



Stochastic Modelling and Analysis of Two Unit Standby System with Two Repair Facilities and Varying Physical Conditions of the Regular Repairman

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Abstract: The present paper deals with stochastic behaviour of a two non-identical units in standby configuration, each unit having two modes, normal and total failure. Two types of repairmen called regular and expert are considered to repair a failed unit on FCFS basis. The regular repairman may be in good or poor physical condition at the time of need. The expert repairman is called if and only if the physical condition of the regular repairman is poor and he is unable to repair the failed unit within a specified time. The failure time distributions of units in both the physical conditions of regular repairman are taken to be exponential with different parameters whereas all the repair time distribution are taken as general. Various reliability characteristics of interest have been studied along with their graphical behaviour.

Keywords: Reliability; MTSF; Availability; Busy period; Expected number of Repairs; Graphical study of Model.

I. INTRODUCTION

In literature, many researchers have discussed the reliability and availability behaviour of standby systems in detail by considering different cases and strategies. Two-unit standby redundant systems have been extensively studied by several authors in the past. In order to improve the reliability or enhance the availability and hence reduce the loss, a two-component redundant system is often employed. Several authors including Goel et al.[1], Gupta and Bhardwaj [4] in the field of reliability theory have analyzed redundant system models under different sets of assumptions such as abnormal weather conditions, waiting time of skilled repairman, imperfect switching device etc. using regenerative point technique. Extensive research is also reported in the area of human reliability, where the researchers have studied man-machine systems. Human plays a pivotal role in the design, development and operational phases of engineering systems. Reliability evaluation of system without taking into consideration the human element does not provide a realistic picture. Different authors have used different techniques for analyzing such system models under different conditions. Commonly used techniques for analyzing such system models are regenerative point technique, supplementary variable technique, Boolean algebra technique etc. In these systems, human error is also a cause of system breakdown beside system's inherent failure. The physical condition of operator is thus an influencing factor on the overall performance of the system. Goel et al [2] have analysed model considering two physical conditions of the operator viz. poor and good. One may expect faster repair during the good physical condition of repairman, as compared to poor physical condition of repairman. Mokaddis et al. [7] analysed a man-machine system model operating under different physical conditions of operator. Rander et al. [6] studied a two unit cold standby system with a perfect master repairman and an imperfect assistant repairman.

Taking this fact into consideration in this paper we investigate two unit standby system model with two repairmen called regular and expert. The regular repairman may be in good or poor physical conditions at the time of need. The expert repairman is called if and only if the physical condition of the regular repairman is poor and he is unable to repair the failed unit within a specified time.

Using the regenerative point technique the following important reliability characteristics of interest are obtained:

1. Transition probabilities and mean sojourn times.
2. Reliability and Mean time to system failure.
3. Point wise and steady-state availabilities of the system.
4. Expected up time of the system.
5. Expected busy time of each repairmen during $(0, t]$ and in the steady-state.
6. Expected number of repairs by each repairmen during $(0, t]$ and in the steady-state.
7. Net expected profit incurred by the system during $(0, t]$ and in the steady-state.

II. SYSTEM DESCRIPTION AND ASSUMPTIONS

1. The system comprises of two non-identical units. Initially, one unit is operative and other is kept as standby.

2. Each unit of the system has two modes- Normal (N) and Failure (F).
3. Two types of repairmen are considered, the first repairman is called regular repairman and other is an expert repairman. The regular repairman may be in good or poor physical conditions at the time of need.
4. Whenever an operating unit fails, it is attended by the regular repairman; he repairs the failed unit if his physical condition is good. If his physical condition is poor and he is not able to repair the failed unit within some specified time, then an expert repairman is called for repair. At that time, regular repairman leaves the system and all other failed units are repaired by the expert repairman.
5. The repair discipline is first come first serve (FCFS).
6. Each repaired unit works as good as new.
7. The failure time distribution of each unit in both the physical conditions of regular repairman and the arrival time distribution of expert repairman are taken to be exponential with different parameters.
8. All the repair time distributions are taken as general.

III. NOTATIONS AND SYMBOLS

α_1/α_2	:	Constant failure rate of 1 st /2 nd unit under poor physical condition of repairman.
α_3/α_4	:	Constant failure rate of 1 st /2 nd unit under good physical condition of repairman.
μ_1	:	Constant arrival rate of an expert repairman.
$H_1(.) / H_2(.)$	:	C.d.f. of time to repair 1 st /2 nd unit when the regular repairman is in poor physical condition.
$H_3(.) / H_4(.)$	:	C.d.f. of time to repair 1 st /2 nd unit when the regular repairman is in good physical condition.
$G_1(.) / G_2(.)$	:	C.d.f. of time to repair 1 st /2 nd unit by an expert repairman.
m_1/m_2	:	Mean repair time for 1 st /2 nd unit when the regular repairman is in good physical condition.
n_1/n_2	:	Mean repair time for 1 st /2 nd unit by the expert repairman.

SYMBOLS FOR THE STATES OF THE SYSTEM

N_{i0}	:	Unit i ($i=1,2$) is in normal mode and operative.
N_{2S}	:	Unit 2 nd is in normal mode and kept as standby.
F_{irg}/F_{iwrg}	:	Unit i ($i=1,2$) is in failure mode and under repair/waiting for repair when the regular repairman is in good physical condition.
F_{irp}/F_{iwrp}	:	Unit i ($i=1,2$) is in failure mode and under repair/waiting for repair when the regular repairman is in poor physical condition.
F_{ire}/F_{iwre}	:	Unit i ($i=1,2$) is in failure mode and under repair/waiting for repair by the expert repairman.

With the help of the above symbols, the possible states of the system are:

$S_0 = [N_{10}, N_{2S}]$	$S_1 = [F_{1rg}, N_{20}]$	$S_2 = [F_{1rp}, N_{20}]$	$S_3 = [F_{1rg}, F_{2wrg}]$
$S_4 = [N_{10}, F_{2rg}]$	$S_5 = [F_{1wrg}, F_{2rg}]$	$S_6 = [F_{1re}, N_{20}]$	$S_7 = [F_{1re}, F_{2wre}]$
$S_8 = [F_{1rp}, F_{2wrg}]$	$S_9 = [N_{10}, F_{2rp}]$	$S_{10} = [N_{10}, F_{2re}]$	$S_{11} = [F_{1wrg}, F_{2rp}]$
$S_{12} = [F_{1wre}, F_{2re}]$			

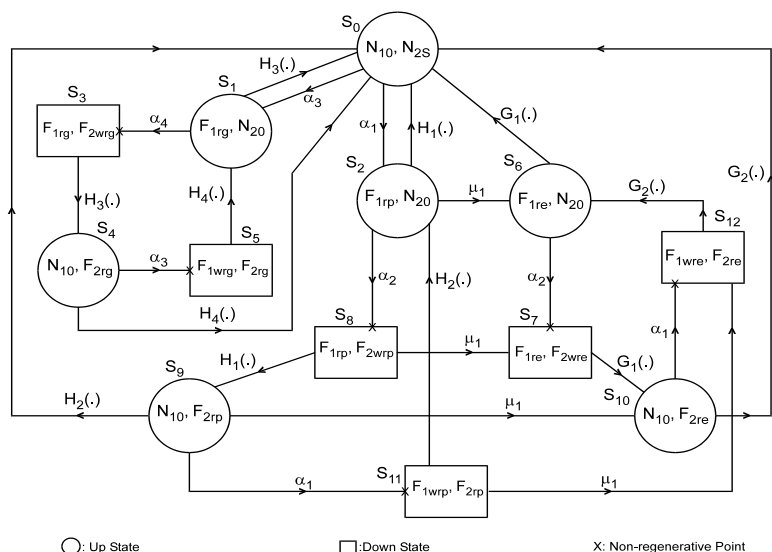


Fig. 1 Transition Diagram

IV. TRANSITION PROBABILITIES AND SOJOURN TIMES

A. STEADY STATE PROBABILITIES:

$$\begin{aligned}
 p_{01} &= \alpha_3 \int e^{-(\alpha_1 + \alpha_3)u} du = \frac{\alpha_3}{\alpha_1 + \alpha_3} = 1 - p_{02} & p_{10} &= \int e^{-\alpha_4 u} dH_3(u) = \tilde{H}_3(\alpha_4) = 1 - p_{14}^{(3)} \\
 p_{20} &= \tilde{H}_1(\mu_1 + \alpha_2) & p_{26} &= \frac{\mu_1}{(\mu_1 + \alpha_2)} [1 - \tilde{H}_1(\mu_1 + \alpha_2)] \\
 p_{27}^{(8)} &= 1 - \tilde{H}_1(\mu_1) - \frac{\mu_1}{(\mu_1 + \alpha_2)} [1 - \tilde{H}_1(\mu_1 + \alpha_2)] & p_{29}^{(8)} &= \tilde{H}_1(\mu_1) - \tilde{H}_1(\mu_1 + \alpha_2) \\
 p_{40} &= \tilde{H}_4(\alpha_3) = 1 - p_{41}^{(5)} & p_{60} &= \tilde{G}_1(\alpha_2) = 1 - p_{6,10}^{(7)} \\
 p_{87} &= 1 - \tilde{H}_1(\mu_1) = 1 - p_{89} & p_{90} &= \tilde{H}_2(\mu_1 + \alpha_1) \\
 p_{92}^{(11)} &= \tilde{H}_2(\mu_1) - \tilde{H}_2(\mu_1 + \alpha_1) & p_{9,10} &= \frac{\mu_1}{(\mu_1 + \alpha_1)} [1 - \tilde{H}_2(\mu_1 + \alpha_1)] \\
 p_{9,12}^{(11)} &= 1 - \tilde{H}_2(\mu_1) - \frac{\mu_1}{(\mu_1 + \alpha_1)} [1 - \tilde{H}_2(\mu_1 + \alpha_1)] & p_{10,0} &= \tilde{G}_2(\alpha_1) = 1 - p_{10,6}^{(12)} \\
 p_{11,2} &= \tilde{H}_2(\mu_1) = 1 - p_{11,12} & p_{34} &= p_{51} = p_{7,10} = p_{12,6} = 1
 \end{aligned}$$

From the obtained steady state probabilities, it can be easily seen that the following results hold good:

$$\begin{aligned}
 p_{01} + p_{02} &= 1 & p_{10} + p_{14}^{(3)} &= 1 & p_{20} + p_{26} + p_{27}^{(8)} + p_{29}^{(8)} &= 1 \\
 p_{40} + p_{41}^{(5)} &= 1 & p_{60} + p_{6,10}^{(7)} &= 1 & p_{87} + p_{89} &= 1 \\
 p_{90} + p_{9,2}^{(11)} + p_{9,10} + p_{9,12}^{(11)} &= 1 & p_{10,0} + p_{10,6}^{(12)} &= 1 & p_{11,2} + p_{11,12} &= 1 \\
 p_{34} = p_{51} = p_{7,10} = p_{12,6} &= 1
 \end{aligned} \tag{1-10}$$

B. MEAN SOJOURN TIMES:

The mean sojourn time in state S_i denoted by ψ_i is defined as the expected time taken by the system in state S_i before transiting to any other state. To obtain mean sojourn time ψ_i , in state S_i , we observe that as long as the system is in state S_i , there is no transition from S_i to any other state. If T_i denotes the sojourn time in state S_i then mean sojourn time in state S_i is

$$\psi_i = E[T_i] = \int P[T_i > t] dt$$

Its values for various states are as follows:

$$\begin{aligned}
 \psi_0 &= \frac{1}{(\alpha_1 + \alpha_3)} & \psi_1 &= \frac{1}{\alpha_4} [1 - \tilde{H}_3(\alpha_4)] & \psi_2 &= \frac{1}{(\mu_1 + \alpha_2)} [1 - \tilde{H}_1(\mu_1 + \alpha_2)] \\
 \psi_3 &= \int \tilde{H}_3(t) dt = m_1 & \psi_4 &= \frac{1}{\alpha_3} [1 - \tilde{H}_4(\alpha_3)] & \psi_5 &= \int \tilde{H}_4(t) dt = m_2 \\
 \psi_6 &= \frac{1}{\alpha_2} [1 - \tilde{G}_1(\alpha_2)] & \psi_7 &= \int \tilde{G}_1(t) dt = n_1 & \psi_8 &= \frac{1}{\mu_1} [1 - \tilde{H}_1(\mu_1)] \\
 \psi_9 &= \frac{1}{(\mu_1 + \alpha_1)} [1 - \tilde{H}_2(\mu_1 + \alpha_1)] & \psi_{10} &= \frac{1}{\alpha_1} [1 - \tilde{G}_2(\alpha_1)] & \psi_{11} &= \frac{1}{\mu_1} [1 - \tilde{H}_2(\mu_1)] \\
 \psi_{12} &= \int \tilde{G}_2(t) dt = n_2
 \end{aligned}$$

V. ANALYSIS OF RELIABILITY AND MTSF

Let the random variable T_i be the time to system failure when system starts up from state $S_i \in E_i$, then the reliability of the system is given by

$$R_i(t) = P[T_i > t]$$

Using the definition of $R_i(t)$ relations among $R_i(t)$ can be developed, taking their Laplace transforms and solving the resultant set of equations for $R_0^*(s)$, we get

$$R_0^*(s) = N_1(s)/D_1(s) \tag{11}$$

Where,

$$N_1(s) = Z_0^* + q_{01}^* Z_1^* + q_{02}^* Z_2^* + q_{02}^* q_{26}^* Z_6^*$$

and

$$D_1(s) = 1 - q_{01}^* q_{10}^* - q_{02}^* q_{20}^* - q_{02}^* q_{26}^* q_{60}^*$$

To get MTSF, we use the well known formula

$$E(T_0) = \int R_0(t) dt = \lim_{s \rightarrow 0} R_0^*(s) = N_1(0)/D_1(0) \tag{12}$$

where,

$$N_1(0) = \psi_0 + p_{01} \psi_1 + p_{02} \psi_2 + p_{02} p_{26} \psi_6 \tag{13}$$

and

$$D_1(0) = 1 - p_{01} p_{10} - p_{02} p_{20} - p_{02} p_{26} p_{60} \tag{14}$$

VI. AVAILABILITY ANALYSIS

Define $A_i(t)$ as the probability that the system is up at epoch t when it initially starts from regenerative state S_i . To obtain recurrence relations among different pointwise availabilities we use the simple probabilistic arguments.

Then taking the Laplace transform of the relations obtained and solving them for $A_0^*(s)$, we have

$$A_0^*(s) = N_2(s)/D_2(s) \tag{15}$$

Where,

$$N_2(s) = Z_0^*(1 - q_{14}^{(3)*} q_{41}^{(5)*})(1 - q_{29}^{(8)*} q_{92}^{(11)*})(1 - q_{6,10}^{(7)*} q_{10,6}^{(12)*}) + q_{01}^*(Z_1^* + q_{14}^{(3)*} Z_4^*)(1 - q_{29}^{(8)*} q_{92}^{(11)*})(1 - q_{6,10}^{(7)*} q_{10,6}^{(12)*}) + q_{02}^*(Z_6^* + q_{6,10}^{(7)*} Z_{10}^*)(1 - q_{14}^{(3)*} q_{41}^{(5)*}) \{q_{26}^* + q_{27}^{(8)*} q_{7,10}^* q_{10,6}^{(12)*} + q_{29}^{(8)*} (q_{9,10}^* q_{10,6}^{(12)*} + q_{9,12}^{(11)*} q_{12,6}^*)\} + q_{02}^*(1 - q_{14}^{(3)*} q_{41}^{(5)*})(1 - q_{6,10}^{(7)*} q_{10,6}^{(12)*})(Z_2^* + q_{27}^{(8)*} q_{7,10}^* Z_{10}^* + q_{29}^{(8)*} Z_9^* + q_{29}^{(8)*} q_{9,10}^* Z_{10}^*)$$

and

$$D_2(s) = (1 - q_{14}^{(3)*} q_{41}^{(5)*})(1 - q_{29}^{(8)*} q_{92}^{(11)*})(1 - q_{6,10}^{(7)*} q_{10,6}^{(12)*}) - q_{01}^*(1 - q_{29}^{(8)*} q_{92}^{(11)*})(1 - q_{6,10}^{(7)*} q_{10,6}^{(12)*})(q_{10}^* + q_{14}^{(3)*} q_{40}^*) - q_{02}^*(1 - q_{14}^{(3)*} q_{41}^{(5)*})(1 - q_{6,10}^{(7)*} q_{10,6}^{(12)*}) \{q_{20}^* + q_{27}^{(8)*} q_{7,10}^* q_{10,6}^{(12)*} + q_{29}^{(8)*} (q_{90}^* + q_{9,10}^* q_{10,6}^{(12)*})\} - q_{02}^*(1 - q_{14}^{(3)*} q_{41}^{(5)*})(q_{60}^* + q_{6,10}^{(7)*} q_{10,6}^{(12)*}) \{q_{26}^* + q_{27}^{(8)*} q_{7,10}^* q_{10,6}^{(12)*} + q_{29}^{(8)*} (q_{9,10}^* q_{10,6}^{(12)*} + q_{9,12}^{(11)*} q_{12,6}^*)\}$$

The steady state availability is given by

$$A_0 = \lim_{t \rightarrow \infty} A_0(t) = \lim_{s \rightarrow 0} s A_0^*(s) = \lim_{s \rightarrow 0} \frac{s N_2(s)}{D_2(s)} = \lim_{s \rightarrow 0} N_2(s) \lim_{s \rightarrow 0} \frac{s}{D_2(s)}$$

where

$$N_2(0) = \psi_0(1 - p_{14}^{(3)} p_{41}^{(5)})(1 - p_{29}^{(8)} p_{92}^{(11)})(1 - p_{6,10}^{(7)} p_{10,6}^{(12)}) + p_{01}(1 - p_{29}^{(8)} p_{92}^{(11)})(1 - p_{6,10}^{(7)} p_{10,6}^{(12)})(\psi_1 + p_{14}^{(3)} \psi_4) + p_{02}(1 - p_{14}^{(3)} p_{41}^{(5)})(\psi_6 + p_{6,10}^{(7)} \psi_{10}) \{p_{26} + p_{27}^{(8)} p_{7,10} p_{10,6}^{(12)} + p_{29}^{(8)} (p_{9,10} p_{10,6}^{(12)} + p_{9,12}^{(11)} p_{12,6})\} + p_{02}(1 - p_{14}^{(3)} p_{41}^{(5)})(1 - p_{6,10}^{(7)} p_{10,6}^{(12)}) \{\psi_2 + p_{27}^{(8)} p_{7,10} \psi_{10} + p_{29}^{(8)} (\psi_9 + p_{9,10} \psi_{10})\}$$

and $D_2(0) = 0$

Hence on using L'Hospital's rule, A_0 becomes

$$A_0 = N_2(0)/D_2'(0) \quad (17)$$

To obtain $D_2'(0)$, we collect the coefficient of relevant m_{ij}^s , where m_{ij}^s is the mean elapsed time of the system in state S_i before transiting to state S_j .

Using the relation $\sum_j m_{ij} = \psi_i$, we get

$$D_2'(0) = (1 - p_{14}^{(3)} p_{41}^{(5)})(1 - p_{6,10}^{(7)} p_{10,6}^{(12)}) [\psi_0(1 - p_{29}^{(8)} p_{92}^{(11)}) + \psi_2 p_{02} + n_1 p_{02} p_{27}^{(8)} + \psi_9 p_{02} p_{29}^{(8)} + n_2 p_{02} p_{29}^{(8)} p_{9,12}^{(11)}] + (1 - p_{29}^{(8)} p_{92}^{(11)})(1 - p_{6,10}^{(7)} p_{10,6}^{(12)}) [\psi_1 p_{01} + \psi_4 p_{01} p_{14}^{(3)}] + [\psi_6 (p_{26} + p_{27}^{(8)} p_{7,10} p_{10,6}^{(12)} + p_{29}^{(8)} p_{10,6}^{(12)} p_{9,10} + p_{29}^{(8)} p_{9,12}^{(11)}) + \psi_{10} (p_{26} p_{6,10}^{(7)} + p_{27}^{(8)} p_{7,10} p_{10,6}^{(12)} + p_{29}^{(8)} p_{9,10} + p_{29}^{(8)} p_{9,12}^{(11)} p_{6,10}^{(7)})] p_{02}(1 - p_{14}^{(3)} p_{41}^{(5)}) \quad (18)$$

Using (16) and (18) in (17), we get the expression for A_0 .

The expected up time of the system during $(0, t]$ is given by

$$\mu_{up}(t) = \int_0^t A_0(u) du \quad (19)$$

so that

$$\mu_{up}^*(s) = A_0^*(s)/s \quad (20)$$

VII. BUSY PERIOD ANALYSIS

BUSY PERIOD ANALYSIS FOR REGULAR / EXPERT REPAIRMAN:

Let us define $B_i^r(t)/B_i^e(t)$ as the probability that the regular/expert repairman is busy in the repairing the failed unit when the system initially starts from state $S_i \in E$. Using probabilistic arguments, the values of $B_0^r(t)/B_0^e(t)$ can be obtained in its L.T as:

$$B_0^{r*}(s) = N_3(s)/D_2(s) \quad \text{and} \quad B_0^{e*}(s) = N_4(s)/D_2(s) \quad (21-22)$$

Where,

$$N_3(s) = q_{01}^*(Z_1^* + q_{14}^{(3)*} Z_4^*)(1 - q_{29}^{(8)*} q_{92}^{(11)*})(1 - q_{6,10}^{(7)*} q_{10,6}^{(12)*}) + (Z_2^* q_{02}^* + Z_9^* q_{02}^* q_{29}^{(8)*})(1 - q_{14}^{(3)*} q_{41}^{(5)*})(1 - q_{6,10}^{(7)*} q_{10,6}^{(12)*})$$

and

$$N_4(s) = (1 - q_{14}^{(3)*} q_{41}^{(5)*})(Z_6^* + q_{6,10}^{(7)*} Z_{10}^*) [q_{02}^* q_{26}^* + q_{02}^* q_{27}^{(8)*} q_{7,10}^* q_{10,6}^{(12)*} + q_{02}^* q_{29}^{(8)*} (q_{9,10}^* q_{10,6}^{(12)*} + q_{9,12}^{(11)*} q_{12,6}^*)] + (1 - q_{14}^{(3)*} q_{41}^{(5)*})(1 - q_{6,10}^{(7)*} q_{10,6}^{(12)*}) [q_{02}^* q_{27}^{(8)*} (Z_7^* + q_{7,10}^* Z_{10}^*) + q_{02}^* q_{29}^{(8)*} (q_{9,10}^* Z_{10}^* + q_{9,12}^{(11)*} Z_2^*)]$$

In the steady state, the probability that the regular/expert repairman will be busy is given by

$$B_0^r = \lim_{t \rightarrow \infty} B_0^r(t) = \lim_{s \rightarrow 0} s B_0^{r*}(s) = N_3(0)/D_2'(0) \quad \text{and} \quad B_0^e = N_4(0)/D_2'(0) \quad (23-24)$$

Where,

$$N_3(0) = p_{01}(\psi_1 + p_{14}^{(3)} \psi_4)(1 - p_{29}^{(8)} p_{92}^{(11)})(1 - p_{6,10}^{(7)} p_{10,6}^{(12)}) + (1 - p_{14}^{(3)} p_{41}^{(5)})(1 - p_{6,10}^{(7)} p_{10,6}^{(12)})(\psi_2 p_{02} + \psi_9 p_{02} p_{29}^{(8)})$$

and

$$N_4(0) = (1 - p_{14}^{(3)} p_{41}^{(5)})(\psi_6 + p_{6,10}^{(7)} \psi_{10}) [p_{02} p_{26} + p_{02} p_{27}^{(8)} p_{7,10} p_{10,6}^{(12)} + p_{02} p_{29}^{(8)} (p_{9,10} p_{10,6}^{(12)} + p_{9,12}^{(11)} p_{12,6})] + (1 - p_{14}^{(3)} p_{41}^{(5)})(1 - p_{6,10}^{(7)} p_{10,6}^{(12)}) [p_{02} p_{27}^{(8)} (n_1 + p_{7,10} \psi_{10}) + p_{02} p_{29}^{(8)} (p_{9,10} \psi_{10} + p_{9,12}^{(11)} n_2)] \quad (26)$$

The expected busy period of the regular/expert repairman during (0, t] is given by

$$\mu_b^r(t) = \int_0^t B_0^r(u) du \quad \text{and} \quad \mu_b^e(t) = \int_0^t B_0^e(u) du \quad (27-28)$$

so that

$$\mu_b^{r*}(s) = B_0^{r*}(s)/s \quad \text{and} \quad \mu_b^{e*}(s) = B_0^{e*}(s)/s \quad (29-30)$$

VIII. EXPECTED NUMBER OF REPAIRS

EXPECTED NUMBER OF REPAIRS BY REGULAR / EXPERT REPAIRMAN:

Let us define $V_i^r(t)/V_i^e(t)$ as the expected number of repairs by the regular/expert repairman during the time interval (0, t] when the system initially starts from regenerative state S_i . Using the definition of $V_i^r(t)/V_i^e(t)$ the recursive relation among them can be easily developed, taking their Laplace Transform and solving for $\tilde{V}_0^r(s)/\tilde{V}_0^e(s)$ we have

$$\tilde{V}_0^r = N_5(s)/D_2(s) \quad \text{and} \quad \tilde{V}_0^e = N_6(s)/D_2(s) \quad (31-32)$$

where

$$N_5(s) = (\tilde{Q}_{01} + \tilde{Q}_{01}\tilde{Q}_{14}^{(3)})(1 - \tilde{Q}_{29}^{(8)}\tilde{Q}_{92}^{(11)})(1 - \tilde{Q}_{6,10}^{(7)}\tilde{Q}_{10,6}^{(12)}) + \tilde{Q}_{02}(1 - \tilde{Q}_{14}^{(3)}\tilde{Q}_{41}^{(5)})(1 - \tilde{Q}_{6,10}^{(7)}\tilde{Q}_{10,6}^{(12)})[\tilde{Q}_{20} + \tilde{Q}_{29}^{(8)}(1 + \tilde{Q}_{90} + \tilde{Q}_{92}^{(11)})]$$

and

$$N_6(s) = \tilde{Q}_{02}\tilde{Q}_{26}(1 - \tilde{Q}_{14}^{(3)}\tilde{Q}_{41}^{(5)})(1 + \tilde{Q}_{6,10}^{(7)}) + \tilde{Q}_{02}(1 - \tilde{Q}_{14}^{(3)}\tilde{Q}_{41}^{(5)})(1 - \tilde{Q}_{6,10}^{(7)}\tilde{Q}_{10,6}^{(12)})[\tilde{Q}_{27}^{(8)}(\tilde{Q}_{7,10} + \tilde{Q}_{7,10}\tilde{Q}_{10,0} + \tilde{Q}_{7,10}\tilde{Q}_{10,6}^{(12)}) + \tilde{Q}_{29}^{(8)}(\tilde{Q}_{9,10} + \tilde{Q}_{9,12}^{(11)}\tilde{Q}_{12,6})]$$

In the long run the expected number of repairs per unit of time by the regular/expert repairman is given by

$$V_0^r = \lim_{t \rightarrow \infty} \frac{[V_0^r(t)]}{t} = \lim_{s \rightarrow 0} s\tilde{V}_0^r(s) = N_5(0)/D_2'(0) \quad (33)$$

Similarly,

$$V_0^e = N_6(0)/D_2'(0) \quad (34)$$

where,

$$N_5(0) = (p_{01} + p_{01}p_{14}^{(3)})(1 - p_{29}^{(8)}p_{92}^{(11)})(1 - p_{6,10}^{(7)}p_{10,6}^{(12)}) + p_{02}(1 - p_{14}^{(3)}p_{41}^{(5)})(1 - p_{6,10}^{(7)}p_{10,6}^{(12)})[p_{20} + p_{29}^{(8)}(1 + p_{90} + p_{92}^{(11)})]$$

and

$$N_6(0) = p_{02}p_{26}(1 - p_{14}^{(3)}p_{41}^{(5)})(1 + p_{6,10}^{(7)}) + p_{02}(1 - p_{14}^{(3)}p_{41}^{(5)})(1 - p_{6,10}^{(7)}p_{10,6}^{(12)})[p_{27}^{(8)}(p_{7,10} + p_{7,10}p_{10,0} + p_{7,10}p_{10,6}^{(12)}) + p_{29}^{(8)}(p_{9,10} + p_{9,12}^{(11)}p_{12,6})]$$

IX. PROFIT FUNCTION ANALYSIS

Two profit functions $P_1(t)$ and $P_2(t)$ can easily be obtained for the system model under study with the help of characteristics obtained earlier. The expected total profits incurred during (0, t] are:

$$P_1(t) = \text{Expected total revenue in (0,t]} - \text{Expected total expenditure in (0,t]} \\ = C_0\mu_{up}(t) - C_1\mu_b^r(t) - C_2\mu_b^e(t) \quad (37)$$

Similarly,

$$P_2(t) = C_0\mu_{up}(t) - C_3V_0^r(t) - C_4V_0^e(t) \quad (38)$$

Where,

C_0 is revenue per unit up time of the system.

C_1 is cost per unit time for which the regular repairman is busy in repairing the failed unit.

C_2 is cost per unit time for which expert repairman is busy in repairing the failed unit.

C_3 is per unit repair cost by regular repairman.

C_4 is per unit repair cost by an expert repairman.

The expected total profits per unit time, in steady state, is

$$P_1 = \lim_{t \rightarrow \infty} [P_1(t)/t] = \lim_{s \rightarrow 0} s^2 P_1^*(s)$$

$$P_2 = \lim_{t \rightarrow \infty} [P_2(t)/t] = \lim_{s \rightarrow 0} s^2 P_2^*(s)$$

So that,

$$P_1 = C_0A_0 - C_1B_0^r - C_2B_0^e \quad (39)$$

and

$$P_2 = C_0A_0 - C_3V_0^r - C_4V_0^e \quad (40)$$

X. GRAPHICAL STUDY OF THE SYSTEM MODEL

For more concrete study of system behaviour, we plot MTSF and Profit functions with respect to α_1 (failure rate of 1st unit) for different values of β_1 (repair rate of 1st unit) and to plot their graphs, the repair time distributions are assumed to be distributed exponentially.

Fig 2 shows the variations in MTSF with respect to α_1 for different values of β_1 as 0.02, 0.04 and 0.06 while the other parameters are fixed as $\alpha_2 = 0.05$, $\alpha_3 = 0.10$, $\alpha_4 = 0.015$, $\mu_1 = 0.004$, $\beta_2 = 0.009$, $\beta_3 = 0.35$, $\beta_4 = 0.65$, $\gamma_1 = 0.025$, $\gamma_2 = 0.35$. It is observed that MTSF decreases with the increase in the failure parameter α_1 .

and for higher values of β_1 , the MTSF is higher i.e., the repair facility has a better understanding of failure phenomenon resulting in longer lifetime of the system.

Fig 3. represents the change in profit function P_1 and P_2 w.r.t. α_1 for different values of β_1 as 0.02, 0.04 and 0.06 while the other parameters are fixed as $\alpha_2 = 0.05$, $\alpha_3 = 0.10$, $\alpha_4 = 0.015$, $\mu_1 = 0.004$, $\beta_2 = 0.009$, $\beta_3 = 0.35$, $\beta_4 = 0.65$, $\gamma_1 = 0.025$, $\gamma_2 = 0.35$, $C_0 = 1100$, $C_1 = 900$, $C_2 = 750$, $C_3 = 250$, $C_4 = 150$. From the graph it is seen that both profit functions decrease with the increase in failure rate α_1 and increase with the increase in β_1 . Thus the better understanding of failure phenomenon by the repairman results in better system performance.

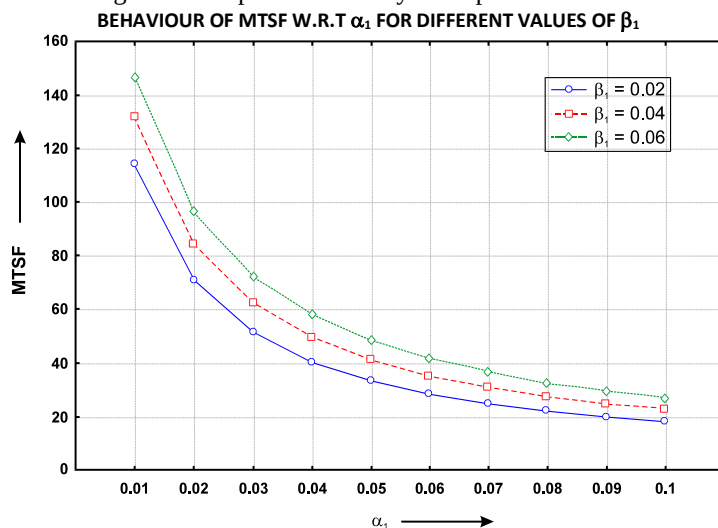


Fig. 2

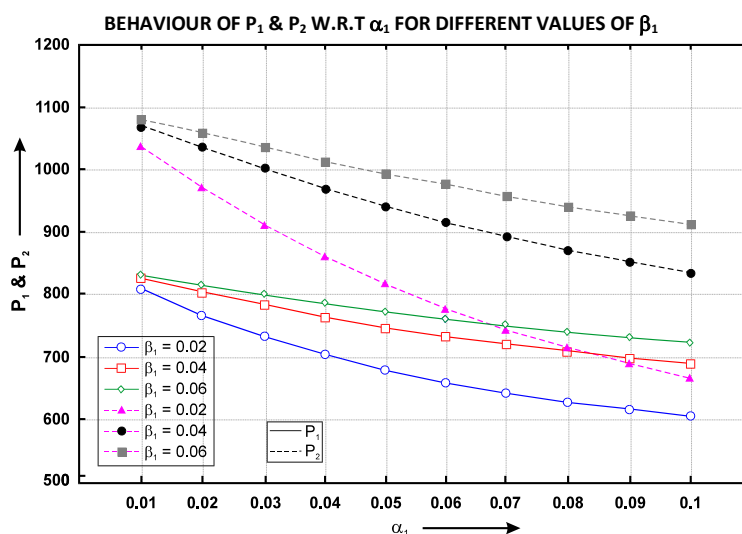


Fig. 3

XI. REFERENCES

- [1] Goel, L.R., A. Kumar and A.K. Rastogi, (1985) "Stochastic behaviour of a man-machine system operating under different weather conditions", *Microelectron Reliab.*, 25(1), 87-91.
- [2] Goel, L.R., Rakesh Gupta and Preeti Srivastava, (1990) "Profit analysis of a two-unit cold standby system with varying physical conditions of the repairman", *Microelectron. Reliab.*, Vol.-30, No. 4, pp. 655-660.
- [3] Goyal, V. and K. Murari (1984): "Cost analysis of a two unit standby system with regular repairman and patience time", *Microelectron Reliab.*, Vol. 24(3), p. 453-459.
- [4] Gupta Rakesh and P. Bhardwaj (2013): "A Two-Unit Standby System with regular repairmen and waiting time of skilled repairmen", *Journal of Mathematics & Computer science*, No. 4, pp. 1115-1130.
- [5] Gupta Rakesh, P. Sharma and Shivakar (2013): "A Two-Unit active redundant system with two physical conditions of repairman and correlated life times", *Journal of Ravishankar University-B*, pp. 24-26, 40-51.
- [6] M.C. Rander, A. Kumar, R.K. Tuteja, (1992): "Analysis of a two unit cold standby system with imperfect assistant repairman and perfect master repairman", *Microelectron. Reliab.* 32 (4), 497-501.
- [7] Mokaddis, G.S., M.L. Tawfek and S.A.M. Elhssia, (2005): "Busy Period Analysis of a Man-machine System Operating Subject To Different Physical Conditions", *Journal of Mathematics and Statistics*, 1 (2), 160-164.
- [8] V. Sridharan and P. Mohanavadi, (1998): "Stochastic behaviour of a two-unit standby system with two types of repairmen and patience time", *Mathematical and Computer Modeling*, 28 (9), 63-71.