



Special Pairs of Pythagorean Triangles and Jarasandha Numbers

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Abstract: We present pairs of Pythagorean triangles, such that in each pair, the difference between their perimeters is six times the Jarasandha number. Also the expression $\frac{2}{3} \left[\frac{A_1}{P_1 - 2z_1} - \frac{A_2}{P_2 - 2z_2} \right]$ represents the Jarasandha number. Also we present the number of pairs of primitive and non-primitive Pythagorean triangles.

Keywords: Pairs of Pythagorean triangles, Jarasandha number, Primitive and Non-primitive Pythagorean triangles.

I. Introduction

The fascinating branch of mathematics is the theory of numbers where in Pythagorean triangles have been a matter of interest to various mathematicians and to the lovers of mathematics, because it is a treasure house in which the search for many hidden connection is a treasure hunt. For a rich variety of fascinating problems one may refer [1-10]. Apart from the polygonal numbers we have some more fascinating patterns of numbers namely Jarasandha numbers, Nasty numbers and Dhuruva numbers. These numbers have been presented in [11-14].

In [15-17], special Pythagorean triangles connected with Polygonal numbers and Nasty numbers are obtained. Recently in [18], special Pythagorean triangles in connection with Hardy Ramanujan number 1729 are exhibited. In [19], Pythagorean triangles in connections with 5-digit Dhuruva numbers are presented. In [20], special pairs of Pythagorean triangles and Dhuruva number are presented.

In this communication, we search for pairs of Pythagorean triangles, such that in each pair, the difference between their perimeters is six times the Jarasandha number and also the expression $\frac{2}{3} \left[\frac{A_1}{P_1 - 2z_1} - \frac{A_2}{P_2 - 2z_2} \right]$

represents the Jarasandha number.

II. Basic Definitions

Definition 1

The ternary quadratic Diophantine equation given by $x^2 + y^2 = z^2$ is known as Pythagorean equation where x, y and z are natural numbers. The above equation is also referred to as Pythagorean triangle and denote it by $T(x, y, z)$

Also, in Pythagorean triangle $T(x, y, z)$: $x^2 + y^2 = z^2$, x and y are called its legs and z its hypotenuse.

Definition 2

Most cited solution of the Pythagorean equation is $x = m^2 - n^2, y = 2mn, z = m^2 + n^2$, where $m > n > 0$. This solution is called primitive, if m, n are of opposite parity and $\gcd(m, n) = 1$.

III. Jarasandha Numbers

In our Indian epic Mahabharatha, we come across a Person named 'JARASANDHA'. He had a boon that if he was split into two parts and thrown apart, the parts would rejoin and return to life. In fact, he was given life by the two halves of his body. In the field of Mathematics, we have numbers exhibiting the same property as Jarasandha.

Consider a number of the form XC . This may split as two numbers X and C and if these numbers are added and squared we get the same number XC again.

$$(i.e) \quad XC = (X + C)^2 = (10^n)(X) + C = XC$$

IV. Method of Analysis

Let PT_1, PT_2 be two distinct Pythagorean triangles with generators m, q ($m > q > 0$), and p, q ($p > q > 0$) respectively. Let P_1, P_2 be the perimeters of PT_1, PT_2 and A_1, A_2 be the areas of PT_1, PT_2 such that

$$P_1 - P_2 = 6 \text{ times the 2-digit Jarasandha number } 81.$$

$$\text{and } \frac{2}{3} \left[\frac{A_1}{P_1 - 2z_1} - \frac{A_2}{P_2 - 2z_2} \right] = 2\text{-digit Jarasandha number } 81.$$

The above two relations leads to the equation

$$(2m + q)^2 - (2p + q)^2 = 972 \quad (1)$$

After performing numerical computations, it is noted that there are 12 distinct values for m, q, p satisfying (1). Provided $m+p+q=\text{Jarasandha number}$.

For simplicity and clear understanding, we have presented below in **Table1** the values of m, q, p, P_1 and P_2 .

Table 1:

S.NO.	m	q	p	P_1	P_2	$\frac{P_1 - P_2}{6}$
1.	30	24	27	3240	2754	81
2.	31	22	28	3286	2800	81
3.	32	20	29	3328	2842	81
4.	33	18	30	3366	2880	81
5.	34	16	31	3400	2914	81
6.	35	14	32	3430	2944	81
7.	36	12	33	3456	2970	81
8.	37	10	34	3478	2992	81
9.	38	8	35	3496	3010	81
10.	39	6	36	3510	3024	81
11.	40	4	37	3520	3034	81
12.	41	2	38	3526	3040	81

For simplicity and clear understanding, we have presented below in **Table2** the values of $m, q, p, z_1, z_2, A_1, A_2, P_1$ and P_2 .

Table 2:

m	q	p	P_1	P_2	z_1	z_2	A_1	A_2	$\frac{2}{3} \left[\frac{A_1}{P_1 - 2z_1} - \frac{A_2}{P_2 - 2z_2} \right]$
30	24	27	3240	2754	1476	1305	233280	99144	81
31	22	28	3286	2800	1445	1268	325314	184800	81
32	20	29	3328	2842	1424	1241	399360	255780	81
33	18	30	3366	2880	1413	1224	454410	311040	81
34	16	31	3400	2914	1412	1217	489600	349680	81
35	14	32	3430	2944	1421	1220	504210	370944	81
36	12	33	3456	2970	1440	1233	497664	374220	81
37	10	34	3478	2992	1469	1256	469530	359040	81
38	8	35	3496	3010	1508	1289	419520	325080	81
39	6	36	3510	3024	1557	1332	347490	272160	81
40	4	37	3520	3034	1616	1385	253440	200244	81
41	2	38	3526	3040	1685	1448	137514	109440	81

Thus it is seen that there are 12- pairs of Pythagorean triangles such that for each pair the difference in the perimeters is six times the 2-digit Jarasandha number 81 and also the expression $\frac{2}{3} \left[\frac{A_1}{P_1 - 2z_1} - \frac{A_2}{P_2 - 2z_2} \right]$ represents the 2-digit Jarasandha number 81. Out of these 12-pairs of Pythagorean triangles 5-pairs are non-primitive and in each of the remaining 7-pairs, one of the triangles is primitive and the other is non-primitive triangle.

A similar observation, regarding 4- digit Jarasandha numbers are exhibited in the **Table 3** below:

Table 3:

Jarasandha Number	Pairs of Pythagorean triangles	Pairs of non-primitive Pythagorean triangles	Pairs of primitive and non-primitive Pythagorean triangles
2025	336	122	214
3025	503	-	503
9801	1632	617	1015

V. Conclusion

To conclude, One may search for the connections between the pairs of Pythagorean triangles and other Jarasandha numbers of higher order and other number patterns.

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