



A Non-Differential Approach for Solving Tri-Level Programming Problems

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Abstract: Tri-level programming is a class of multi-level programming in which there are three independent decision-makers (DMs). Each DM attempts to optimize its objective function and is affected by the actions of the others DMs. This paper studies a three-level large scale linear and non-linear programming problem, in which the objective functions at every level are to be maximized. Three-level programming problem can be thought as a static version of the Stackelberg strategy, which is used leader-follower game in which a Stackelberg strategy is used by the leader. The objective of this paper is to apply weighting approach based on AHP to three-level large scale programming problem. Proposed approach does not require any assumption or information regarding the DMs utility function. The method has no special requirement for the characters of the function and overcome the difficulty discussing the conditions and the algorithms of the optimal solution with the definition of the differentiability of the function. Theoretical results are illustrated with the help of a numerical example.

Keywords: Large scale problems, Three-level programming problem, Analytic Hierarchy Process, Optimal solution, Satisfactory solution.

I. Introduction

Multi-level programming is a powerful technique for solving hierarchical decision-making problems. Multi-level optimization plays an important role in engineering design, management, and decision making in general. Ultimately, a designer or decision maker needs to make tradeoffs between disparate and conflicting design objectives. The field of multi-level optimization defines the art and science of making such decisions. The prevailing approach for address this decision-making task is to solve an optimization problem, which yields a candidate solution. A tri-level programming problem (TLPP) is a special case of multi-level programming problem (MLPP). Multi-level programming problem can be defined as a p -person, non-zero sum game with perfect information in which each player moves sequentially from top to bottom. This problem is a nested hierarchical structure. When $p = 3$ we call the system a tri-level programming problem.

Hierarchical optimization or multi-level programming techniques are extension of Stackelberg games for solving decentralized planning problem with multiple DMs in a hierarchical organization. The Stackelberg solution has been employed as a solution concept to bi-level programming problems (BLPPs), and a considerable number of algorithms for obtaining the solution have been employed. TLPP can be thought as a static version of the Stackelberg strategy, which is used leader-follower game in which a Stackelberg strategy is used by the leader. When the Stackelberg solution is employed, it is assumed that there is no communication among the DMs, or they do not make any binding agreement even if there exists such communication. However, the above assumption is not always reasonable when we model decision-making problems in a decentralized firm as a BLPP or TLPP because it is supposed that there exists a cooperative relationship between them. Consider a computational aspect to the Stackelberg solution we note that the problem for obtaining the Stackelberg solution is non-convex programming problem with special structure. From such difficulties a new solution concept, which is easy to compute and reflects the structure of multi-level (or bi-level, tri-level etc.) programming problem is expected. However a few algorithms have been proposed to solve TLPP, several algorithms have been proposed to BLPP [4,5,6,7]. Recently, as an extension of BLPP, the tri-level programming problem has received increasing attention in the literature [1,3,11]. These algorithms are divided into the following classes: global techniques, enumeration methods, transformation methods, meta-heuristic approaches, fuzzy methods, primal-dual interior methods. Also, it has been proven that the MLPPs,

especially TLPPs, are NP-Hard problems. The literature shows a few attempts for solving using TLPP. Eghbal and Kamalabadilisa [1] solved linear TLPP using approaches based on line search and approximate algorithm. They attempted to develop two effective approaches, one based on Taylor theorem and the other based on the hybrid algorithm by combining the penalty function and the line search algorithm for solving the linear TLPP. Lasunon and Remsungnen [3] proposed an algorithm for solving TLPP, which outperforms the K-th-best algorithm in iteration. Taghreed [11] presented an interactive fuzzy goal programming approach to determine the preferred compromise solution to linear TLPP considering the imprecise nature of the decision makers' judgements for the objectives. This paper studies a three-level large scale linear programming problem, in which the objective functions at every level are to be maximized. We deal with the tri-level programming problem with the essentially co-operative decision makers and propose an approach to solve TLPP by weighting method using AHP.

The estimation of the relative weights of criteria plays an important role in a multiple criteria decision analysis (MCDA) process. Mishra [4] consider the solution of a bi-level linear fractional programming problems (BLFFPP) by Weighting method. A non-dominated solution set is obtained by this method. In this paper we deal with the TLPP with the essentially cooperative DMs and propose an approach, to solve TLPP by weighting method using AHP [2,4,6,9,10]. In proposed approach the hierarchical system will be converted into scalar optimization problem (SOP) by finding proper weights for all three objective functions using AHP so that objective functions of all levels can be combined into one objective function. Here the relative weights represent the relative importance of the objective functions. AHP and similar methods often use pairwise comparison matrices for determining the scores of alternatives with respect to a given criterion, or determining values of weight vector. It is a simple method to apply to the tri-level systems compared to the other transformation method. The proposed approach really depends on the configuration of the system, it's over all management and the relative importance of a DM with respect to other DMs in the system. Finally, a numerical example is given to clarify the main results developed in this paper.

II. Tri-level programming problem (TLPP)

Consider a programming problem in which the government is at first level. During the planning period, the government proposes certain goals. In order to optimize the achievement of such goals, it formulates certain policy measures such as taxes and subsidies. The industries at the second level design their course of action keeping such policy measures in mind so that their objectives are fulfilled. The industries supply their products to the consumers in a certain area. The customers at the third level are at liberty to make their purchases from any industries. In doing so, the customers will consider economic criteria such as cost optimization. This is a three level programming problem [8] in which the government's objectives are at least in partial conflict with the two sectors industry and consumers, the policy makers face an optimization problem subject to the optimization problems for industries as well as for the consumers.

Consider a linear/ nonlinear TLP problem of maximization-type objectives at each level. Mathematically, it can be formulated as follows:

$$\max_{x_1} z_1(\bar{x}) \quad (1)$$

$$\max_{x_2} z_2(\bar{x}) \quad (2)$$

$$\max_{x_3} z_3(\bar{x}) \quad (3)$$

subject to

$$x \in S = \{A_1x_1 + A_2x_2 + A_3x_3 (\geq, =, \leq) B, x \geq 0\}$$

$$\bar{x}_1 \geq 0, \bar{x}_2 \geq 0, x_3(\bar{x}) \geq 0. \quad (4)$$

Where, In the tri-level linear/ nonlinear programming problem:

$z_1(\bar{x}) = z_1(x_1, x_2, x_3)$, $z_2(\bar{x}) = z_2(x_1, x_2, x_3)$ and $z_3(\bar{x}) = z_3(x_1, x_2, x_3)$ respectively represent linear/ nonlinear objective functions of DM1, DM2, DM3 and $\bar{x}_1 \geq 0, \bar{x}_2 \geq 0, x_3(\bar{x}) \geq 0$ are decision vectors under the control of DM1, DM2 and DM3 respectively.

$A_i x_i (\geq, =, \leq) B$, are linear or nonlinear constraints.

Let $X =$ set of feasible solutions $= \{ \bar{x} : \bar{x} \in R^n, A_i x_i (\geq, =, \leq) B \}$,

$\bar{x}$ = decision vector in n-dimensional Euclidean space $= x = x_1 \cup x_2 \cup x_3$.

Also let DM1 denote the decision maker at the first level, DM2 denote the decision maker at the second level and DM3 denote the decision maker at the third level.

III. Generation of non-dominated solution by weighting method using AHP

A. Weighting method

Weighting method has been widely studied, experimented and applied in many fields in engineering worlds. Not only does weighting method provide an alternate method to solving problem, it consistently outperforms other traditional methods in the most of the problem link. Weighting method has no special requirement for the characters and differentiability of the function. Perhaps the most creative task in making a decision with the hierarchical decision making with the hierarchical decision making situations is to choose the factors that are important for that decision. The basic idea of assigning weights to the various objective functions, combining these into a single objective function and parametrically varying the weights to generate the non-dominated set was first proposed by Zadeh in 1963. Mathematically, the weighting method can be stated as follows:

$$\max/\min \quad w_1 z_1(\bar{x}) + w_2 z_2(\bar{x}) + \dots + w_p z_p(\bar{x}) \quad (5)$$

Subject to $\bar{x} \in X$ where X is the feasible region. Thus, a multiple objective problem has been transformed into a single objective optimization problem for which solution methods exist. The coefficient w_p operating on the p^{th} objective function, $z_p(\bar{x})$, is called the weight and can be interpreted as “the relative weight or worth” of that objective function when compared to the other objectives. These weights can be obtained by Analytic Hierarchy Process (AHP).

B. Determination of Weight using Analytic Hierarchy Process (AHP)

Thomas Saaty [10], developed the AHP, it becomes a useful tool for estimating judgment elements by quantifying subjective decisions. It is used to derive relative weights of decision elements and then synthesize them to obtain the corresponding weights for alternatives and criteria. It is a building block for decision making. There are many papers applied AHP to solve decision problem. For example, Zanakakis et al. [12] studied over 100 applications of AHP in the service and government sectors. On the other hand, other researchers provided different approach to enhance the theoretical background of AHP to refine its analytical derivation. One open question in AHP is how to derive the relative weights from a comparison matrix. The majority practitioner agreed to use the eigenvector method proposed by Saaty [10].

AHP [4,6,9,10] is a mathematical technique developed for incorporating multi criteria decision making and designed to solve its complex problems. AHP and similar methods often use pairwise comparison matrices for determining the scores of alternatives with respect to a given criterion, or determining values of a weight vector. AHP can be conducted in three steps:

- *perform pairwise comparisons,
- * assess consistency of pairwise judgments,
- * compute the relative weights and then, it enables DM to make pairwise comparisons of importance between objectives according to the scale.

Because human is not always consistent, the theory of AHP does not demand perfect consistency and allows some small inconsistency in judgment and provides a measure of inconsistency. Before computing the weights based on pairwise judgments, the degree of inconsistency is measured by the Consistency Index (CI) [6,9,10]. Perfect consistency implies a value of zero for CI.

If the weights of the various objectives are interpreted as the representing the relative preference of some DM, then the solution to (5) is equivalent to the best compromise solution, i.e., the optimal solution relative to a particular preference structure. Moreover, the optimal solution to (5) is a non-dominated solution provided all the weights are positive. Allowing negative weights would be equivalent to transforming the maximizing problem to a minimizing one, for which a different set of non-dominated solutions will exist. The trivial case where all the weights are zero will simply identify $\bar{x} \in X$ as an optimal solution and will not distinguished between dominated and non-dominated solutions.

C. Method for generating non-dominated solutions

A non-dominated solution is one in which no one objective function can be improved without a simultaneous detriment to at least one of the other objectives of the vector maximum problem [VMP]. A given multiple objective mathematical problem which contains only maximization type objective functions is called the VMP.

A feasible solution $\bar{x}^* \in X$ (decision space) is a non-dominated solution to the VMP iff there does not exist any other feasible solution $\bar{x} \in X$ such that $z_p(\bar{x}^*) \leq z_p(\bar{x})$, $p = 1, 2$ and $z_p(\bar{x}^*) < z_p(\bar{x})$, for at least one p .

A non-dominated solution is one for which there is no other solution giving equal or greater values of each and every objective function. But in even the smallest problem, the number of non-dominated solutions generated

may be infinite. This is because all points on the line joining two non-dominated and extreme points are themselves non-dominated. A preferred solution is a non-dominated solution which is chosen by the DM, through some additional criteria, as the final decision. As such it lies in the region of acceptance of all the criteria values for the problem. A preferred solution is also known as the 'best' solution.

The most common strategy for finding non-dominated solutions of a tri-level problem is to convert it into a scalar optimization problem (SOP). This class of method does not require any assumption or information regarding the DMs utility function.

The concept of non-dominated solution was introduced by *Pareto*, an economist in 1896. A preferred (best) solution is a non-dominated solution which is chosen by the DM his self that is lies in the region of acceptance of all DMs. Non-dominated solution is to design the best alternative by considering the various interactions within the design constraints that best satisfy the DM by way of obtaining some acceptable levels of quantifiable objective functions. This method be distinguished with; a set of quantifiable objective functions, a set of well defined constraints and a process of obtaining some trade-off information, between the stated quantifiable objective functions. The most common strategy for finding non-dominated solutions of TLPPs is to convert it into a SOP. DMs provide their preference and converting TLP problem into a SOP by finding vector of weights for objectives.

IV. Solution of Tri-level Linear and Non-Linear Programming Problems using AHP

Let a TLPP be represented as:

$$\max_{x_1} z_1(\bar{x}) \quad (6)$$

$$\max_{x_2} z_2(\bar{x}) \quad (7)$$

$$\max_{x_3} z_3(\bar{x})$$

subject to

$$x \in S = \{A_1 x_1 + A_2 x_2 + A_3 x_3 (\geq, =, \leq) B, x \geq 0\} \quad , \quad \bar{x}_1 \geq 0, \bar{x}_2 \geq 0, \bar{x}_3 \geq 0. \quad (8)$$

Where, $z_1(\bar{x})$, $z_2(\bar{x})$, $z_3(\bar{x})$ and $A_i x_i (\geq, =, \leq) B$, are linear or non-linear objective functions and linear or non linear constraints respectively. $\bar{x}_1 \geq 0, \bar{x}_2 \geq 0, \bar{x}_3 \geq 0$ are decision vectors under the control of the first level, second level and third level decision maker (DM) respectively.

Let X = set of feasible solutions = $\{ \bar{x} : \bar{x} \in R^n, A_i x_i (\geq, =, \leq) B \}$,

$\bar{x}$ = decision vector in n-dimensional Euclidean space = $x = x_1 \cup x_2 \cup x_3$.

In the weighting problem $\bar{p}(w)$ in the absence explicit preference structure, the strategy is to generate all or representative subsets of non-dominated solutions from which a DM can select the suitable solution.

Solving the SOP involves finding $\bar{x}^* \in X$ such that $z(\bar{x}^*) \geq z(\bar{x}) \quad \forall \quad \bar{x} \in X$. The point $\bar{x}^*$ is said to be global optimum. If strict inequality holds for the objective functions, then $\bar{x}^*$ is the unique global optimum. If the inequality holds for some neighborhood of $\bar{x}^*$, then $\bar{x}^*$ is a local or relative optimum while it is strict local optimum if strict inequality holds in a neighborhood of $\bar{x}^*$.

By using the AHP pairwise comparison process, weights or priorities are derived from a set of judgments. While it is difficult to justify weights that are arbitrarily assigned, it is relatively easy to justify judgments and the basis (hard data, knowledge, experience) for the judgments. suppose already the relative weights of 3-objective functions are known, for 3-level hierarchical objective functions a complete pairwise comparison matrix A can be expressed as; $A = [a_{ij}]$, $i, j = 1, 2, 3$ is a matrix of size 3×3 with the following properties;

$a_{ij} > 0, a_{ii} = 1$, and $a_{ij} = 1/a_{ji}, i, j = 1, 2, 3$ where a_{ij} is the numerical answer given by the each DM for the question "How many times objective i is more important than objective j ?"

$$\tilde{A} = \begin{matrix} & \begin{matrix} z_1 & z_2 & z_3 \end{matrix} \\ \begin{matrix} z_1 \\ z_2 \\ z_3 \end{matrix} & \begin{bmatrix} 1 & a_{12} & a_{13} \\ a_{21} & 1 & a_{23} \\ a_{31} & a_{32} & 1 \end{bmatrix} \end{matrix}$$

After the normalized matrix, N of pairwise comparison matrix A for a hierarchical 3-level structure is designed, the normalized principal priority vector can be obtained by some ways such as averaging across the rows where, it shows the relative weights for objectives. The weighting problem is to find the weight vector $W = (w_1, w_2, w_3)$ such that the appropriate ratios of the components of W reflect or, at least, approximate all the a_{ij} values ($i, j = 1, 2, 3$) given by DMs.

Then, the weighting problem for TLPP problem is formulated as follows:

$$P(\bar{w}) = \max_{\bar{x} \in X} \sum_{n=1}^3 \bar{w}_p z_p(\bar{x}) \quad (9)$$

subject to

$$g_i(x) \leq b_i, \quad i = 1, 2, \dots, m \quad (10)$$

$$\bar{L}_1 \leq \bar{x}_1 \leq \bar{U}_1 \quad (11)$$

$$\bar{L}_2 \leq \bar{x}_2 \leq \bar{U}_2 \quad (12)$$

$$\bar{L}_3 \leq \bar{x}_3 \leq \bar{U}_3 \quad (13)$$

Where, $\bar{w} \in W = \{ \bar{w} : \bar{w} \in R^P, w_n \geq 0, P = 1, 2, 3 \text{ and } \sum_{n=1}^3 \bar{w}_p = 1 \}$

$\bar{U}_p$ and $\bar{L}_p$ are the upper and lower bounds of decision vector provided by the respective DM. Finally the linear or non-linear programming problem (9) - (13), with a single objective function is solved. Here the weighting coefficients convey the importance attached to an objective function. Suppose that the relative importance of the both objective functions is known and is constant. Then the preferred solution is obtained by

solving $p(\bar{w})$ where $\bar{w}_p$'s ≥ 0 are the weighting coefficients. The $\bar{w}_p$'s are normalized since, $\sum_{n=1}^3 \bar{w}_p = 1$.

This method can be used to generate non-dominated solutions by utilizing various values of $\bar{w}$. In such a case the weighting coefficients $\bar{w}$ do not reflect the relative importance of the objective functions in the proportional sense, but are only parameters varied to locate the non-dominated points.

V. Numerical example

In this section we present numerical example to demonstrate the solution procedures by proposed non-differential weighting approach to solve TLPP. The following example considered by Narang and Arora [8] is again used to demonstrate the solution procedures and clarify the effectiveness of the proposed approach. Consider the following Tri-Level Programming Problem [8]:

$$\max_{x_1} z_1 = (x_1 + x_2 + 2x_3 + 4)(-x_1 - x_2 + x_3 + 2x_4 + 1)$$

where x_2 , solves

$$\max_{x_3} z_2 = 2x_2 + x_3 + 3x_4$$

Where, $x_3, x_4, x_5, x_6, x_7, x_8$ solves, for a given x_1 and x_2

$$\max z_3 = \frac{2x_1 + 3x_2 + 2x_3 - 3x_4}{5x_1 + 11x_2 + x_5 + 29}$$

subject to

$$-3x_1 + 7x_2 + x_3 + x_5 = 10; \quad 14x_1 + 4x_2 + x_6 = 6;$$

$$x_1 + x_2 + x_3 - x_4 + x_7 = 5; \quad 2x_1 + x_2 + 2x_4 + x_8 = 8;$$

$$\bar{x}_1 \geq 0, \bar{x}_2 \geq 0, \bar{x}_3 \geq 0, \bar{x}_4 \geq 0, \bar{x}_5 \geq 0, \bar{x}_6 \geq 0, \bar{x}_7 \geq 0, \bar{x}_8 \geq 0.$$

Solution by proposed Solution approach: Let the bounds provided by the respective DMs be $0 \leq x_1 \leq 5$, $0 \leq x_2 \leq 2$, $5 \leq x_3 \leq 20$, $4 \leq x_4 \leq 30$, $1 \leq x_5 \leq 20$, $0 \leq x_6 \leq 30$, $0 \leq x_7 \leq 15$ and $0 \leq x_8 \leq 20$. The pair wise comparison matrix $\tilde{A}$ of order 3 and its normalized matrix $\tilde{N}$ [4, 6] for the hierarchical objective functions are given as:

$$\tilde{A} = \begin{matrix} & \begin{matrix} f_1 & f_2 & f_3 \end{matrix} \\ \begin{matrix} f_1 \\ f_2 \\ f_3 \end{matrix} & \begin{bmatrix} 1 & 2 & 5 \\ 1/2 & 1 & 3 \\ 1/5 & 1/3 & 1 \end{bmatrix} \end{matrix}; \quad \tilde{N} = \begin{bmatrix} 1/1.7 & 2/3.33 & 5/9 \\ 0.5/1.7 & 1/3.33 & 3/9 \\ 0.2/1.7 & 0.333/3.33 & 1/9 \end{bmatrix} = \begin{bmatrix} 0.588 & 0.6 & 0.556 \\ 0.294 & 0.3 & 0.333 \\ 0.118 & 0.1 & 0.111 \end{bmatrix}$$

Thus the normalized weights are:

$$w_1 = \frac{(0.588+0.6+0.556)}{3} = 0.581,$$

$$w_2 = 0.309, \quad w_3 = 0.110$$

$$\tilde{A}\bar{w} = \begin{bmatrix} 1 & 2 & 5 \\ 1/5 & 1 & 3 \\ 1/5 & 1/3 & 1 \end{bmatrix} \begin{bmatrix} 0.581 \\ 0.309 \\ 0.110 \end{bmatrix} = \begin{bmatrix} 1.749 \\ 0.930 \\ 0.329 \end{bmatrix}$$

$$P_{\max} = 1.749 + 0.930 + 0.329 = 3.008 \text{ and } P = 0.3.$$

Thus, $CI = (3.008-3)/3-1 = 0.008/2 = 0.004$ and $RI = 1.98(3-2)/3 = 0.66$ which gives

$$CR = CI/RI = 0.004/0.66 = 0.006.$$

The weighting problem is therefore formulated as:

$$P(\bar{w}) = \max_x \left[0.581(x_1 + x_2 + 2x_3 + 4)(-x_1 - x_2 + x_3 + 2x_4 + 1) + 0.309(2x_2 + x_3 + 3x_4) + 0.110 \frac{2x_1 + 3x_2 + 2x_3 - 3x_4}{5x_1 + 11x_2 + x_3 + 29} \right]$$

The non-dominated solution set is generated by parametrically varying the weights and is tabulated below:

w_1, w_2, w_3	$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$	$P(\bar{w})$
0.1,0.4,0.5	0,0,9,4,1,6,0,0	48.10
0.1,0.5,0.4	0,0,9,4,1,6,0,0	50.18
0.2,0.3,0.5	0,0,9,4,1,6,0,0	85.60
0.3,0.4,0.3	0,0,9,4,1,6,0,0	127.26
0.4,0.4,0.2	0,0,9,4,1,6,0,0	166.84
0.5,0.3,0.2	0,0,9,4,1,6,0,0	204.34
0.581,0.309,0.110	0,0,9,4,1,6,0,0	236.59
0.6,0.3,0.1	0,0,9,4,1,6,0,0	243.92
0.7,0.2,0.1	0,0,9,4,1,6,0,0	281.42
0.9,0,0.1	0,0,9,4,1,6,0,0	356.42
0.9,0.1,0	0,0,9,4,1,6,0,0	358.5

Solution of the above problem obtained by Narang and Arora [8] using Enumerative Algorithm is $(\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4, \bar{x}_5, \bar{x}_6, \bar{x}_7, \bar{x}_8) = (0,0,9,4,1,6,0,0)$. Using our weighting approach, we got exactly the same solution. In this example, we see that even we vary the weight vector, the solution remains more or less the same. Thus the non-dominated solution set reduces to a point (almost).

VI. Conclusion

In this paper, a powerful and robust method which is based on AHP is proposed to solve TLPPs. The proposed approach is effective tool for finding a satisfactory solution where it can produces results which are very close

or improved to the results obtained by most of the other existing methods with observing that even though varying the weights vectors, the solutions remain more or less the same. Thus the non-dominated solution set reduces to a point. From the numerical result, the results by the method in this paper accord with the results in the references [8]. The numerical result shows the proposed algorithm is feasible and efficient, can find global optimal solutions with less computational burden. We convert the hierarchical system into scalar optimization problem by finding proper weights using AHP so that objective functions of both levels can be combined into one objective function. Since, the objectives of all level decision makers are potentially conflicting in nature, a possible relaxation of each level decisions are considered by providing weights to objective functions for avoiding decision deadlock. The procedure is not excessively interactive, which most DMs prefer. AHP gives the relative weights to form a super objective function. This class of approach is purely non-differential approach as it does not require any assumption or information regarding the DMs utility function. The method has no special requirement for the characters of the function and overcome the difficulty discussing the conditions and the algorithms of the optimal solution with the definition of the differentiability of the function. The main advantage of the proposed approach is that the possibility of rejecting the solution again and again by the fuzzy approach and re-evaluation of the problem repeatedly, by redefining the elicited membership functions, needed to reach to the satisfactory decision does not arise. The problem can never be infeasible and unbounded. Proposed method facilitates computation to reduce the complexity in solving problem and is much more effective.

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