



Modulational Dispersion and Amplification in Semiconductor Plasma

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Abstract: The modulational dispersion and amplification in piezoelectric semiconductor plasma is studied by the coupled mode theory and using the hydrodynamic model of a semiconductor plasma. In the present paper we analytically investigated in a doped III-V semiconductor, viz. n-InSb, the frequency modulation interaction between copropagating high-frequency electromagnetic beams and acoustic waves and the consequent amplification of the modulated waves. The threshold value of pump electric field ($|E_0|$) and modulational gain coefficient (a_{eff}) are obtained by the nonlinear effective susceptibility χ_{eff} . Modulational Dispersion and Amplification in doped III-V Semiconductors like n-InSb crystal at 77 K duly irradiated by a nanosecond-pulsed $10.6 \mu\text{m}$ CO_2 laser. The magnitude of χ_{eff} can be increased considerably in a heavily doped medium by increasing the strength of the d.c. magnetic field.

Keywords: Acoustic waves, Modulational instability, Semiconductor Plasma, Optical susceptibility.

I. Introduction

The phenomena of modulation interaction exhibit a distinctive role in non linear optics. The modulational interaction between propagating laser beam Modulational instability and generated acoustic mode is analysed by using coupled mode theory and nonlinear induced current density.

Modulational instability refers to instability of a wave propagating in nonlinear dispersive media such that the steady state becomes unstable and evolves into a temporally modulated state [1]. The concept of transverse modulational instability originates from a space time analogy that exists when the dispersion is replaced by diffraction [2]. The well-known instability of a plane wave in a self-focusing Kerr medium [3] is an example of transverse modulational instability. There are number of papers published in this area is that modulational instability of a laser beam found in a piezoelectrically active semiconducting medium with a high dielectric constant.

The transmission, display and processing of information is possible by modulation [4]. The fabrication of some optical devices, such as acousto-optic modulators, is based on the interaction of an acoustic wave or low-frequency electromagnetic waves with the incident laser beam. The propagation of the acoustic wave creates a refractive index grating, which in turn diffracts the incident laser beam resulting in the modulation of the incident beam. At small diffraction efficiencies the diffracted light intensity is proportional to the acoustic intensity. This fact is used in acoustic modulation of optical radiation. The information signal is used to modulate the intensity of the acoustic beam. This modulation is then transferred, as intensity modulation, onto the diffracted optical beam [5]. The most important application of acousto-optic interactions is the deflection of optical beams. This can be achieved by changing the sound frequency while operating near the Bragg diffraction condition [5].

Most optoelectronics devices for control the relaxation time of material are used the semiconductors [6]. Semiconductors are used in most of the sophisticated, sensitive and ultrafast optoelectronic devices [7] due to their compactness, provision of control of material relaxation time and highly advanced fabrication technology. In nonlinear optical processes, large number of free electrons/holes act as majority charged carriers is doped with semiconductors [8]. Scattering of light beam from free electrons in piezoelectric semiconductors was reported by Guha and Sen [9] and by other workers. Many such investigations are based upon the nonlinear optical response in a semiconductor medium [10, 11].

II. Theoretical Formulation

In order to study the transverse modulational amplification in a magnetized piezoelectric semiconductor arising due to nonlinear effective susceptibility χ_{eff} , the hydrodynamic model of semiconductor plasma is considered.

A Spatially uniform pump electric field is applied along the x-axis parallel to propagation vector k and externally applied D.C. magnetic field B_0 , is taken along the z-axis normal to E_0 and k .

The momentum and energy exchange between these waves can be described by phase-matching condition: $\hbar\omega_0 = \hbar\omega_s + \hbar\omega_a$. The phase-matching conditions enable one to consider $k_s + k_a = k$. We could neglect the nonuniformity of the high-frequency electric field under the dipole approximation when the wavelength of the excited sound wave is very small compared to the scale length of the electromagnetic field variation [12].

A uniform pump electric field $\vec{E} = E_0 e^{-i\omega t}$ (pump vector $k_0=0$)

$$\frac{\partial v_0}{\partial t} + \omega_0 = -\frac{e}{m} \bar{E}_0(t) \quad (1)$$

$$\frac{\partial v_1}{\partial t} + \omega_1 = -\frac{e}{m} \bar{E}_1(t) - \left(v \frac{\partial}{\partial x} \right) v_1 \quad (2)$$

$$\frac{\partial n_1}{\partial t} + n_e \frac{\partial v_1}{\partial x} = -v_0 \left(\frac{\partial n_1}{\partial x} \right) \quad (3)$$

$$\frac{\partial E_a}{\partial x} + \frac{\beta}{\epsilon} \frac{\partial^2 u}{\partial x^2} = -\frac{n_1 e}{\epsilon} \quad (4)$$

$$\rho \frac{\partial^2 u}{\partial t^2} + 2\gamma\rho \frac{\partial u}{\partial t} + \beta \frac{\partial E_a}{\partial x} = C \frac{\partial^2 u}{\partial t^2} \quad (5)$$

Eqs. (1) and (2) are the zeroth- and first-order momentum transfer equations in which v_0 and v_1 are, respectively, the zeroth-order oscillatory fluid velocities of the electron of effective mass m and charge $-e$, v being the electron collision frequency. The basic nonlinearity induced in the motion of the charge carriers is due to the convective derivative $(\mathbf{v} \cdot \nabla) \mathbf{v}$ depends on the total intensity of illumination and the Lorentz force on the electrons in equation (2) [19]. Eq. (3) is the continuity equation, where n_0 and n_1 are the initial electron concentrations in the n-type-doped semiconductor and perturbed electron concentration, respectively. In Poisson's equation (4), E_a represents the perturbed electric field component and ϵ , and β are the dielectric constant and piezoelectric constant of the crystal material, respectively. Equation (5) describes the motion of the lattice in the piezoelectric crystal with ρ , u , γ and C being, respectively, the mass density of the crystal, displacement of the lattice, phenomenological damping parameter of acoustic mode and crystal elastic constant. The acoustic perturbation created in the medium under the influence of a strong pump source gives rise to an electron density perturbation at the acoustic frequency, which couples nonlinearly with the pump wave and drives the acoustic wave at modulated frequencies in a modulational instability process.

$$\frac{\partial^2 n_1}{\partial t^2} + v \frac{\partial n_1}{\partial t} + \omega_p^2 n_1 + \frac{n_e e \beta}{m \epsilon} \times \frac{\partial^2 u}{\partial x^2} = -\frac{e E_0}{m} \frac{\partial n_1}{\partial x} \quad (6)$$

Where $\omega_c = eB_0/m$ being the cyclotron frequency and $\omega_p = [(n_e/m\epsilon)^{1/2}]$ is electron plasma frequency. In Eq. (6), we have neglected the Doppler shift assuming $\omega_0 \gg kv_0$. We consider the cyclotron frequency (ω_c) and electron plasma frequency (ω_p) to be comparable to the incident pump frequency (ω_0) and the pump field amplitude and the magnetic field due to the pump wave (B) is neglected.

The modulation process under consideration must fulfill the phase matching conditions –

$$k_0 = k_1 \pm k_s$$

$$\omega_0 = \omega_1 \pm \omega_s$$

$$\text{Such that } |k| = |k_0 \pm k_s| \approx |k_s| = k$$

We considered only the resonant sideband frequencies and higher order terms are neglected. These two equations exhibit the coupling between the high and low-frequency components of the density perturbations n_{1s} and n_{1a} via the pump electric field.

Under rotating-wave approximation, Eq. (6) yields the two-coupled equations for n_{1s} and n_{1a} as-

$$n_{1s} = \frac{in_e e \beta^2 k^3 E_s}{m \epsilon \rho (w_s^2 - k^2 v_s^2 + 2i \gamma w_s)} \times [w_p^2 - w_+^2 - i v w_+ + i k_s \vec{E}]^{-1} \quad (7(a))$$

$$n_{1a} = \frac{in_e e \beta^2 k^3 E_s}{m \epsilon \rho (w_s^2 - k^2 v_s^2 + 2i \gamma w_s)} \times [w_p^2 - w_-^2 - i v w_- + i k_s \vec{E}]^{-1} \quad (7(b))$$

Where $w_+ = w_s + w_0$ and $w_- = w_s - w_0$ and $v_s = (C / \rho)^{1/2}$ is the velocity of the acoustic wave in the lattice. It is clear that from equation (7) n_{1s} and n_{1a} is depend upon the magnitude of the pump intensity (I). We consider only the Stokes component of the nonlinear current density $J(w)$ associated with the Stokes mode arising due to the coupling of the nonlinear current densities n_{1s} and n_{1a} expressed as -

$$J(w_+) = -n_{1s} e v_0 \quad (8)$$

$$J(w_-) = -n_{1a} e v_0^* \quad (9)$$

The induced polarization at the modulated frequencies $P(w_{\pm})$ as the time integral of the nonlinear current density $J(w_{\pm})$ is -

$$P(w_{\pm}) = \int J(w_{\pm}) dt \quad (10)$$

The effective nonlinear polarization of the modulated wave is -

$$P_{eff} = P(w_+) + P(w_-) \quad (11)$$

Therefore, total effective polarization is -

$$P_{eff} = \frac{i w_p^2 w_0 e \epsilon A k E_0 E_s}{m (w_s^2 - k^2 v_s^2 + 2i w_s \gamma) (w_0^2 - w_c^2)} \times \left[\frac{1}{w_+} (\Delta_1^2 - i v w_+ + i k \vec{E})^{-1} - \frac{1}{w_-} (\Delta_2^2 - i v w_- + i k \vec{E})^{-1} \right] \quad (12)$$

Where $\Delta_1^2 = w_p^2 - w_+^2$, $\Delta_2^2 = w_p^2 - w_-^2$, $A = \alpha^2 k^2 v_s^2$, $\zeta^2 = \beta^2 / \epsilon C$ and $\Delta = w_0 - w_p$

$$P_{eff} = \frac{2 w_p^2 e^2 \epsilon A k^2 E_0^2 E_s (\Delta^2 - v^2)}{m^2 (w_s^2 - k^2 v_s^2 + 2i w_s \gamma) (w_0^2 - w_c^2)^2} \times \left[\left(\Delta^2 + v^2 - \frac{k^2 \vec{E}^2}{w_0^2} \right)^2 + \frac{4 k^2 \Delta^2 \vec{E}^2}{w_0^2} \right]^{-1} \quad (13)$$

In the presence of pump fields, the transverse components of the oscillatory electron fluid velocity v_0 is -

$$\left. \begin{aligned} v_{ox} &= \frac{\vec{E}}{v - i w_0} \\ v_{oy} &= \frac{-(e/m) E_0 w_c}{w_0^2 - w_c^2} \end{aligned} \right\} \quad (14)$$

The induced polarization due to nonlinearities at modulated frequencies (w_+) is defined as -

$$P_{eff} = \epsilon_0 \chi_{eff} |E_0|^2 E_s \quad (15)$$

This gives -

$$\chi_{eff} = \frac{2 w_p^2 e^2 \epsilon_1 A k^2 (\Delta^2 - v^2)}{m^2 (w_s^2 - k^2 v_s^2 + 2i w_s \gamma) (w_0^2 - w_c^2)^2} \times \left[\left(\Delta^2 + v^2 - \frac{k^2 \vec{E}^2}{w_0^2} \right)^2 + \frac{4 k^2 \Delta^2 \vec{E}^2}{w_0^2} \right]^{-1} \quad (16)$$

We assume the semiconductor medium to be dispersionless for the acoustic waves. This analysis are made in the highly doped regime –

$$w_p \approx w_0 \approx w_{\pm} \text{ And } w_p \gg v(w_s)$$

Consequently, the effective nonlinear susceptibility of the semiconductor medium given by Eq. (16) reduces to the form

$$\chi_{eff} = \frac{2w_p^2 e^2 \epsilon_1 A k^2 (\Delta^2 - v^2) (w_s^2 - k^2 v_s^2)}{m^2 \{ (w_s^2 - k^2 v_s^2)^2 + 4w_s^2 \gamma^2 \} (w_0^2 - w_c^2)^2} \times \left[\left(\Delta^2 + v^2 - \frac{k^2 \bar{E}^2}{w_0^2} \right)^2 + \frac{4k^2 \Delta^2 \bar{E}^2}{w_0^2} \right]^{-1} \quad (17)$$

In the presence of a transverse magnetostatic field, the effective nonlinear susceptibility (eqn (17)) characterizes the steady state optical response of the medium and governs the nonlinear wave propagation through the medium. It is clear that the total crystal susceptibility is affected by the equilibrium carrier concentration through $w_p \neq 0$ and the external d.c. magnetic field through $w_c \neq 0$.

In order to investigate the modulational amplification coefficient α_{eff} in a doped semiconductor, we employ the relation –

$$\alpha_{eff} = \frac{k}{2\epsilon_1} \chi_{eff} |E_0|^2 \quad (18)$$

If α_{eff} is negative, then the nonlinear growth of the modulated signal is possible. Therefore, the nonlinear growth of the modulated signal is possible only if χ_{eff} is negative.

The nonlinear modulational gain of the signal as well as the idler waves can be possible only if α_{eff} is negative for pump electric field $|E_0| > |E_{0th}|$.

In general, to determine the threshold value of the pump amplitude of the modulational amplification is we set $P_{eff}=0$ and is –

$$|E_{0th}| = \frac{\Delta m (w_0^2 - w_c^2)}{ekw_0^2} \quad (19)$$

It is cleared from eqn (18) even in absence of damping the transverse modulational instability of the signal wave has a nonzero intensity threshold.

Thus, the growth rate of modulated beam is –

$$g = \frac{w_p^2 e^2 \epsilon_1 A k^3 (\Delta^2 - v^2) (w_s^2 - k^2 v_s^2) E_0^2}{m^2 \epsilon_1 \{ (w_s^2 - k^2 v_s^2)^2 + 4w_s^2 \gamma^2 \} (w_0^2 - w_c^2)^2} \times \left[\left(\Delta^2 + v^2 - \frac{k^2 \bar{E}^2}{w_0^2} \right)^2 + \frac{4k^2 \Delta^2 \bar{E}^2}{w_0^2} \right]^{-1} \quad (20)$$

III. Results And Discussion

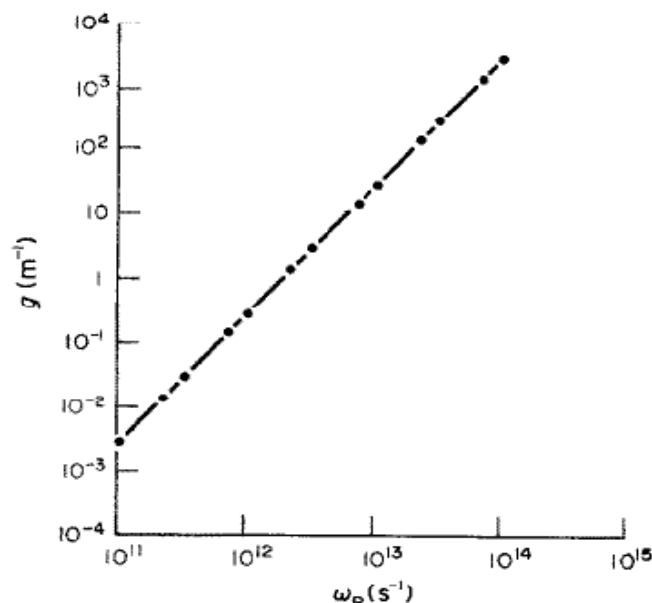
Modulational Dispersion and Amplification in doped III–V Semiconductors like n-InSb crystal at 77 K duly irradiated by a nanosecond-pulsed 10.6 μm CO₂ laser. The physical constants of the n-type InSb crystals are – $m = 0.014m_0$ (m_0 the free electron mass), $\epsilon_1 = 17.9$, $\beta = 0.054\text{cm}^{-2}$, $\gamma = 5 \times 10^{-10} \text{ s}^{-1}$, $\rho = 5.8 \times 10^3 \text{ kgm}^{-3}$ and $v = 4 \times 10^{11} \text{ s}^{-1}$.

The expression of the growth rate as obtained from equation (20) has predicted by Drake et al. [13].

It is cleared that from equation (20) the gain constant g has dependence on the wave vector k has following –

- (i) For lower magnitudes of k (such that $w_s \gg kv_s$), g increases with k
- (ii) For a non dispersive acoustic mode, at $w_s \sim kv_s$, g is maximum.
- (iii) At $w_s \ll kv_s$, then g shows a steep decline with increasing k .

In Figure we have plotted the growth rate of the signal as a function of the electron plasma frequency w_p , for the dispersion less regime of the low-frequency acoustic mode w_c at $E_0 = 2 \times 10^7 \text{ Vm}^{-1}$, $w_c = 0.9w_0$ and $k = 2.5 \times 10^7 \text{ m}^{-1}$. It is cleared that from this paper the growth rate of the transversely modulated wave increases with a rise in electron density of the medium The nature of the curve is similar to the conclusions arrived at by Salimullah and Singh [14] who considered the modulational interaction of an extraordinary mode subjected to perturbations parallel to the magnetic field.



When the carrier concentration of the medium by n-type doping in the crystal is increases then higher amplification of the waves is obtained. It is condition that the doping should not exceed the limit for which the plasma frequency ω_p exceeds the input pump frequency ω_0 , because, in the regime when $\omega_p > \omega_0$, the electromagnetic pump wave will be reflected back by the intervening medium.

The present work deals with the analytical investigations of modulational dispersion and amplification in n-InSb semiconductor plasmas. The pump electric field produces a shift in the resonance frequency in induced polarization term and it plays an important role in the enhancement of both modulational dispersion and amplification.

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