



AN LRS BIANCHI TYPE V MODEL WITH VARYING G AND Λ ALLOWING LATE TIMES ACCELERATIONS

Sanjay Kumar¹ and A. Prasad²

¹Department of Mathematics, S.R.M.S. College of Engineering and Technology, Bareilly (UP) India

²Department of Applied Mathematics, Faculty of Engineering and Technology,
M.J.P. Rohilkhand University, Bareilly (UP) India

Abstract: $G(t)$ and $\Lambda(t)$ are explicitly determined in an LRS Bianchi type V model filled with perfect fluid satisfying barotropic equation of state. Expanding behaviour of the resulting model includes late times accelerations in a particular situation.

Key words: LRS Bianchi V, Barotropic, Varying G and Λ , Late Times Accelerations.

I. INTRODUCTION

Bianchi type-V cosmological models are interesting in the study because its structure is physically and geometrically richer than the standard Friedmann-Robertson-Walker (FRW) models. Bianchi type-V cosmological models have been investigated by a number of researchers, for example, Fransworth, [1]; Collins, [2]; Maartens and Nel, [3]; Wainwright et al., [4]. Roy and Singh, [5] investigated an locally rotationally symmetric (LRS) Bianchi type-V cosmological model in general relativity. Later, Banerjee and Sanyal, [6] investigated the Bianchi type-V cosmological models for viscous fluid distribution and heat flow. Coley, [7] investigated the Bianchi type-V cosmological imperfect fluid cosmological models for the equation of state, i.e. $p = \omega\rho$ where p is the isotropic pressure, ρ the energy density of matter and $0 \leq \omega \leq 1$. Roy and Prasad, [8] obtained some LRS Bianchi type-V cosmological models filled with heat conduction and radiation. Nayak and Sahoo, [9] investigated Bianchi type-V with matter distribution admitting anisotropic pressure and heat flow. Bali and Meena, [10] investigated Bianchi type-V conformally flat tilted cosmological model for perfect fluid distribution in general relativity. Recently Bali and Prateek, [12] have investigated the Bianchi type-V cosmological model for barotropic perfect fluid distribution $p = \omega\rho$ with variable G and Λ , where p is the isotropic pressure, ρ the energy density of matter and $0 \leq \omega \leq 1$.

There have been numerous modifications of general relativity to allow a variable G . Dirac, [13] first considered the possibility of a variable G . These theories have not gained wide acceptance. However, recently a modification (Kalligas, [14]; Rehman, [15]; Berman, [16], [17]; Beesham, [18]) linking the variation of G with that of Λ has been considered within the frame work of general relativity. The cosmological constant problem is one of the outstanding problem in cosmology (Weinberg, [19]), Zel'dovich, [20] and Linde, [21] have studied its significance from time to time. The cosmological term, which is a measure of energy of empty space, provides a repulsive force opposing the gravitational pull between the galaxies. If the cosmological term exists, the energy represents counts as a mass because Einstein has shown that mass and energy are equivalent but recent researches suggest that the cosmological term corresponds to a very small value of order of 10^{-58} cm^{-2} (Johari and Chandra, [22]).

The two universal constants, G (gravitational constant) and Λ (cosmological constant), which appear in Einstein's field equations, if allowed to vary with time, have great significance, specially in the context of an expanding universe. Variations, either in G , or in Λ , or in both, have been a matter of interesting concern while trying to explain the nature of the expansion. Recent examples of such variations are available in the works of Bali and Singh [23]; Singh, Pradhan and Singh [24]; Pradhan and Kumar [25]; Prasad and Kumar [26]; Bahera and Tripathi [27]; Jamil and Devnath [28] and others. A positive Λ or a negative G contributes to the expansion whereas reverse is the situation for a positive G or negative Λ . The overall expansion (whether it is uniform or it is decelerated or accelerated) with the passage of time may be explained in terms of relative values of G and Λ . This requires in the first place that G and Λ must be determined explicitly.

Explicit determination of Λ and G has been achieved in this paper in the framework of an L.R.S. (Locally Rotationally symmetric) Bianchi type V space- time which is filled with perfect fluid. A particular solution of

Einstein's field equations in this framework determines each of $G(t)$ and $\Lambda(t)$ if the cosmic fluid is allowed to obey a barotropic equation of state.

The scale factor (**S**) of the resulting model is arbitrary which we select for discussions conveniently. The usual power law form (viz., $S = t^n$, $n > 0$) renders the model incompatible with the observational fact (vide works of Knop *et al.* [29]; Riess *et al.* [30]; Spergel *et al.* [31]; Tegmark *et al.* [32], and others) of late times accelerations. Instead, it provides a situation of uniformly decelerating expansion. Another specific

form $S = (e^{nat} - 1)^{\frac{1}{n}}$, $a > 0$, $n > 1$; which corresponds to earlier work of Ellis and Madsen, [33], puts the model in tune with above observational fact. The expansion in both the cases is isotropic throughout.

II. DERIVATION OF THE MODEL

We take an LRS Bianchi type V space-time in the form:

$$ds^2 = -dt^2 + A^2(x,t) dx^2 + B^2(x,t) e^{2x} (dy^2 + dz^2), \quad (1)$$

which is filled with a co-moving perfect fluid.

The energy-momentum tensor T_{ij} of the fluid is given by

$$T_{ij} = (\rho + p) v_i v_j + p g_{ij}, \quad (2)$$

in which ρ , p and v_i are respectively the energy density, the pressure and the unit flow vector of the fluid.

The Einstein's field equations:

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi G(x,t) T_{ij} + \Lambda(x,t) g_{ij} \quad (3)$$

Then lead to

$$8\pi G p = \frac{1}{A^2} \left[\left(\frac{B_1}{B} \right)^2 + \frac{2B_1}{B} + 1 \right] - \left(\frac{B_4}{B} \right)^2 - \frac{2B_{44}}{B} + \Lambda, \quad (4)$$

$$8\pi G p = \frac{1}{A^2} \left[\frac{B_{11}}{B} - \frac{A_1 B_1}{AB} + \frac{2B_1}{B} - \frac{A_1}{A} + 1 \right] - \frac{A_4 B_4}{AB} + \frac{A_{44}}{A} + \frac{B_{44}}{B} + \Lambda, \quad (5)$$

$$8\pi G \rho = \frac{-1}{A^2} \left[2 \frac{B_{11}}{B} - 2 \frac{A_1 B_1}{AB} - \frac{2A_1}{A} + \frac{6B_1}{B} + 3 + \left(\frac{B_1}{B} \right)^2 \right] + \left(\frac{B_4}{B} \right)^2 + 2 \frac{A_4 B_4}{AB} - \Lambda, \quad (6)$$

$$\text{and } 0 = \frac{B_{14}}{B} - \frac{A_4 B_1}{AB} - \frac{A_4}{A} + \frac{B_4}{B}. \quad (7)$$

$$\text{Vanishing divergence of } T_{ij} \text{ leads to } \rho_4 + (\rho + p) \left(\frac{A_4}{A} + 2 \frac{B_4}{B} \right) = 0, \quad p_1 = 0 \quad (8)$$

whereas that of $(-8\pi G T_{ij} + \Lambda g_{ij})$ gives

$$0 = -8\pi (G_1 p - G_4 \rho) + 8\pi G \left\{ -p_1 + \rho_4 + (p + \rho) \left(\frac{A_4}{A} + 2 \frac{B_4}{B} \right) \right\} + \Lambda_1 + \Lambda_4, \quad (9a)$$

$$\text{in which } 8\pi (G_1 p + G p_1) = \Lambda_1 \quad (9b)$$

$$\text{and } 0 = \Lambda_4 + 8\pi \left[G_4 \rho + G \left\{ \rho_4 + (\rho + p) \left(\frac{A_4}{A} + 2 \frac{B_4}{B} \right) \right\} \right]. \quad (9c)$$

The suffixes '1' and '4' in the above stand for differentiation with respect to x and t respectively.

Equations (8) and (9) simplify equivalently to

$$\rho_4 + (\rho + p) \left(\frac{A_4}{A} + 2 \frac{B_4}{B} \right) = 0, \quad (10)$$

$$\text{and } \Lambda_4 + 8\pi G_4 \rho = 0. \quad (11)$$

Equations (4), (5), (6), (7), (10) and (11), when considered simultaneously, put in the metric (1) in the form:

$$ds^2 = -dt^2 + e^{2g(t)} [dX^2 + e^{2gX} (dy^2 + dz^2)], \quad (12)$$

in which $dX = e^k dx$ and $s = (m+1)e^{-k}$; where k and m are constants, the functions $g(t)$ being arbitrary.

The form (12) then determines, for barotropic equation of state [$p = w\rho$, $0 \leq w \leq 1$], the quantities ρ , Λ and G as follows:

$$\rho = \ell \cdot e^{-3(w+1)g}, \quad \ell > 0 \quad (13)$$

$$\Lambda = \frac{2}{w+1} \left[\frac{(m+1)^2}{e^{2g+2k}} + g_{44} \right] - \frac{3(m+1)^2}{e^{2g+2k}} + 3g_4^2, \quad (14)$$

$$\text{and } G = -\frac{e^{3(w+1)S}}{4\pi\ell(w+1)} \left[\frac{(m+1)^2}{e^{2g+2k}} + g_{44} \right]. \quad (15)$$

III. DISCUSSION

The model (12) reduces, for $m = -1$, to Friedmann-Robertson Walker model with zero curvature. Also, it reduces to, (1) for $0 \neq m+1 = e^k$. Leaving aside these two trivial cases, we discuss the model in two particular situations of the scale factor.

Firstly, when the scale factor has some positive power of t , i.e. when

$$S = e^{g(t)} = t^n, \quad n > 0; \quad (16)$$

we obtain the following expression :

$$\text{Expansion scalar} = \theta = v_{;i}^i = 3g_4 = \frac{3n}{t}, \quad (17a)$$

the semi-colon standing for covariant differentiation;

$$\text{Shear} = \sigma = 0 \text{ (identically),}$$

$$(17b)$$

$$\text{Deceleration parameter} = q = \frac{-SS_{44}}{(S_4)^2} = \frac{1}{n} - 1,$$

$$(17c)$$

$$\rho = \ell t^{-3(w+1)n},$$

$$(17d)$$

$$\Lambda = \frac{2}{w+1} \left[\frac{(m+1)^2}{e^{2k} t^{2n}} - \frac{n}{t^2} \right] - \frac{3(m+1)^2}{e^{2k} t^{2n}} + \frac{3n^2}{t^2},$$

$$(17e)$$

$$G = \frac{t^{3n(w+1)}}{4\pi\ell(w+1)} \left[\frac{n}{t} - \frac{(m+1)^2}{e^{2k} t^{2n}} \right].$$

$$(17f)$$

Each of θ and ρ , as above, tends to ∞ and 0 when t tends to 0 and ∞ respectively, whereas $\sigma = 0$ throughout. Thus the model starts with a bigbang at $t = 0$ and continues expanding isotropically till $t = \infty$.

In the matter dominated case ($w = 0$), $G \rightarrow \infty$ and $-\infty$ when $t \rightarrow 0$ and ∞ respectively, provided $n < 2/3$. In between, it takes the value zero at $t = \left[\frac{(m+1)^2}{ne^{2k}} \right]^{\frac{1}{2(n+1)}} \equiv t_0$ (say). Also, $\Lambda \rightarrow -\infty$ and 0 respectively when

$t \rightarrow 0$ and ∞ . Thus initially, both $G (>0)$ and $\Lambda (<0)$ oppose the bigbang expansion till $t = t_0$, when G ceases to do so, while Λ continues. After $t = t_0$, $G (<0)$ starts contributing to the expansion whereas Λ retains its earlier role, ofcourse with diminishing strength. But, the overall effect over the expansion is to decelerate it throughout uniformly, as is indicated by the constant value of $q (= 1/n - 1)$ in which $n < 2/3$. In the radiation dominated ($w = 1/3$) situation, all the above discussions hold to be the same qualitatively but for $n < 1/2$.

Next, we choose the scale factor in a form which allows both the decelerated and accelerated phases of expansion. As done in our previous paper [Prasad and Kumar (2009)].

we take the Hubble parameter (H) in the form

$$H = \frac{S_4}{S} = a(S^{-n} + 1), \quad (18)$$

in which $a > 0$ and $n > 1$ are constants. This is in the lines already set by Ellis and Madsen [9].

In view of (18) the function $g(t)$ comes out to be

$$g(t) = \frac{1}{n} \log(e^{nat} - 1), \quad (19)$$

and hence the expressions corresponding to (17) are obtained as

$$\theta = 3a \cdot \frac{e^{nat}}{e^{nat} - 1},$$

$$\sigma = 0,$$

$$q = \frac{n}{e^{nat}} - 1,$$

$$\rho = \frac{\ell}{(e^{\text{nat}} - 1)^{3(\omega+1)n}},$$

$$\Lambda = -\frac{(3w+1)(m+1)^2}{(w+1)e^{2g+2k}} - \frac{2na^2e^{\text{nat}}}{(w+1)(e^{\text{nat}} - 1)^2} + \frac{3a^2e^{2\text{nat}}}{(e^{\text{nat}} - 1)^2},$$

$$G = \frac{e^{3(w+1)S}}{4\pi\ell(w+1)} \left[\frac{na^2e^{\text{nat}}}{(e^{\text{nat}} - 1)^2} - \frac{(m+1)^2}{e^{2g+2k}} \right].$$

This time, θ tends to ∞ and $3a$ when t tends to 0 and ∞ respectively, the corresponding limits for ρ being ∞ and 0 . We have $\sigma = 0$ (again), which is really independent of S . So, the general set-up of the expansion is almost similar to that in the first case. But, Λ , G and q behave somewhat differently.

q is now a function of t and it tends to $n-1$ when $t \rightarrow 0$ and to -1 when $t \rightarrow \infty$. In the matter dominated case ($w=0$), we find that $G \rightarrow \infty$ and $-\infty$ when $t \rightarrow 0$ and ∞ respectively, provided $n=2$, $k=0$, and $2a^2 < (m+1)^2$. Under the same restrictions over the constants, $\Lambda \rightarrow -\infty$ and $3a^2$ respectively when $t \rightarrow 0$ and ∞ . Thus Λ and G initially behave in ways similar to those in the first choice of the scale factor. But, their late time behaviours this

time are really in tune with the observational fact of an accelerated universe. For, at $t = \frac{1}{2a} \log 2 = t^*(\text{say})$

corresponding to $q=0$, both Λ and G are positive and the two are equal provided further that $(m+1)^2 < 4a^2$ and $8\pi\ell = 1$. And beyond $t = t^*$, each of Λ and G tends to contribute to the expansion, which is in tune with the negative limits of q . All these go in favour of the observed (late times) accelerations of the real universe. In the radiation dominated case ($w=1/3$), the above limiting values are the same except that $G \rightarrow 3a^2/\ell$ (finite) when $t \rightarrow 0$, and hence late times accelerations are also justified.

The model can be discussed similarly for other choices of the scale factor which must, of course, be chosen wisely.

IV. CONCLUSION

A particular solution of Einstein's field equations containing variable G and Λ yields explicit determination of these quantities in an LRS Bianchi type V universe filled with perfect fluid provided barotropic equation of state is satisfied. The resulting model has an arbitrary scale factor whose specification in a particular form renders the model allow late times accelerations. Other suitable specifications of the scale factor may reveal other interesting features of the model.

REFERENCES

- [1] Fransworth, D.L. (1967). Some New General Relativistic Dust Metrics Possessing String Isometries. *Journal of Mathematical Physics*. Vol. 8: 2315-2317.
- [2] Collins, C.B. (1974). Tilting at Cosmological Singularities. *Communications in Mathematical Physics*. Vol.39: 131-151.
- [3] Maartens, R. and Nel, S.D. (1978). Decomposable Differential Operators in a Cosmological Context. *Communications in Mathematical Physics*. Vol. 59: 273-283.
- [4] Wainwright, J., Ince, W.C.W. and Marsman, B.J. (1979). Spatially Homogeneous and Inhomogeneous Cosmologies with Equation of State $p=\mu$. *General Relativity and Gravitation*. Vol.10: 259-271.
- [5] Roy, S.R. and Singh, J.P. (1983). LRS Bianchi Type-V Universe Filled with Matter and Radiation. *Astrophysics and Space Science*. Vol. 96: 303-312.
- [6] Banerjee, A. and Sanyal, A.K. (1988). Irrotational Bianchi Type-V Viscous Fluid Cosmology with Heat Flux. *General Relativity and Gravitation*. Vol. 20: 103-113.
- [7] Coley, A.A. (1990). Coley, A.A. (1990) Bianchi V Imperfect Fluid Cosmology. *General Relativity and Gravitation* Vol.22: 3-18.
- [8] Roy, S.R. and Prasad, A. (1994). Some L.R.S. Bianchi type-V Cosmological Models of Local Embedding Class One *General Relativity and Gravitation*. Vol. 26(10): 939-950.
- [9] Nayak, B.K. and Sahoo, B.K. (1996). Exact Bianchi Type-V imperfect Fluid Solution *Gen Relativity and Gravitation*. Vol. 28 (3): 251-261.
- [10] Bali, R. and Meena, B.L. (2004). Conformally Flat Tilted Bianchi Type-V Cosmological Models in General Relativity. *Pramana*, 62, 1007-1014.
- [11] Bali, R. and Yadav, M.K. (2005). Bianchi Type-IX Viscous Fluid Cosmological Model in General Relativity *Pramana J. Phys.* Vol. 64(2): 187-196.
- [12] Bali, R. and Pareek, S. (2007). Bianchi Type -I Cosmological Model For Barotropic Perfect Fluid Distribution With Variable G and Λ *Bulletin of Pure and Applied Sciences (India)* Vol. 26E(2): 327-334.
- [13] Dirac, P.A.M. (1937). The Cosmological Constants. *Nature*. Vol.139 (3512): 323.
- [14] Kalligas, D., Wesson, P. and Everitt, C.W.F. (1992). Flat FRW Models With Variable G and Λ . *General Relativity and Gravitation*. Vol.24: 351.
- [15] Abdel-Rehman, A.M.M. (1990). A critical Density Cosmological Model With Varying Gravitational and Cosmological "Constants". *General Relativity and Gravitation*. Vol.22(6): 665.
- [16] Berman, M.S. (1991a). Cosmological Models with Variable Gravitational and Cosmological "Constants". *General Relativity and Gravitation*. Vol.23: 465-469.
- [17] Berman, M.S. (1991b). Cosmological Models with Variable Cosmological Term. *Physical Review D*. Vol. 43: 1075-1078.
- [18] Beesham, A. (1986). Variable- G Cosmology and Creation. *International Journal Theoretical Physics*. Vol. 25(12): 1295-1298.

- [19] Weinberg, S. (1989). The Cosmological Constant Problem. *Reviews of Modern Physics*. Vol.61: 1-23.
- [20] Zel'dovich, Ya B. (1968). The Cosmological Constant and The Theory of Elementary Particles. *Soviet Physics Usp.*. Vol. 11: 381-393.
- [21] Linde, A.D. (1974). Is the Cosmological Constant a Constant. *ZETP Lett.* Vol. 19(5): 183.
- [22] Johri, V.B. and Chandra, R. (1983). Vacuum Field, Modified Cosmical Term and Non-Singular Cosmology. *Journal of Math. Phys. Sci. India*. Vol.17: 73-86.
- [23] Bali, R. and Singh, J.P. (2008). Bulk Viscous Bianchi Type I Cosmological Models. With Time Dependent Cosmological Term Λ *International Journal of Theoretical Physics*. Vol.47: 3288-3297.
- [24] Singh, J.P. *et al.* (2008). Bianchi Type-I Cosmological Models With Variable G and Λ -term in General Relativity *Astrophysics and Space Science*. Vol.314: 83-88.
- [25] Pradhan, A. and Kumar, S.S. (2009). LRS Bianchi type-III Bulk Viscous Fluid Universe With Decaying Vacuum Energy Density Λ . *International Journal of Theoretical Physics*. Vol. 48: 1466-1477.
- [26] Prasad, A. and Kumar, S. (2009). Late Times Accelerations Viewed Through An LRS Bianchi Type I Cosmological Model. *International Journal of Theoretical Physics*. Vol. 48: 3030-3034.
- [27] Bahera, D. and Tripathi, S. K. (2010). Bulk Viscous Bianchi-III Models With Time Dependent G and Λ . *International Journal of Theoretical Physics*. Vol. 49: 2569-2581.
- [28] Jamil, M and Devnath, U. (2011). FRW Cosmology With Variable G and Λ . *International Journal of Theoretical Physics*. Vol. 50: 1602-1613.
- [29] Knop, R. A. *et al.* (2003). New Constraints on Ω_M and Ω_Λ and ω from an Independent Set of II High Red Shift Supernovae Observed with the Hubble space Telescope. *Astrophysics Journal* Vol.598: 102-137.
- [30] Riess, A.G. *et al.* (2004). Type Ia Supernovae Discoveries at $z > 1$ from the Hubble space Telescope: Evidence for past Deceleration and constraints on Dark energy Evolution. *Astrophysics Journal*. Vol. 607: 665-687.
- [31] Spergel, D.N. *et al.* (2006). : WMAP three years results : Implications for cosmology, preprint, astro-ph/0603449.
- [32] Tegmark, M. *et al.* (2004). Cosmological Parameters from SDSS and WMAP. *Physical Review D*. Vol. 69: 103501-103528.
- [33] Ellis, G.F.R. and Madsen, M. (1991). Exact Scalar field Cosmologies *Class.Quantum and Gravity*. Vol.8: 667-676.c