



Analysis of Step Bearing of Hydrodynamic Lubrication

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Abstract: The hydrodynamic theory of viscous lubrication was studied from the Navier-stokes equations by the method of successive approximations, which was based upon the smallness of film thickness. It was founded that the first approximation gives the Reynolds equation. The second order rotatory theory of hydrodynamic lubrication was founded on the expression obtained by retaining the terms containing first and second powers of rotation number in the extended generalized Reynolds equation. In the present paper, there are some new excellent fundamental solutions with the help of geometrical figure, expressions, calculated tables and graphs for the step bearing in the second order rotatory theory of hydrodynamic lubrication. The analysis of equations for pressure and load capacity, tables and graphs reveal that pressure and load capacity are not independent of viscosity. Also the pressure and load capacity both increase with increasing values of rotation number. The relevant tables and graphs confirm these important investigations in the present paper.

Keywords: Continuity, Fluid Film, Pressure, Reynolds equation, Rotation number, Viscosity.

I. Introduction

The analysis of the lubrication film was originally worked out by Osborne Reynolds in 1886, on the fluid flow phenomenon through converging passages, and it was for long accepted that these passages were necessary for film lubrication [11]. After few years in 1946, Fogg analyses the use of thrust bearings with parallel faces, and given the explanation that the thermal expansion of the lubricant generates the thermal wedge. More analysis regarding this has been given by Bower in 1946 and Shaw in 1947 [14]. The temperature distribution in the bearings was analyzed by Christopherson in 1941 and Cameron & Wood in 1946.

The fundamental equations of hydrodynamics were expressed by Cope in 1942 by assuming all the physical properties of the fluid as variables. The flow of lubricants obeys the basic laws of fluid mechanics i.e., the equation of conservation of mass and the momentum conservation equations. The assumption of incompressibility is perfectly adequate in most of the cases. The equation of continuity or the mass conservation equation for an incompressible fluid in Cartesian coordinates is given by

$$\text{div } \mathbf{v} = \partial v_x / \partial x + \partial v_y / \partial y + \partial v_z / \partial z = 0 \quad (1)$$

The equation of momentum conservation or Navier-Stokes equation for a Newtonian fluid also in rectangular Cartesian coordinates (x_1, x_2, x_3) is the statement of the balance of momentum along each of the three the x_i directions

$$\rho \frac{\partial v_{x_i}}{\partial t} + \mathbf{v} \cdot \nabla v_{x_i} = - \frac{\partial p}{\partial x_i} + \eta \nabla^2 v_{x_i} + \rho g_{x_i}, i = 1, 2, 3 \quad (2)$$

Here t is time, $\mathbf{v} = (v_{x_1}, v_{x_2}, v_{x_3})$ is the velocity field vector and $\mathbf{g} = (g_{x_1}, g_{x_2}, g_{x_3})$ is the gravitational acceleration vector. For the analysis of fluid flow in lubricating films the following assumptions are commonly made.

- (a) Negligible body forces
- (b) Steady state conditions
- (c) Constant pressure through film
- (d) Negligible inertia forces
- (e) Newtonian fluid
- (f) Laminar flow
- (g) Constant fluid density
- (h) Rigid and smooth solid surfaces
- (i) No slip at boundaries
- (j) Constant viscosity through film

The equations resulting from the introduction of the above assumptions into the original governing equations of fluid mechanics constitute the statement of lubrication theory. The governing equations of fluid mechanics i.e. the equations of continuity and motion can be combined under the assumptions of lubrication theory to yield a single equation for computing the pressure inside the film, which is said to be Reynolds equation. Consider a lubricating film constrained between two solid surfaces. In a Cartesian coordinate system let the z -axis be located along the direction of the film thickness $h(x, y)$, while the span of the liquid layer on the x - y plane is much larger than its thickness. Moreover, let the fluid motion be driven by the relative velocity (U, V) and be restricted to the x - y plane.

II. Second Order Rotatory Theory of Hydrodynamic Lubrication

In the theory of hydrodynamic lubrication, two dimensional classical theories [4, 10] were first given by Osborne Reynolds [11]. In 1886, in the wake of a classical experiment by Beauchamp Tower [12], he formulated an important differential equation, which was known as: Reynolds Equation [11]. The formation and basic mechanism of fluid film was analyzed by that experiment on taking some important assumptions given as:

- (a) The fluid film thickness is very small as compare to the axial and longitudinal dimensions of fluid film.
- (b) If the lubricant layer is to transmit pressure between the shaft and the bearing, the layer must have varying thickness.

Later Osborne Reynolds himself derived an improved version of Reynolds Equation known as: "Generalized Reynolds Equation" [7, 10], which depends on density, viscosity, film thickness, surface and transverse velocities. The rotation [1] of fluid film about an axis that lies across the film gives some new results in lubrication problems of fluid mechanics [13]. The origin of rotation can be traced by certain general theorems related to vorticity in the rotating fluid dynamics. The rotation induces a component of vorticity in the direction of rotation of fluid film and the effects arising from it are predominant, for large Taylor's Number, it results in the streamlines becoming confined to plane transverse to the direction of rotation of the film [14].

The new extended version of "Generalized Reynolds Equation" [7, 10] is said to be "Extended Generalized Reynolds Equation" [1,3], which takes into account of the effects of the uniform rotation about an axis that lies across the fluid film and depends on the rotation number M [1], i.e. the square root of the conventional Taylor's Number. The generalization of the classical theory of hydrodynamic lubrication is known as the "Rotatory Theory of Hydrodynamic Lubrication" [1, 3]. The "First Order Rotatory Theory of Hydrodynamic Lubrication" and the "Second Order Rotatory Theory of Hydrodynamic Lubrication" [3, 8] was given by retaining the terms containing up to first and second powers of M respectively by neglecting higher powers of M .

III. Governing Equations and Boundary Conditions

In the second order rotatory theory of hydrodynamic lubrication the "Extended Generalized Reynolds Equation" [7] is given by equation (1). Let us consider the mathematical terms as follows:

$$\begin{aligned} & \frac{\partial}{\partial x} \left[-\sqrt{\frac{2\mu}{M\rho}} \frac{1}{M} \left(\frac{\sinh h \sqrt{\frac{M\rho}{2\mu}} - \sin h \sqrt{\frac{M\rho}{2\mu}}}{\cosh h \sqrt{\frac{M\rho}{2\mu}} + \cos h \sqrt{\frac{M\rho}{2\mu}}} \right) \frac{\partial P}{\partial x} \right] + \frac{\partial}{\partial y} \left[-\sqrt{\frac{2\mu}{M\rho}} \frac{1}{M} \left(\frac{\sinh h \sqrt{\frac{M\rho}{2\mu}} - \sin h \sqrt{\frac{M\rho}{2\mu}}}{\cosh h \sqrt{\frac{M\rho}{2\mu}} + \cos h \sqrt{\frac{M\rho}{2\mu}}} \right) \frac{\partial P}{\partial y} \right] \\ & + \frac{\partial}{\partial x} \left[-\frac{h}{M} + \sqrt{\frac{2\mu}{M\rho}} \frac{1}{M} \left(\frac{\sinh h \sqrt{\frac{M\rho}{2\mu}} + \sin h \sqrt{\frac{M\rho}{2\mu}}}{\cosh h \sqrt{\frac{M\rho}{2\mu}} + \cos h \sqrt{\frac{M\rho}{2\mu}}} \right) \frac{\partial P}{\partial y} \right] \\ & - \frac{\partial}{\partial y} \left[-\frac{h}{M} + \sqrt{\frac{2\mu}{M\rho}} \frac{1}{M} \left(\frac{\sinh h \sqrt{\frac{M\rho}{2\mu}} + \sin h \sqrt{\frac{M\rho}{2\mu}}}{\cosh h \sqrt{\frac{M\rho}{2\mu}} + \cos h \sqrt{\frac{M\rho}{2\mu}}} \right) \frac{\partial P}{\partial x} \right] \\ & = -\frac{U}{2} \frac{\partial}{\partial x} \left[\rho \sqrt{\frac{2\mu}{M\rho}} \left(\frac{\sinh h \sqrt{\frac{M\rho}{2\mu}} + \sin h \sqrt{\frac{M\rho}{2\mu}}}{\cosh h \sqrt{\frac{M\rho}{2\mu}} + \cos h \sqrt{\frac{M\rho}{2\mu}}} \right) \right] \\ & - \frac{U}{2} \frac{\partial}{\partial y} \left[-\rho \sqrt{\frac{2\mu}{M\rho}} \left(\frac{\sinh h \sqrt{\frac{M\rho}{2\mu}} - \sin h \sqrt{\frac{M\rho}{2\mu}}}{\cosh h \sqrt{\frac{M\rho}{2\mu}} - \cos h \sqrt{\frac{M\rho}{2\mu}}} \right) \right] - \rho W^* \end{aligned} \quad (3)$$

Where x, y and z are coordinates, U is the sliding velocity, P is the pressure, ρ is the fluid density, μ is the viscosity and W^* is fluid velocity in z -direction. The Extended Generalized Reynolds Equation in view of second order rotatory theory of hydrodynamic lubrication, in ascending powers of rotation number M and by retaining

the terms containing up to second powers of M and neglecting higher powers of M , can be written as equation (2). For the case of pure sliding $W^* = 0$, so we have the equation as given:

$$\begin{aligned} & \frac{\partial}{\partial x} \left[-\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \rho \frac{\partial P}{\partial x} \right] + \frac{\partial}{\partial y} \left[-\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \rho \frac{\partial P}{\partial y} \right] \\ & + \frac{\partial}{\partial x} \left[-\frac{M\rho^2h^5}{120\mu^2} \left(1 - \frac{31M^2\rho^2h^4}{3024\mu^2} \right) \frac{\partial P}{\partial y} \right] - \frac{\partial}{\partial y} \left[-\frac{M\rho^2h^5}{120\mu^2} \left(1 - \frac{31M^2\rho^2h^4}{3024\mu^2} \right) \frac{\partial P}{\partial x} \right] \\ & = -\frac{\partial}{\partial x} \left[\frac{\rho U}{2} \left\{ h - \frac{M^2\rho^2h^5}{120\mu^2} \left(1 - \frac{31M^2\rho^2h^4}{3024\mu^2} \right) \right\} \right] \\ & - \frac{\partial}{\partial y} \left[\frac{M\rho^2U}{2} \left\{ -\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \right\} \right] \quad (2) \\ & \frac{\partial}{\partial y} \left[-\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \rho \frac{\partial P}{\partial y} \right] + \frac{\partial}{\partial x} \left[-\frac{M\rho^2h^5}{120\mu^2} \left(1 - \frac{31M^2\rho^2h^4}{3024\mu^2} \right) \frac{\partial P}{\partial y} \right] \\ & = -\frac{\partial}{\partial x} \left[\frac{\rho U}{2} \left\{ h - \frac{M^2\rho^2h^5}{120\mu^2} \left(1 - \frac{31M^2\rho^2h^4}{3024\mu^2} \right) \right\} \right] \\ & - \frac{\partial}{\partial y} \left[\frac{M\rho^2U}{2} \left\{ -\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \right\} \right] \quad (3) \end{aligned}$$

The step bearing was first used by Lord Rayleigh [4] in 1918. He used the calculus of variation to see which film shape had the biggest load-carrying capacity. He found the best was two parallel zones.

The geometry of step bearing is given by the figure- 1. The figure shows that the entry zone has a gap h_1 and the exit gap is h_2 .

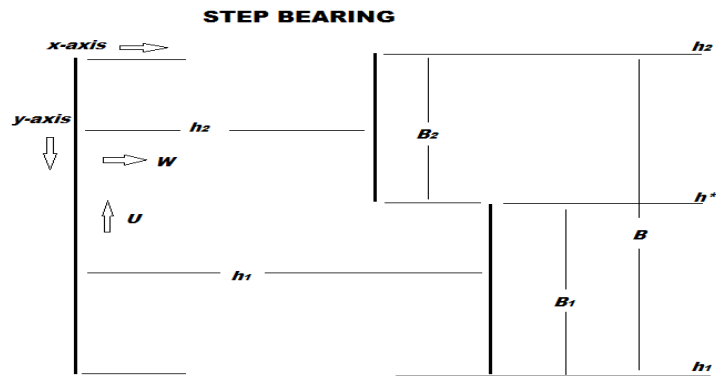


Figure 1: Geometry of Step Bearing

The figure shows that the runner move in the (-y) direction, which implies that the variation of pressure in x-direction is very small as compared to the variation of pressure in y-direction. So the terms containing pressure gradient $\partial p/\partial x$ can be neglected as compared to the terms containing $\partial p/\partial y$ in the differential equation of pressure, hence P may be taken as function of y alone.

Taking $h=h(y)$, $U=-U$, $P=P(y)$ (4)

$$\begin{aligned} & \frac{d}{dy} \left[-\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \rho \frac{dP}{dy} \right] \\ & = \frac{\partial}{\partial y} \left[\frac{M\rho^2U}{2} \left\{ -\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \right\} \right] \end{aligned} \quad (5)$$

When h is constant then the resultant pressures will be zero. Hence a bearing having a constant film thickness has no load capacity. However, if the film is parallel but has step in it such as shown in the figure, the bearing will produce hydrodynamic forces.

The film thicknesses are taken as:

$$h=h_1 \text{ in the region } B_1, \quad (6)$$

$$h=h_2 \text{ in the region } B_2. \quad (7)$$

The boundary conditions for the determination of pressure are:

$$P=0, \text{ when } y=0 \quad (8)$$

$$P=P_c, \quad dP/dy=0 \text{ at } h=h^* \quad (9)$$

Where h^* is determined by equating the two values of P_c derived in regions B_1 and B_2 respectively.

IV. Determination of Pressure

Integrating equation (5) under the boundary conditions (8) and (9), we get the equations for pressure. The pressure for the region B_1 is given by

$$P = M \frac{\rho U B_2}{2} \left\{ \frac{h_1^3 - h_2^3}{h_1^3 B_2 + h_2^3 B_1} \right\} y + M^2 \left[\frac{17 B_2 U \rho^3 y (h_1^3 - h_2^3)(h_1^7 B_2 - h_2^7 B_1) - (h_1^7 - h_2^7)(h_1^3 B_2 + h_2^3 B_1)}{3360 \mu^2} \frac{(h_1^3 B_2 + h_2^3 B_1)}{\left\{ M \left(1 + \frac{17 \rho^2 (h_1^7 B_2 - h_2^7 B_1)}{1680 \mu^2 (h_1^3 B_2 + h_2^3 B_1)} M^2 \right) \right\}} \right] \quad (10)$$

The pressure for the region B_2 is given by

$$P = M \frac{\rho U B_1}{2} \left\{ \frac{h_1^3 - h_2^3}{h_1^3 B_2 + h_2^3 B_1} \right\} y + M^2 \left[\frac{17 B_1 U \rho^3 y (h_1^3 - h_2^3)(h_1^7 B_2 - h_2^7 B_1) - (h_1^7 - h_2^7)(h_1^3 B_2 + h_2^3 B_1)}{3360 \mu^2} \frac{(h_1^3 B_2 + h_2^3 B_1)}{\left\{ M \left(1 + \frac{17 \rho^2 (h_1^7 B_2 - h_2^7 B_1)}{1680 \mu^2 (h_1^3 B_2 + h_2^3 B_1)} M^2 \right) \right\}} \right] \quad (11)$$

V. Determination of Load Capacity

The load capacity W for step bearing is given by

$$W = M \frac{\rho U L}{2} \left\{ 1 - \frac{(17 M^2 \rho^2 h_2^7 - 1680 \mu^2 h_2^3)(B_1 + B_2)}{17 M^2 \rho^2 (h_1^7 B_2 - h_2^7 B_1) - 1680 \mu^2 (h_1^3 B_2 + h_2^3 B_1)} \right\} \int_0^{B_1} y dy + M \frac{\rho U L}{2} \left\{ \frac{(17 M^2 \rho^2 h_1^7 - 1680 \mu^2 h_1^3)(B_1 + B_2)}{17 M^2 \rho^2 (h_1^7 B_2 - h_2^7 B_1) - 1680 \mu^2 (h_1^3 B_2 + h_2^3 B_1)} - 1 \right\} \int_0^{B_2} y dy \quad (12)$$

$$W = M \frac{\rho U L B_1 B_2 (B_1 + B_2)}{4} \left\{ \frac{h_1^3 - h_2^3}{(h_1^3 B_2 + h_2^3 B_1)} + M^2 \frac{17 \rho^2 (h_1^3 - h_2^3)(h_1^7 B_2 - h_2^7 B_1) - (h_1^7 - h_2^7)(h_1^3 B_2 + h_2^3 B_1)}{1680 \mu^2 (h_1^3 B_2 + h_2^3 B_1)} \left(1 + \frac{17 \rho^2 (h_1^7 B_2 - h_2^7 B_1)}{1680 \mu^2 (h_1^3 B_2 + h_2^3 B_1)} M^2 \right) \right\} \quad (13)$$

VI. Numerical Simulation

By taking the values of different mathematical terms in C.G.S. system the calculated tables and graphical representations are as follows:

$$U = 80, \rho = 1, \frac{L}{B} = 1, h_1 = 0.0269, h_2 = 0.0167, e = 0.2, \mu = 0.0002, B_1 = 1, B_2 = 0.5 \quad (14)$$

S.NO.	M	P	W
1.	0.1	2.0565596	2.3136315
2.	0.2	4.1131680	4.6172160
3.	0.3	6.1337808	6.8946132
4.	0.4	8.1358800	9.1528792
5.	0.5	10.100488	11.363063

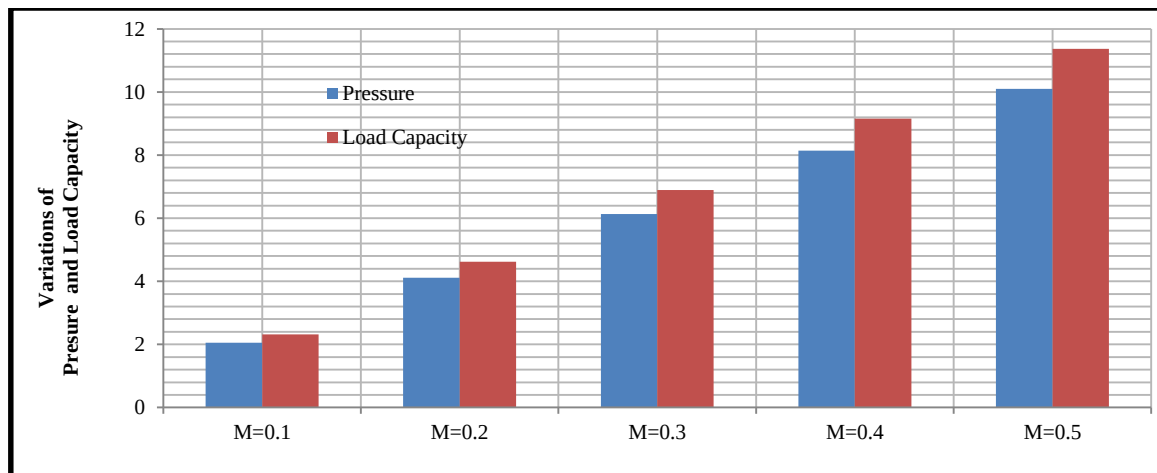


Figure 2: Variations of Pressure and Load Capacity with respect to rotation number M

VI. Conclusions

The variation of pressure and load capacity for *Step Bearings* with respect to rotation number M , when viscosity is constant; are shown by table and graph. Hence in the second order rotatory theory of hydrodynamic lubrication, the pressure and load capacity for step bearings both increases with increasing values of M , when viscosity is taken as constant. The equations for pressure and load capacity show that both of them are not independent of viscosity, it varies with μ .

VII. References

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