



Invariant Lagrangian for Neutrinos Field in Terms of Divisors

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Abstract: In previous works, Weyl's equation for neutrinos has been written in tensor form, in the form of non-linear Maxwell's like equations, through isotropic complex vector $\vec{F} = \vec{E} + i\vec{H}$. Complex vector $\vec{F} = \vec{E} + i\vec{H}$ satisfies non-linear condition $\vec{F}^2 = 0$, equivalent to two conditions for real quantities $\vec{E}^2 - \vec{H}^2 = 0$ and

$\vec{E} \cdot \vec{H} = 0$, obtained by equating to zero separately real and imaginary parts in the equality $\vec{F}^2 = 0$.

In development of the above ideas, in this work, we wrote the Lagrange function for neutrinos field in terms of divisors.

Keywords: Invariant Lagrangian, neutrinos field, divisors.

I. Introduction

In previous works, Weyl's equation for neutrinos has been written in tensor form, in the form of non-linear Maxwell's like equations through isotropic complex vector $\vec{F} = \vec{E} + i\vec{H}$. It has been proved, that complex vector $\vec{F} = \vec{E} + i\vec{H}$ satisfies non-linear condition $\vec{F}^2 = 0$, equivalent to two conditions for real quantities $\vec{E}^2 - \vec{H}^2 = 0$ and $\vec{E} \cdot \vec{H} = 0$, obtained by equating to zero separately real and imaginary parts in equality $\vec{F}^2 = 0$. It has been proved, that the vectors $\vec{E}$ and $\vec{H}$ have the same properties as those of vectors $\vec{E}$ and $\vec{H}$, components of electromagnetic field. For example, under Lorentz relativistic transformations, they are transformed as components of a second rank tensor $F_{\mu\nu}$. In addition, it has been proved that, the solution of these non-linear equations for free particle as well fulfils Maxwell's equations for vacuum (with zero at the right side).

In development of the above ideas, in this work, we shall write Lagrange function for neutrinos field in terms of divisors.

II. Research Method

In this work, we shall investigate the possibility of elaboration of Lagrange formalism for neutrinos field in tensor formalism, through divisors. Via Cartan map, spinor Weyl's equation for neutrinos has been written in tensor form, in the form of non-linear Maxwell's like equations, through isotropic vector $\vec{F} = \vec{E} + i\vec{H}$. Further, these non-linear equations have been written in covariant form, through divisors K_μ and m_μ . In this work, using the method of tensor analysis, we shall write the Lagrange function for neutrinos field in tensor form, through divisors K_μ and m_μ .

Lagrange Function for Neutrinos Field through Divisors

In previous works, Weyl's equation for neutrinos

$$p_0 \xi = (\vec{p} \vec{\sigma}) \xi, \quad (1)$$

has been written in vector form, through isotropic complex vector $\vec{F} = \vec{E} + i\vec{H}$ as follows

$$D_0 \vec{F} = i\vec{D} \times \vec{F} - (\vec{D} F_i) v_i. \quad (2)$$

Where $D_0 = \frac{i}{2} \frac{\partial}{\partial t}$

$$\vec{D} = \frac{i}{2} \vec{\nabla} \quad (3)$$

$$\vec{v} = \frac{\vec{E} \times \vec{H}}{|\vec{E}|^2}.$$

Here we use the natural system of units in which $c = \hbar = 1$.

Separating real and imaginary parts in equation (2), we obtain

$$\begin{cases} \text{rot}\vec{E} + \frac{\partial\vec{H}}{\partial t} = v_i(\vec{\nabla}H_i) \\ \text{rot}\vec{H} - \frac{\partial\vec{E}}{\partial t} = -v_i(\vec{\nabla}E_i) \\ \text{div}\vec{E} = -v_i\frac{\partial E_i}{\partial t} \\ \text{div}\vec{H} = -v_i\frac{\partial H_i}{\partial t} \end{cases} \quad (4)$$

In terms of complex isotropic vectors, Maxwell's equations for vacuum take the form

$$\begin{cases} D_0\vec{F} = i\vec{D} \times \vec{F} \\ \vec{D} \vec{F} = 0 \end{cases} \quad (5)$$

However, in general, the solution of Maxwell's equations does not satisfy isotropic condition $\vec{F}^2 = 0$, whereas the solution of neutrinos equations always satisfies this condition.

Let us introduce bivector field $F_{\mu\nu}$, satisfying isotropic condition

$$F_{\mu\nu} \tilde{F}_{\mu\nu} = 0. \quad (6)$$

Here $F_{\mu\nu}$ is chosen in the following form

$$F_{\mu\nu} = K_\mu \wedge m_\nu. \quad (7)$$

Where K_μ is a real four vector and m_μ is a complex four vector. In formula (7) the sign " $\wedge$ " means antisymmetrisation over indices μ and ν . The four vectors K_μ and m_μ are called divisors and are expressed through $\vec{E}$ and $\vec{H}$ as follows

$$K_\mu = \left(|\vec{E}|, \frac{\vec{E} \times \vec{H}}{|\vec{E}|} \right), \quad m_\mu = \left(0, \frac{\vec{F}}{|\vec{E}|} \right). \quad (8)$$

Then, equations (4) can be written through divisors in the following covariant form

$$D^\nu F_{\mu\nu} = K^\nu \partial_\mu m_\nu. \quad (9)$$

In spinor formalism, Weyl's equation (1) can be obtained using variation principle from Lagrange function

$$L = \frac{i}{2} (\xi \sigma^\mu \partial_\mu \xi^* - \partial^\mu \xi \sigma_\mu \xi^*). \quad (10)$$

Written through isotropic vectors, formula (10) takes the form

$$L = \frac{i}{2} \{ [D_0\vec{F} - i\vec{D} \times \vec{F} + v_i(\vec{D}F_i)]\vec{F}^* - [D_0\vec{F}^* + i\vec{D} \times \vec{F}^* + v_i(\vec{D})F_i^*]\vec{F} \} \left(\frac{\vec{F}\vec{F}^*}{2} \right)^{1/2}. \quad (11)$$

Formula (11) can be written through divisors as follows

$$L = \frac{i}{2} \{ [D^\nu F_{\mu\nu} - K^\nu \partial_\mu m_\nu] m^{\mu*} - [D^\nu F_{\mu\nu}^* + K^\nu \partial_\mu m_\nu^*] m^\mu \}. \quad (12)$$

III. Discussion and Conclusion

In this work, we solved the problem of elaboration of Lagrange formalism for neutrinos field in tensor formalism through divisors. Via Cartan map, spinor Weyl's equation for neutrinos has been written in tensor form, in the form of non-linear Maxwell's like equations, through strengths $\vec{E}$ and $\vec{H}$. Further, these non-linear equations have been written in covariant form, through divisors K_μ and m_μ . In this work, we elaborated the Lagrange formalism for neutrinos field in tensor formalism. We found the Lagrange function for neutrinos field, written through divisors K_μ and m_μ . It is clear that, applying Noether's theorem, we can derive from the obtained Lagrange function, expressions for fundamental dynamical variables (energy, momentum, charge and spin) conserved in time. These expressions for fundamental dynamical variables have been obtained in previous works.

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