



Construction of Interquartile range (IQR) control chart using process capability for range

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Abstract: Any course of action examine by control charts is a quite popular practice in statistical process control. In this research article a new control chart based on robust IQR using process capability for range is proposed instead of Shewhart chart for range when the underlying normality assumption is not met and it enables easier detection of outliers. Furthermore the performances of these charts compared based on numerical examples under the assumptions of normal and exponential distributions. The proposed robust control chart using IQR show to a better performance than the Shewhart control chart for range with moderate sample sizes.

Keywords: Control chart, Control limit interval, Interquartile range and process capability.

I. INTRODUCTION

Statistical process control has been used to great effect in the manufacturing industry to increase productivity in processes by specifically identifying and reducing variation (Deming, 1982). In such a way that the variation fluctuates in a natural or expected manner, a stable pattern of many chance causes of variation develops. Chance causes of variation are inevitable. When an assignable cause of variation is present, the variation will be excessive, and the process is classified as out of control or beyond the expected natural variation. Determination of the common or assignable causes of variation in control chart is possible with the use of control limits. Shewhart (1931) control chart which is one of the most widely used statistical process control technique developed under the normality to monitor the process in order to control the process variability. In this research article a new control chart based on robust IQR using process capability for range is proposed instead of Shewhart (1931) chart for range when the underlying normality assumption is not met and shows to a better performance than the Shewhart control chart for range.

A. Robust methods

This is one of the most commonly used statistical methods when the underlying normality assumption is violated. These methods offer useful and viable alternative to the traditional statistical methods and can provide more accurate results, often yielding greater statistical power and increased sensitivity and yet still be efficient if the normal assumption is correct (Moustafa Omar Ahmed Abu-Shawiesh, 2008). The standard deviation measures spread about the mean. Therefore, it is not practical to calculate the standard deviation when using the median as the measure of central tendency. Other statistics may be more useful when calculating the spread about the median. One statistic that is often used to measure the spread is to calculate the range. The range is found by subtracting the smallest value in the sample, y_1 , from the largest value, y_n . The problem with the range is that it shares the worst properties of the mean and the median. Like the mean, it is not resistant. Any outlier in any direction will significantly influence the value of the range. Like the median, it ignores the numerical values of most of the data. That is not to say that the range does not provide any useful information and it is a relatively easy statistic to compute. In order to avoid the problem of dealing with the outliers, however, we can calculate a different measure of dispersion called the interquartile range (IQR). The interquartile range can be found by subtracting the first quartile value (q_1) from the third quartile value (q_3). For a sample of observations, we define q_1 to be the order statistic below which 25% of the data lies. Similarly, q_3 is defined to be the order statistic, below which 75% of the data lies.

The population IQR for a continuous distribution is defined to be $IQR=Q_3-Q_1$, where Q_3 and Q_1 are found by solving the following integrals $0.75 = \int_{-\infty}^{Q_3} f(x)dx$ and $0.25 = \int_{-\infty}^{Q_1} f(x)dx$. The function $f(x)$ is continuous over the support of X that satisfies the two properties, (i) $f(x) \geq 0$ and $\int_{-\infty}^{\infty} f(x)dx = 1$.

B. Terms and concepts

a. Upper specification limit (USL)

It is the greatest amount specified by the producer for a process or product to have the acceptable performance.

b. Lower specification limit (LSL)

It is the smallest amount specified by the producer for a process or product to have the acceptable performance.

c. Tolerance level (TL)

It is a statistical interval within which, with some confidence **level**, a specified proportion of a sampled population falls. It is the difference between USL and LSL, $TL = USL - LSL$

d. Process capability (C_p)

Process capability compares the output of an in-control process to the specification limits by using capability indices. The comparison is made by forming the ratio of the spread between the process specifications to the spread of the process values, as measured by 6 process standard deviation units. i.e. $C_p = \frac{TL}{6\sigma} = \frac{USL - LSL}{6\sigma}$.

e. Interquartile range (IQR)

The interquartile range (IQR) is a measure of variability, based on dividing a data set into quartiles. These quartiles divide a rank-ordered data set into four equal parts. The values that divide each part are called the first, second, and third quartiles; and they are denoted by Q_1 , Q_2 , and Q_3 , respectively.

- Q_1 is the "middle" value in the first half of the rank-ordered data set
- Q_2 is the median value in the set
- Q_3 is the "middle" value in the second half of the rank-ordered data set

The interquartile range is equal to Q_3 minus Q_1 .

f. Robust quality control constant

The quality control constant σ_{RPC} introduced in this research article to determine the robust control limits based on IQR using process capability for range chart.

II. CONSTRUCTION OF IQR CONTROL CHART USING PROCESS CAPABILITY FOR RANGE

In this division a method to build an IQR control chart using process capability for range and suitable Table – A (APPENDIX I) is also obtained and presented for the companies to take quick decisions. Fix the tolerance level (TL) and process capability (C_p) to find out the process standard deviation (σ_{RPC}). Apply the value of σ_{RPC} in the control limits $\bar{X} \pm \left(\frac{3\sigma_{RPC}}{\sqrt{n}}\right)$, to get the robust control limits using process capability for range (Radhakrishnan et al., 2011), where σ_{RPC} is replaced instead of σ from the Shewhart 3-Sigma.

A. Determination of robust quality control constant under normality

The quality control constant σ_{RPC} is coined by the control limits of Inter quartile range (IQR) in the course of “z-score” that corresponds to the areas under the normal curve of 0.25 and 0.75 respectively. Thus we have $Q_3 = 0.6745 + \mu$ and $Q_1 = -0.6745 + \mu$ implies that $IQR_{norm} \approx 1.3490\sigma$ because of the central limit theorem motivates the use of the normal distribution $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$, $-\infty < x < \infty$.

B. Determination of robust quality control constant under exponential

The quality control constant σ_{RPC} is coined by the control limits of Inter quartile range (IQR) in the course of $0.75 = \int_0^{Q_3} \frac{1}{\beta} e^{-x/\beta} dx$, subsequently solving for Q_3 , becomes $Q_3 = -\beta \ln.25$ and $Q_1 = -\beta \ln.75$ implies that $IQR_{exp} = 1.0986\beta$, where λ is the mean number of occurrences in a unit interval, then $\beta = (1/\lambda)$ is the mean time between events. The density function for the exponential distribution is given by $f(x) = \frac{1}{\beta} e^{-x/\beta}$, $x > 0$. the shape of an exponential distribution is skewed right. Figure 2 is an illustration of an exponential density function when $\beta=1$. For a specified TL and C_p of the process, the value of σ (termed as σ_{RPC}) is calculated from $C_p = (TL/6\sigma)$ using a Visual Basic (VB) program and presented in Table – A (APPENDIX I) for various combinations of TL and C_p .

III. ASSUMPTIONS FOR THE STUDY

- Production managers involved in the study will be willing and able to learn the principles of evaluating control charts
- Production managers involved in the study will have adequate knowledge and experience to make adjustments to an activity to improve the productivity of a process based on the data conveyed in the control charts
- The activities to be studied will feature crews comprised of the same labourers and operators during the pre-intervention and intervention periods

IV. CONDITIONS FOR APPLICATION

- Robust control limits will be used if the data is found to be non-normal
- Companies adopt the concept of IQR using process capability in its processes

V. DETERMINATION OF CONTROL CHARTS

A. Determination of control charts under normality

The example provided by E.L. Grant (1952, Page No. 79) is considered here. The following data gives 120 drawings from the bowl of chips. The drawings are arranged in subgroups of four.

Table I: Drawing from Shewhart's normal bowl of chips arranged

Number of drawings	Makings on chips in subgroup				Range	IQR _{zs}
1-4	47	32	44	35	15	7.78
5-8	33	33	34	34	1	0.74
9-12	34	34	31	34	3	0.56
13-16	12	21	24	47	35	8.15
17-20	35	23	38	40	17	4.82
21-24	19	37	31	27	18	5.56
25-28	23	45	26	37	22	10.19
29-32	33	12	29	43	31	7.97
33-36	25	22	37	33	15	7.23
37-40	29	32	30	13	19	4.08
41-44	40	18	30	11	29	12.05
45-48	21	18	36	34	18	10.56
49-52	26	35	31	29	9	2.78
53-56	52	29	21	18	34	10.75
57-60	26	20	30	20	10	5.19
61-64	19	1	30	30	29	11.49
65-68	28	34	39	17	22	7.41
69-72	29	25	24	30	6	3.34
73-76	21	37	32	25	16	6.86
77-80	24	22	16	35	19	4.63
81-84	28	39	23	21	18	6.12
85-88	41	32	46	12	34	11.30
89-92	14	23	41	42	28	15.20
93-96	32	28	46	27	19	5.74
97-100	42	34	22	34	20	3.71
101-104	20	38	27	32	18	6.12
105-108	30	14	37	43	29	9.27
109-112	28	29	32	35	7	2.97
113-116	35	30	37	26	11	4.82
117-120	51	13	45	55	42	11.12
					$\bar{R} = 19.8$	$\overline{IQR}_{zs} = 6.95$

a. Shewhart control chart for range

The 3σ control limits suggested by Shewhart (1931) are $\bar{R} \pm \left(\frac{3d_3\bar{R}}{d_2}\right)$

$$UCL_{\bar{R}} = \bar{R} + \left(\frac{3d_3\bar{R}}{d_2}\right) = 19.8 + \left(\frac{3 \times 0.88 \times 19.8}{2.059}\right) = 45.19$$

$$CL_{\bar{R}} = \bar{R} = 19.8$$

$$LCL_{\bar{R}} = \bar{R} - \left(\frac{3d_3\bar{R}}{d_2} \right) = 19.8 - \left(\frac{3 \times 0.88 \times 19.8}{2.059} \right) = -5.59$$

However the control limit interval hereafter refers to as CLI, is the difference between the control limits value. Therefore, for the IQR control chart for range, the control limit interval will be determined using the expression:

$$CLI_{\bar{R}} = \left(\frac{3d_3\bar{R}}{d_2} \right) = \left(\frac{3 \times 0.88 \times 19.8}{2.059} \right) = 50.77$$

b. Inter quartile range (IQR) control chart for range

The 3σ control limits based on IQR are $(d_2\overline{IQR}_{z_S})/d_3 \pm (3\overline{IQR}_{z_S})$

$$UCL_{\bar{R},Rob} = 40.67, CL_{\bar{R},Rob} = 19.82 \text{ and } LCL_{\bar{R},Rob} = -1.03$$

However the control limit interval hereafter refers to as CLI, is the difference between the control limits value. Therefore, the IQR control chart for range, the control limit interval will be determined using the expression:

$$CLI_{\bar{R},Rob} = 6\overline{IQR}_{z_S} = 6 \times 6.95 = 41.70$$

c. Proposed inter quartile range (IQR) control chart using process capability for range

Difference between upper specification and lower specification limits is 14.64 (USL - LSL = 15.20 - 0.56), which termed as tolerance level (TL) and choose the process capability (Cp) is 2.0, it is found from the Table - A that the value of σ_{RPC} is 1.22. The control limits of inter quartile range (IQR) using process capability for range for a specified tolerance level with the control limits $(d_2\sigma_{RPC})/d_3 \pm (3\sigma_{RPC})$

$$UCL_{\bar{R},RPC} = 7.14, CL_{\bar{R},RPC} = 3.48 \text{ and } LCL_{\bar{R},RPC} = -0.18$$

However the control limit interval hereafter refers to as CLI, is the difference between the control limits value. Therefore, the IQR control chart using process capability for range, the control limit interval will be determined using the expression: $CLI_{\bar{R},RPC} = 6\sigma_{RPC} = 6 \times 1.22 = 7.32$. From the resulting Figure III, it is clear that the process is out of control, since the group numbers 1, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27 and 30 goes above the upper control limit and the control limit interval is 7.32 for n=4.

Table II: Assessment of Shewhart, IQR and IQR using process capability control charts

Control limits	Shewhart control chart	IQR	IQR using process capability
LCL	-5.59	-1.03	-0.18
CL	19.80	19.82	3.48
UCL	45.19	40.67	7.14
CLIs	50.77	41.70	7.32

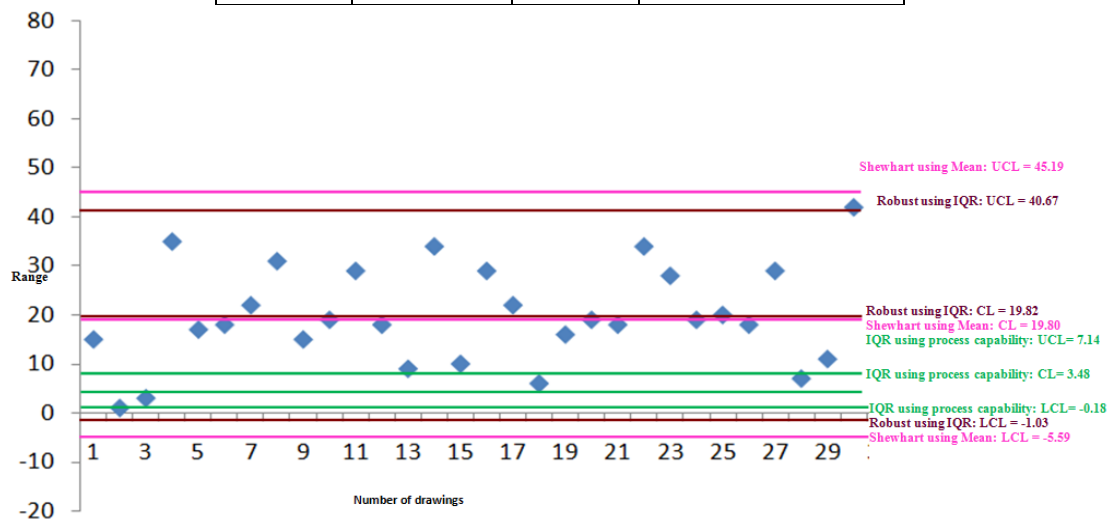


Figure IV: Assessment of Shewhart, IQR and IQR using process capability control chart

It is found from the Figures I that the process is in control when the control limits of Shewhart 3 – Sigma and IQR are adopted and also the process is out of control when the control limits of IQR using process capability are used. The control limits interval of IQR using process capability is smaller than the control limits interval of Shewhart and IQR. It is clear that the product/service is not in good quality as expected, accordingly a modification and improvement is needed in the process/system.

B. Determination of control charts under exponential

A randomly produced data by EasyFit5.6-standard from the exponential distribution is used to illustrate the following method. The randomly generated data consisting 20 subgroups of size n = 5 observations. The performance of the proposed control chart of IQR using process capability for range is compared with Shewhart and IQR under exponential distribution.

Table III: Assessment of Shewhart, IQR and IQR using process capability control charts under exponential

Control limits	Shewhart control chart	IQR	IQR using process capability
LCL	-0.18	-0.18	-0.03
CL	1.59	1.59	0.30
UCL	3.36	3.36	0.63
CLIs	3.54	3.54	0.66

It is found from the Table III, many points fall outside the control limits compared than the control limits of Shewhart 3-Sigma and IQR when the control limits of IQR using process capability adopted. The control limits interval of IQR using process capability is smaller than the control limits interval of Shewhart and IQR control charts. It is clear that the product/service is not in good quality as expected, accordingly a modification and improvement is needed in the process/system.

VI. CONCLUSION

Usually, Shewhart (1931) range control chart for monitoring the process variability is based on some assumptions with standard deviation (σ) which is not a robust estimator. For this cause, we offered the control chart based on robust IQR using process capability for range. The outcome of numerical example and randomly selected data under exponential show that the proposed robust method leads better to the performance in the presence of normality and exponential, such as many points fall outside the control limits than the existing control charts and the control limits interval of IQR using process capability is smaller than the control limits interval of Shewhart and IQR. It is clear that the product/service is not in good quality as expected, accordingly a modification and improvement is needed in the process/system. Furthermore, in the case of non normality, it is recommended to use proposed robust control charts as an alternative to Shewhart control chart for range. The proposed control chart based on IQR using process capability for range will not only assist the producer in providing better quality but also increase the fulfilment and self-assurance of the consumers. Furthermore, this procedure is designed not only to the manufacturing industries alone, but also to the other industries such as Health care, Software, E-commerce industry and so on.

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WEBSITE

- <http://www.mathwave.com/>

Table- A: σ_{RPC} Values for a specified Cp and TL

TL Cp	0.0001	0.0002		0.001	0.002		1	2		50	100	200
1	0	0		0.0002	0.0003		0.17	0.33		8.33	16.67	33.33
1.1	0	0		0.0002	0.0003		0.15	0.30		7.58	15.15	30.30
1.2	0	0		0.0001	0.0003		0.14	0.28		6.94	13.89	27.78
1.3	0	0		0.0001	0.0003		0.13	0.27		6.41	12.82	25.64
1.4	0	0		0.0001	0.0002		0.12	0.24		5.95	11.90	23.81
1.5	0	0		0.0001	0.0002		0.11	0.22		5.56	11.11	22.22
1.6	0	0		0.0001	0.0002		0.10	0.21		5.21	10.42	20.83
1.7	0	0		0.0001	0.0002		0.10	0.20		4.90	9.80	19.61
1.8	0	0		0	0.0002		0.09	0.19		4.63	9.26	18.52
1.9	0	0		0	0.0002		0.09	0.18		4.39	8.77	17.54
2	0	0		0	0.0002		0.08	0.17		4.17	8.33	16.67
2.1	0	0		0	0.0002		0.08	0.16		3.97	7.94	15.87
2.2	0	0		0	0.0002		0.08	0.15		3.79	7.58	15.15
2.3	0	0		0	0.0001		0.07	0.14		3.62	7.25	14.49
2.4	0	0		0	0.0001		0.07	0.14		3.47	6.94	13.89
2.5	0	0		0	0.0001		0.07	0.13		3.33	6.67	13.33