $Sc = \frac{\nu}{D}$ is the Schmidt Number, θ is the dimensionless temperature and ϕ is the dimensionless concentration. f is the dimensionless stream function,

It is noteworthy that the local parameters, Gr and Gc in Equations are functions of x . However, in order to have a similarity solution all the parameters, Gr, Gc, Ec

IV. NUMERICAL SIMULATION AND COMPUTATION :

The Spectral collocation method is adopted to find the numerical solution of the nonlinear coupled differential Equations (17) –(19) under the boundary condition (10)-(11). Solution is done computationally using FORTRAN 77 programming language. The comparison is also made with the finite difference technique which is available in literature.

V. RESULT AND DISCUSSION

As a result of the numerical calculations, the dimensionless velocity, temperature and concentration distributions for the flow under consideration are obtained and their behavior have been discussed for variations in the governing parameters viz., the thermal Grashof number Gr , solutal Grashof number Gc , magnetic field parameter M , Radiation parameter F , the parameter of relative difference between the temperature of the sheet and temperature far away from the sheet r , Prandtl number Pr , Eckert number Ec , heat generation parameter Q and Schmidt number Sc . In the present study, the following default parametric values are adopted. $Gr = 2.0$, $Gc = 2.0$, $M = 0.5$, $Pr = 0.71$, $F = 1.0$, $r = 0.05$, $Q = 0.1$, $Kr = 0.5$, $Sc = 0.6$, $Ec = 0.01$. All graphs therefore correspond to these unless specifically indicated on the appropriate graph.

In order to get a physical insight into the problem, extensive numerical computation has been carried out for different values of the parameters entering the problem and the corresponding results are depicted in figs. 1-10. Throughout we have assumed the following default values: $Sc = 0.2$, $Re^2 = 25$, $Gr = 5$, $M = 1.0$, $Nr = 0.5$, $\Phi = 0.1$, $Ec = 0.2$, $Pr = 0.72$, $K = 0.5$, $R = 0.5$ (unless otherwise indicated).

The effect of suction/blowing parameter on the velocity and temperature profile has been shown in Fig. 1 and fig.2 respectively. As expected, sucking decelerated fluid particles through the porous wall leading to a reduction in the velocity and temperature boundary layers.

Fig. 3 represents the combined effect of magnetic field and chemical reaction parameter on the concentration profile. The presence of magnetic field in the flow field creates a drag force called the Lorentz force which has a tendency to slow down the flow motion. This decrease in flow motion lead to an increase in solute concentration in the fluid flow and hence an increase in concentration boundary layer as shown by figure 3. Similar effect is shown on increasing the chemical reaction parameter as expected.

Figs. 4-6 show the effect of Buoyancy ratio parameter on velocity, temperature and concentration profile. It is observed that an increase in buoyancy parameter lead to a decrease in velocity profile whereas a reverse effect is seen for the temperature and concentration profile. By fig. 4, it is found that an increase in buoyancy ratio lead

to an increase in velocity. This increase in velocity profile results in an increase in heat and concentration transport thus leading to a decreased temperature and concentration profile as depicted by fig 5. and fig. 6.

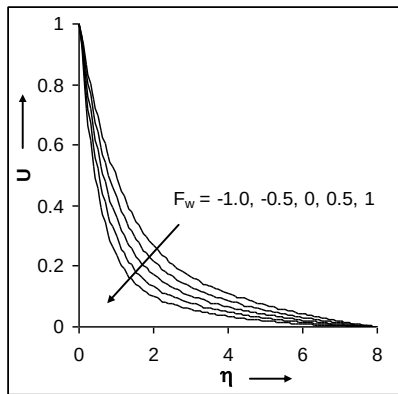


Fig. 1. Velocity for different value of suction parameter

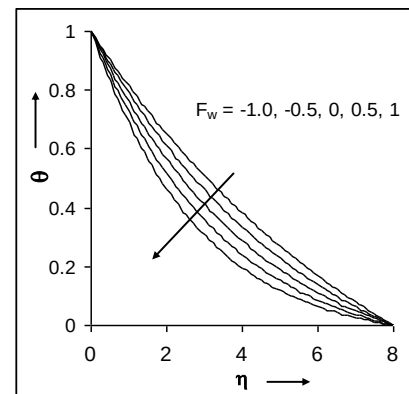


Fig. 2. Temperature for different value of suction parameter

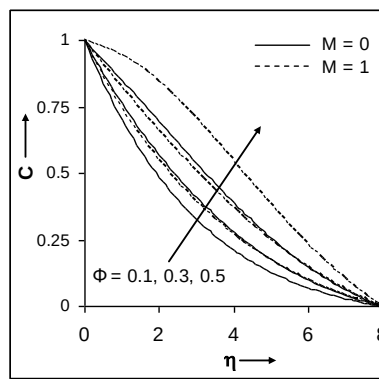


Fig. 3. Concentration for different value of Chemical reaction

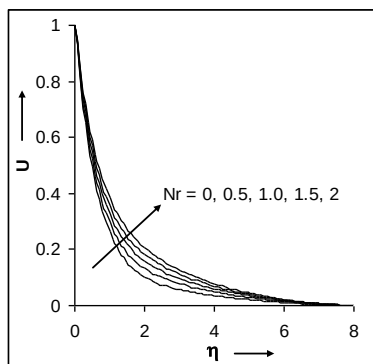


Fig. 4. Velocity for different value of Buoyancy ratio parameter in the presence and absence of magnetic field

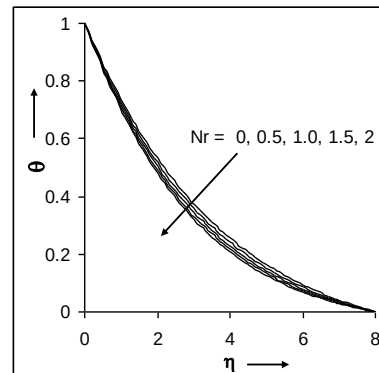


Fig. 5. Temp. for different value of Buoyancy ratio parameter

The effect of Radiation parameter R on the velocity and temperature profiles is shown in figs. 7-8. An increase in radiation parameter seems to have a deceleration effect on both the velocity and temperature profile. This results quantitatively agrees with the expectations, since the effect of radiation is to decrease the rate of energy transport to the fluid, thereby decreasing the temperature of the fluid and hence a decrease in velocity also as depicted by figs. 7 and 8.

Fig.9 shows the effect of Schmidt number on concentration boundary layer. Schmidt number is the relative measure of the effectiveness of momentum and the mass transport in the hydrodynamic and concentration fields. Thus a rise in Schmidt number lead to an increase in mass transport and hence a decrease in the concentration boundary layer as depicted by fig 9

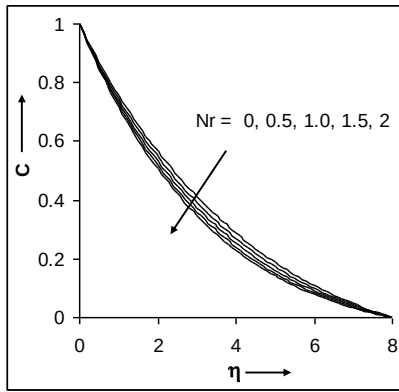


Fig. 6. Temp. for different value of Buoyancy ratio parameter

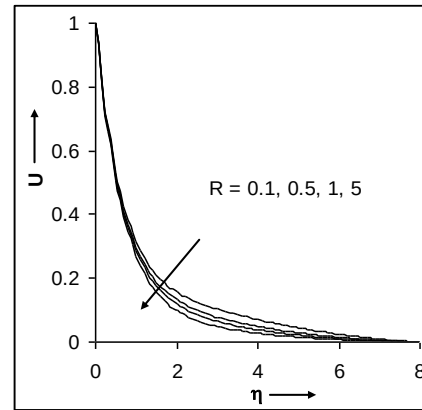


Fig. 7. Velocity for different value of Radiation parameter

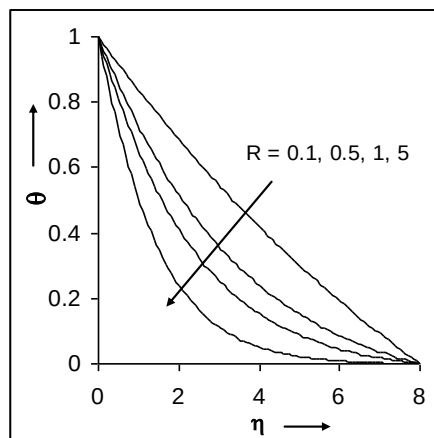


Fig. 8. Temperature for different value of Radiation parameter

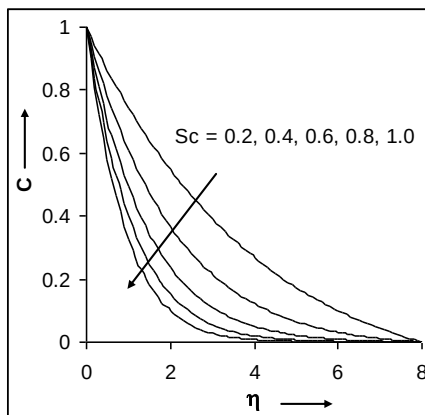


Fig. 9. Concentration for different value of Schmidt number

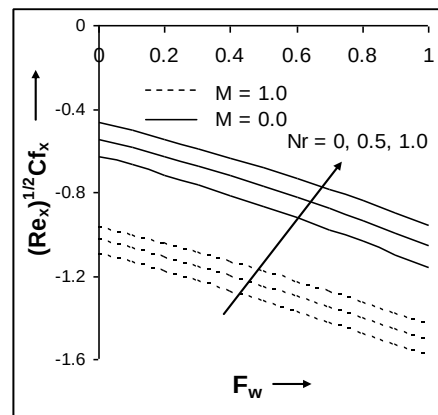


Fig. 10. Skin friction for different value of Magnetic number & Buoyancy ratio parameter with the variation in suction parameter

Numerical results representing the value of skin friction coefficient $U'(0)$ versus Suction parameter (at the vertical surface) for different values of Buoyancy ratio Parameter in the presence and absence of magnetic field is depicted in Fig. 10. Both magnetic field and Suction parameter lead to a decrease in flow velocity and hence this retardation in flow velocity results in the decrease of skin friction at the vertical surface as shown by figure 10. On the other hand, it is also observed that an increase in Buoyancy ratio parameter results in an increase in velocity profile. Hence, an increase in skin friction is observed with an increase in Buoyancy parameter.

VI. CONCLUSION

From the above study we conclude the following.

- i. The Spectral collocation method gives much similar result as we obtained in Finite difference method
- ii. The velocity profiles are found to increase to a certain maximum point and then reduce asymptotically to zero. As Prandtl number increases, the temperature profile and the thermal boundary layer thickness decrease.
- iii. Radiation parameter has a significant effect on the velocity as well as temperature distribution.
- iv. Magnetic field can be used to control the flow characteristics and heat transfer.
- v. Increasing Chemical reaction parameter increases the dimensionless concentration function.
- vi. Increasing Schmidt number decrease dimensionless mass transfer function values.
- vii. Increasing Buoyancy parameter boosts dimensionless velocity function as well as skin friction but has a retarding effect on temperature and mass transfer function.

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