



A BULK ARRIVAL QUEUEING MODEL OF THREE STAGES OF SERVICE WITH DIFFERENT VACATION POLICIES, SERVICE INTERRUPTION AND DELAY TIME

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Abstract: We study a M/G/1 queueing system with bulk arrival. Here the service is given in three stages of service. All the three stages of service are compulsory to all the arriving customers. After the completion of service the server takes a vacation. An added assumption Extended vacation is considered in this model. Moreover service interruption is considered. Repair process does not start immediately. There is a delay in repair. Service and all the other parameters follows general distribution. Breakdown is exponentially distributed. Using supplementary variable technique we derive the probability generating functions for the number of customers in the system. Some of the performance measures are calculated.

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I. Introduction

An overview on the conceptual aspects for the stage service in queueing model is provided in this model. The motivation of this model comes from numerous versatile applications. Many authors have studied queues with server vacations and breakdowns. Le et al [2] studied reliability analysis of an M/G/1 queue with breakdowns and vacations. Khalaf., Madan & Lucas.[4] studied about $M^{[x]}/G/1$ Queue with Bernoulli schedule, General vacation times, random breakdowns, and general repair times. Borthakur and Choudhury [6] discussed about generalized vacation in his batch arrival Poisson queue. Choudhury[8] in his batch arrival queueing system considered the aspect of additional service channel and derived the performance measures of the queueing system Badamchi Zadeh and Shahkar. G.H.[10] investigated a two phase queue system with Bernoulli feedback and Bernoulli schedule server vacation. Maragathasundari and Srinivasan, .[12]made a analysis in M/G/1 feedback queue with three stage multiple sever vacation.

In this paper, we consider a batch arrival queueing system with three stages of service. After the completion of three stages of service completion the server goes for vacation. The server after completion of completion, he might take the optional extended vacation with probability r or may return to the system to serve the customers with probability $1 - r$. Breakdown arrives at random. Due to various reasons, repair process does not follow immediately. There is a delay in repair Service time, vacation time, extended vacation & Repair time follows general distribution. This paper is organized as follows. Model assumptions are given in section 2. Steady state condition is given in section 3. Queue size distribution at a random epoch is given in section 4. The average queue size and the average waiting time are computed in section 5. Conclusion is given in section 6.

II. Model Assumptions

Customers arrive at the system in batches of variable sizes in a compound Poisson process, and one by one service on a 'first come-first served' basis is implemented. Let $\lambda c_i dt (i = 1, 2, 3 \dots)$ be the first order probability that a batch of i customers arrives at the system during a short duration of time $(t, t + dt)$, where $0 \leq c_i \leq 1$ and $\sum_{i=1}^{\infty} c_i = 1$ and $\lambda > 0$ is the mean arrival rate of the batches. There is a single server and the service time follows general(arbitrary) distribution with distribution function $M_i(v)$ and density function $m_i(v)$. Let $\mu_i(x)dx$ be the conditional probability density of service completion of the i^{th} stage of service during the interval $(x, x + dx)$, given that the elapsed time is x , so that

$$\mu_i(x) = \frac{m_i(x)}{1 - M_i(x)} \quad i = 1, 2, 3 \quad (1)$$

$$\text{and } m_i(v) = \mu_i(v) e^{-\int_0^v \mu_i(x) dx} \quad (2)$$

As soon as a service is completed, the server may take a vacation. The server's vacation time follows a general(arbitrary) distribution, with the distribution function $B_1(s)$ and density function $b_1(s)$. Let $\beta(x)dx$ be

the conditional probability of a completion of a vacation during the interval $(x, x + dx)$, so that

$$\beta(x) = \frac{b_1(x)}{1 - B_1(x)} \quad (3)$$

$$\text{and } b_1(s) = \beta(s)e^{-\int_0^s \beta(x)dx} \quad (4)$$

After the completion of vacation, the server has the option of taking extended vacation with probability r . The server's Extended vacation time follows a general(arbitrary) distribution, with the distribution function $B_2(s)$ and density function $b_2(s)$. Let $\theta(x)dx$ be the conditional probability of a completion of a Extended vacation during the interval $(x, x + dx)$, so that

$$\theta(x) = \frac{b_2(x)}{1 - B_2(x)} \quad (3a)$$

$$b_2(s) = \theta(s)e^{-\int_0^s \theta(x)dx} \quad (4a)$$

The server may breakdown at random, and breakdowns are presumed to arise according to the Poisson stream, with the mean breakdown rate $\alpha > 0$. Further, we assume that once the system breaks down, the customer whose service is interrupted goes back to the head of the queue. The server's delay time follows a general(arbitrary) distribution, with the distribution function $B_3(s)$ and density function $b_3(s)$. Let $\delta(x)dx$ be the conditional probability of a completion of a delay during the interval $(x, x + dx)$, so

$$\text{that } \delta(x) = \frac{b_3(x)}{1 - B_3(x)} \quad (5a)$$

$$b_3(s) = \delta(s)e^{-\int_0^s \delta(x)dx} \quad (5b)$$

The server's repair time follows a general(arbitrary) distribution, with the distribution function $B_4(s)$ and density function $b_4(s)$. Let $\gamma(x)dx$ be the conditional probability of a completion of a repair during the interval $(x, x + dx)$, so that

$$\gamma(x) = \frac{b_4(x)}{1 - B_4(x)} \quad (6)$$

$$b_4(s) = \gamma(s)e^{-\int_0^s \gamma(x)dx}$$

III. Steady State Condition

Then, connecting the states of the system at time $t + dt$ with those at time t and then taking the limit as $t \rightarrow \infty$, we obtain the following set of steady state equations governing the system

$$\frac{\partial}{\partial x} P_n^{(1)}(x) + (\lambda + \mu_1(x) + \alpha)P_n^{(1)}(x) = \lambda \sum_{i=1}^{n-1} C_i P_{n-1}^{(1)}(x) \quad (7)$$

$$\frac{\partial}{\partial x} P_0^{(1)}(x) + (\lambda + \mu_1(x) + \alpha)P_0^{(1)}(x) = 0 \quad (8)$$

$$\frac{\partial}{\partial x} P_n^{(2)}(x) + (\lambda + \mu_2(x) + \alpha)P_n^{(2)}(x) = \lambda \sum_{i=1}^{n-1} C_i P_{n-1}^{(2)}(x) \quad (9)$$

$$\frac{\partial}{\partial x} P_0^{(2)}(x) + (\lambda + \mu_2(x) + \alpha)P_0^{(2)}(x) = 0 \quad (10)$$

$$\frac{\partial}{\partial x} P_n^{(3)}(x) + (\lambda + \mu_3(x) + \alpha)P_n^{(3)}(x) = \lambda \sum_{i=1}^{n-1} C_i P_{n-1}^{(3)}(x) \quad (11)$$

$$\frac{\partial}{\partial x} P_0^{(3)}(x) + (\lambda + \mu_3(x) + \alpha)P_0^{(3)}(x) = 0 \quad (12)$$

$$\frac{\partial}{\partial x} V_n(x) + (\lambda + \beta(x))V_n(x) = \sum_{i=1}^{n-1} \lambda C_i V_{n-i}(x) \quad (13)$$

$$\frac{\partial}{\partial x} V_0(x) + (\lambda + \beta(x))V_0(x) = 0 \quad (14)$$

$$\frac{\partial}{\partial x} E_n(x) + (\lambda + \theta(x))E_n(x) = \sum_{i=1}^{n-1} \lambda C_i E_{n-i}(x) \quad (15)$$

$$\frac{\partial}{\partial x} E_0(x) + (\lambda + \theta(x))E_0(x) = 0 \quad (16)$$

$$\frac{\partial}{\partial x} D_n(x) + (\lambda + \delta(x))D_n(x) = \sum_{i=1}^{n-1} \lambda C_i D_{n-i}(x) \quad (17)$$

$$\frac{\partial}{\partial x} D_0(x) = 0 \quad (18)$$

$$\frac{\partial}{\partial x} R_n(x) + (\lambda + \gamma(x))R_n(x) = \sum_{i=1}^{n-1} \lambda C_i R_{n-i}(x) \quad (19)$$

$$\frac{\partial}{\partial x} R_0(x) + (\lambda + \gamma(x))R_0(x) = 0 \quad (20)$$

$$\begin{aligned} \lambda Q &= \int_0^\infty R_0(x)\gamma(x)dx + \int_0^\infty P_0^{(3)}(x)\mu_3(x)dx \\ &\quad + \int_0^\infty V_0(x)\beta(x)dx + \int_0^\infty E_0(x)\theta(x)dx \end{aligned} \quad (21)$$

The above equations are to be solved subject to the boundary conditions

$$P_n^{(1)}(0) = \int_0^\infty P_{n+1}^{(3)}(x)\mu_3(x)dx + (1 - r) \int_0^\infty V_{n+1}(x)\beta(x)dx$$

$$+ \int_0^\infty E_{n+1}(x)\theta(x)dx + \int_0^\infty R_{n+1}(x)\gamma(x)dx + \lambda C_{n+1}Q \quad (22)$$

$$P_n^{(2)}(0) = \int_0^\infty P_n^{(1)}(x)\mu_1(x)dx \quad (23)$$

$$P_n^{(3)}(0) = \int_0^\infty P_n^{(2)}(x)\mu_2(x)dx \quad (24)$$

$$V_n(0) = \int_0^\infty P_n^{(3)}(x)\mu_3(x)dx \quad (25)$$

$$E_n(0) = r \int_0^\infty V_n(x)\beta(x)dx \quad (26)$$

$$D_n(0) = \alpha \sum_{i=1}^3 \int_0^\infty P_{n-1}^{(i)}(x)dx = \alpha \sum_{i=1}^n P_{n-1}^{(i)} \quad (27)$$

$$R_n(0) = \int_0^\infty D_n(x)\delta(x)dx \quad (28)$$

$$D_0(0) = 0 \quad (29)$$

$$R_0(0) = 0 \quad (30)$$

IV. Queue Size distribution at a random epoch

Now multiply equation (7) by Z^n & summing over n from 1 to ∞ , adding the result to equation (8), and using generating function we get

$$\frac{\partial}{\partial x} P_q^{(1)}(x, z) + (\lambda - \lambda c(z) + \mu_1(x) + \alpha) P_q^{(1)}(x, z) = 0 \quad (31)$$

Similarly,

$$\frac{\partial}{\partial x} P_q^{(2)}(x, z) + (\lambda - \lambda c(z) + \mu_2(x) + \alpha) P_q^{(2)}(x, z) = 0 \quad (32)$$

$$\frac{\partial}{\partial x} P_q^{(3)}(x, z) + (\lambda - \lambda c(z) + \mu_3(x) + \alpha) P_q^{(3)}(x, z) = 0 \quad (33)$$

$$\frac{\partial}{\partial x} V_q(x, z) + (\lambda - \lambda c(z) + \beta(x)) V_q(x, z) = 0 \quad (34)$$

$$\frac{\partial}{\partial x} D_q(x, z) + (\lambda - \lambda c(z) + \delta(x)) D_q(x, z) = 0 \quad (35)$$

$$\frac{\partial}{\partial x} R_q(x, z) + (\lambda - \lambda c(z) + \gamma(x)) R_q(x, z) = 0 \quad (36)$$

Applying the same process for the boundary conditions, we get

$$z P_q^{(1)}(0, z) = \int_0^\infty P_q^{(3)}(x, z) \mu_3(x) dx + (1-r) \int_0^\infty V_q(x, z) \beta(x) dx + \int_0^\infty E_q(x, z) \theta(x) dx + \int_0^\infty R_q(x, z) \gamma(x) dx + \lambda (C(z) - 1) Q \quad (37)$$

Similarly we have,

$$P_q^{(2)}(0, z) = \int_0^\infty P_q^{(1)}(x, z) \mu_1(x) dx \quad (38)$$

$$P_q^{(3)}(0, z) = \int_0^\infty P_q^{(2)}(x, z) \mu_2(x) dx \quad (39)$$

$$V_q(0, z) = \int_0^\infty P_q^{(3)}(x, z) \mu_3(x) dx \quad (40)$$

$$D_q(0, z) = \alpha z \sum_{i=1}^3 P_q^{(i)}(z) \quad (41)$$

$$R_q(0, z) = \int_0^\infty D_q(x, z) \delta(x) dx \quad (42)$$

Integrating equation (31) from 0 to x , we get

$$P_q^{(1)}(x, z) = P_q^{(1)}(0, z) e^{-(\lambda - \lambda c(z) + \alpha)x - \int_0^x \mu_1(t) dt} \quad (43)$$

Again integrating Equation (43) by parts with respect to x yields,

$$P_q^{(1)}(z) = P_q^{(1)}(0, z) \left[\frac{1 - \bar{M}_1(s)}{s} \right], \quad s = \lambda - \lambda c(z) + \alpha \quad (44)$$

Where $\bar{M}_1(s) = \int_0^\infty e^{-(\lambda - \lambda c(z) + \alpha)x} d\bar{M}_1(x)$ the Laplace Stieltjes is transform of first stage of service time $M_1(x)$.

Now multiply both sides of the Equation (43) by $\mu_1(x)$ & integrating

$$\int_0^\infty P_q^{(1)}(x, z) \mu_1(x) dx = P_q^{(1)}(0, z) \bar{M}_1(s) \quad (45)$$

Similar process to be carried out for equations (26)-(31), Hence we have

$$P_q^{(2)}(z) = P_q^{(2)}(0, z) \left[\frac{1 - \bar{M}_1(s)(1 - \bar{M}_2(s))}{\theta} \right] \quad (46)$$

$$\text{And } \int_0^\infty P_q^{(2)}(x, z) \mu_1(x) dx = P_q^{(1)}(0, z) \bar{M}_1(s) \bar{M}_2(s) \quad (47)$$

$$P_q^{(3)}(z) = P_q^{(1)}(0, z) \left[\frac{\bar{M}_1(s) \bar{M}_2(s)(1 - \bar{M}_3(s))}{s} \right] \quad (48)$$

$$\text{And } \int_0^\infty P_q^{(3)}(x, z) \mu_3(x) dx = P_q^{(1)}(0, z) \bar{M}_1(s) \bar{M}_2(s) \bar{M}_3(s) \quad (49)$$

$$V_q(z) = V_q(0, z) \left[\frac{(1 - \bar{B}_1(h))}{h} \right], \quad h = \lambda - \lambda c(z) \\ = P_q^{(1)}(0, z) \bar{M}_1(s) \bar{M}_2(s) \bar{M}_3(s) \frac{(1 - \bar{B}_1(h))}{h} \quad (50)$$

$$\int_0^\infty V_q(x, z) \beta(x) dx = P_q^{(1)}(0, z) \bar{M}_1(s) \bar{M}_2(s) \bar{M}_3(s) \bar{B}_1(h) \quad (51)$$

$$E_q(z) = E_q(0, z) \left[\frac{(1 - \bar{B}_2(h))}{h} \right]$$

$$= rP_q^{(1)}(0, z) \bar{M}_1(s) \bar{M}_2(s) \bar{M}_3(s) \bar{B}_1(h) \frac{(1 - \bar{B}_2(h))}{h} \quad (52)$$

$$\int_0^\infty E_q(x, z) \theta(x) dx = rP_q^{(1)}(0, z) \bar{M}_1(s) \bar{M}_2(s) \bar{M}_3(s) \bar{B}_1(h) \bar{B}_2(h) \quad (53)$$

$$D_q(z) = D_q(0, z) \left[\frac{1 - \bar{B}_3(h)}{h} \right] \\ = \alpha z P_q^{(1)}(0, z) \left[\frac{1 - \bar{M}_1(s) \bar{M}_2(s) \bar{M}_3(s)}{s} \right] \left[\frac{1 - \bar{B}_3(h)}{h} \right] \quad (54)$$

$$\int_0^\infty D_q(x, z) \delta(x) dx = \frac{\alpha z P_q^{(1)}(0, z) [1 - \bar{M}_1(s) \bar{M}_2(s) \bar{M}_3(s)] \bar{B}_3(h)}{s} \quad (55)$$

$$R_q(z) = R_q(0, z) \left[\frac{1 - \bar{B}_4(h)}{h} \right] \\ = \frac{\alpha z P_q^{(1)}(0, z) [1 - \bar{M}_1(s) \bar{M}_2(s) \bar{M}_3(s)] \bar{B}_3(h)}{s} \left[\frac{1 - \bar{B}_4(h)}{h} \right] \quad (56)$$

$$\int_0^\infty R_q(x, z) \gamma(x) dx = \frac{\alpha z P_q^{(1)}(0, z) [1 - \bar{M}_1(s) \bar{M}_2(s) \bar{M}_3(s)] \bar{B}_3(h) \bar{B}_4(h)}{s} \quad (57)$$

Therefore from equation 37, we have

$$P_q^{(1)}(0, z) = \frac{s \lambda Q(C(z) - 1)}{s[z - \sum_{i=1}^3 \bar{M}_i(s) [1 + \bar{B}_1(h) + r \bar{B}_1(h) \bar{B}_2(h)] - \alpha z [1 - \sum_{i=1}^3 \bar{M}_i(s)] \bar{B}_3(h) \bar{B}_4(h)]} \quad (58)$$

Let $S_q(z)$ be the generating function of the queue size

$$S_q(z) = P_q^{(1)}(0, z) + P_q^{(2)}(0, z) + P_q^{(3)}(0, z) + V_q(z) + E_q(z) + R_q(z) + D_q(z) \\ = -Q \left[\frac{(1 - \bar{M}_1(s) \bar{M}_2(s) \bar{M}_3(s)) (s + \alpha h z - \alpha h z \bar{B}_3(h) \bar{B}_4(h))}{(1 - \bar{B}_2(h))} \right] \\ = \frac{s[z - \bar{M}_1(s) \bar{M}_2(s) \bar{M}_3(s) (1 - \bar{B}_1(h) + r \bar{B}_1(h) \bar{B}_2(h))] - \alpha z \bar{B}_3(h) \bar{B}_4(h) [1 - \bar{M}_1(s) \bar{M}_2(s) \bar{M}_3(s)]}{s[z - \bar{M}_1(s) \bar{M}_2(s) \bar{M}_3(s) (1 - \bar{B}_1(h) + r \bar{B}_1(h) \bar{B}_2(h))] - \alpha z \bar{B}_3(h) \bar{B}_4(h) [1 - \bar{M}_1(s) \bar{M}_2(s) \bar{M}_3(s)]} \quad (59)$$

$$\text{In order to find } Q, \text{ we use the normalization condition } S_q(1) + Q = 1 \quad (60)$$

And hence the utilization factor ρ is determined.

$\rho < 1$ is the stability condition under which the steady state exists.

V. The Average Queue Size and Average Waiting Time

Let L_q denote the mean number of customers in the queue under the steady state. Then

$$L_q = \left. \frac{d}{dz} S_q(z) \right|_{z=1} \quad (61)$$

Since the formula gives 0/0 for m , then we write $S_q(z)$ given in (59) as

$S_q(z) = N(z)/D(z)$ Where $N(z)$ and $D(z)$ are the numerator and denominator of the right hand side of (59) respectively. Then we use

$$L_q = \frac{D'(1)N''(1) - N'(1)D''(1)}{2(D'(1))^2} \quad (62)$$

$$N'(1) = -Q[\alpha(\lambda E(I)) \bar{M}'_1(\alpha) \bar{M}'_2(\alpha) \bar{M}'_3(\alpha) - \alpha \lambda E(I) - r s \lambda E(s) E(I)] \quad (63)$$

$$N''(1) = -Q[\lambda E(I) [\bar{M}'_1(\alpha) \bar{M}'_2(\alpha) \bar{M}'_3(\alpha)] + (\lambda E(I) (\bar{M}'_1(\alpha) \bar{M}'_2(\alpha) \bar{M}'_3(\alpha))) - \alpha - \alpha \lambda E(I) \\ + r[(\lambda E(I)^2(E(s) + \alpha \bar{M}'_1(\alpha) E(s) + \alpha \bar{M}'_2(\alpha) E(s) + \alpha \bar{M}'_3(\alpha) E(s) + \alpha E(v) E(s)))] \\ + ((\lambda E(I)^2 E(s) - \lambda E(I(I - 1)) \alpha E(s) \\ + (\lambda E(I)^2 (\alpha E(s) \bar{M}'_1(\alpha) + \alpha E(s) \bar{M}'_2(\alpha) + \alpha E(s) \bar{M}'_3(\alpha) - \alpha E(s) E(v) \\ + \alpha E(s^2)))] \quad (64)$$

$$D'(1) = (-\lambda E(I)) [1 - r] \\ + \alpha \left[1 + r + \lambda E(I) \left(\frac{\bar{M}'_1(\alpha) \bar{M}'_2(\alpha) \bar{M}'_3(\alpha)}{\bar{M}'_1(\alpha) \bar{M}'_2(\alpha) \bar{M}'_3(\alpha) + \bar{M}'_1(\alpha) \bar{M}'_2(\alpha) \bar{M}'_3(\alpha)} \right) \right] \\ + (\lambda E(I)) [E(v) + r E(v) + r E(s)] \quad (65)$$

$$D''(1) = -\lambda E(I(I - 1)) [1 - r] + (-\lambda E(I)) [1 - (-\lambda E(I)) r (\bar{M}'_1(\alpha) \bar{M}'_2(\alpha) \bar{M}'_3(\alpha) + \bar{M}'_2(\alpha) \bar{M}'_3(\alpha) + \bar{M}'_1(\alpha) \bar{M}'_3(\alpha))] \\ + [(-\lambda E(I)) (1 - \alpha - \lambda E(I) r) (\bar{M}'_1(\alpha) \bar{M}'_2(\alpha) \bar{M}'_3(\alpha) + \bar{M}'_1(\alpha) \bar{M}'_2(\alpha) \bar{M}'_3(\alpha) + \bar{M}'_1(\alpha) \bar{M}'_3(\alpha) \bar{M}'_2(\alpha))] \\ + (\lambda E(I))^2 [E(v) + r E(v) + r E(s)] - \alpha [-\lambda E(I(I - 1)) \bar{M}'_1(\alpha) \bar{M}'_2(\alpha) \bar{M}'_3(\alpha) r + \\ + 2(\lambda E(I))^2 \bar{M}'_1(\alpha) \bar{M}'_2(\alpha) \bar{M}'_3(\alpha) r + 2(\lambda E(I))^2 \bar{M}'_1(\alpha) \bar{M}'_2(\alpha) \bar{M}'_3(\alpha) r + (-\lambda E(I)) [\bar{M}'_1(\alpha) \bar{M}'_2(\alpha) \bar{M}'_3(\alpha) + \bar{M}'_2(\alpha) \bar{M}'_3(\alpha) + \bar{M}'_3(\alpha) \bar{M}'_1(\alpha) \bar{M}'_2(\alpha))] \\ + (-\lambda E(I)) [E(v) + E(s)] - \lambda E(I(I - 1)) \bar{M}'_1(\alpha) \bar{M}'_2(\alpha) \bar{M}'_3(\alpha) r + (-\lambda E(I)) [\bar{M}'_2(\alpha) \bar{M}'_1(\alpha) \bar{M}'_3(\alpha) - \lambda E(I(I - 1)) \bar{M}'_1(\alpha) \bar{M}'_2(\alpha) \bar{M}'_3(\alpha)] \\ + \alpha [(\lambda E(I))^2 (E(D) + E(R)) ((\bar{M}'_1(\alpha) \bar{M}'_2(\alpha) \bar{M}'_3(\alpha) + \bar{M}'_1(\alpha) \bar{M}'_2(\alpha) \bar{M}'_3(\alpha) + \bar{M}'_1(\alpha) \bar{M}'_2(\alpha) \bar{M}'_3(\alpha)))] +$$

$$\lambda E(I)[\overline{M}'_1(\alpha)\overline{M}_2(\alpha)\overline{M}_3(\alpha) + \overline{M}_1(\alpha)\overline{M}'_2(\alpha)\overline{M}'_3(\alpha) + \overline{M}_1(\alpha)\overline{M}_2(\alpha)\overline{M}'_3(\alpha)][1 - \lambda E(I)E(D) - \lambda E(I)E(R)] + \lambda E(I(I-1))[\overline{M}''_1(\alpha)\overline{M}_2(\alpha)\overline{M}_3(\alpha) + \overline{M}_1(\alpha)\overline{M}'_2(\alpha)\overline{M}_3(\alpha) + \overline{M}_1(\alpha)\overline{M}_2(\alpha)\overline{M}'_3(\alpha)] \quad (66)$$

Where primes and double primes in (62) denote the first and second derivatives at $z = 1$, respectively. Carrying out the derivatives at $z = 1$ and if we substitute the values of $N'(1), N''(1), D'(1)$ and $D''(1)$ into (62), we obtain L_q in closed form. Further, the mean waiting time of a customer could be found using $W_q = L_q/\lambda$ and other performance measures can be determined using Little's formula.

VI. Conclusions

In this paper we have investigated a batch arrival queueing model of three stage heterogeneous service with different types of vacation policies. All the three stages of service are compulsory to all the arriving customers. The server has the option of undergoing an extended vacation. The Steady state results and some performance measures of the queueing system are derived. As a future work, repair process in terms of stages and stand by server could also be considered as additional aspects in this model.

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