



Solution of Cubic Equation

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Abstract: In this paper, we solved cubic equation with the help of Cardan's rule.

Keywords: Cubic Equation, Real and Complex roots, Exact Solution.

1. Introduction

Roots of cubic equation $x^3 + px^2 + qx + r = 0$, say x_1, x_2, x_3 are

If $\beta^2 - \mu \geq 0$, where $\beta = 9pq - 2p^3 - 27r$ and $\mu = 4(p^2 - 3q)^3$

$$x_1 = u + v - \frac{p}{3}$$

$$x_2 = \frac{-\frac{1}{3}\{2p + 3(u+v)\} + \sqrt{\frac{1}{3}\{4p^2 - 9(u+v)^2\} - 4q}}{2}$$

$$x_3 = \frac{-\frac{1}{3}\{2p + 3(u+v)\} - \sqrt{\frac{1}{3}\{4p^2 - 9(u+v)^2\} - 4q}}{2}$$

Where u and v are given by

$$u^3 = \frac{\beta + \sqrt{\beta^2 - \mu}}{54} \text{ and } v^3 = \frac{\beta - \sqrt{\beta^2 - \mu}}{54}$$

If $\beta^2 - \mu < 0$, where $\beta = 9pq - 2p^3 - 27r$ and $\mu = 4(p^2 - 3q)^3$

$$x_1 = \frac{2}{3}\sqrt{p^2 - 3q} \cos \frac{\theta}{3} - \frac{p}{3}$$

$$x_2 = \frac{2}{3}\sqrt{p^2 - 3q} \cos \left(\frac{2\pi + \theta}{3} \right) - \frac{p}{3}$$

$$x_3 = \frac{2}{3}\sqrt{p^2 - 3q} \cos \left(\frac{4\pi + \theta}{3} \right) - \frac{p}{3}$$

Where θ is given by $\cos \theta = \frac{\beta}{\sqrt{\mu}}$

Method and Proof. Consider a cubic equation in x of the form

$$x^3 + px^2 + qx + r = 0 \quad \dots (1), \text{ where } p, q \text{ and } r \text{ are all real numbers}$$

Putting $x = y + h$ in equation (1), then

$$(y+h)^3 + p(y+h)^2 + q(y+h) + r = 0$$

$$y^3 + h^3 + 3y^2h + 3yh^2 + py^2 + ph^2 + 2pyh + qy + qh + r = 0$$

$$y^3 + (3h + p)y^2 + (3h^2 + 2ph + q)y + (h^3 + ph^2 + qh + r) = 0 \quad \dots(2)$$

Take $3h + p = 0$, we get

$$h = \frac{-p}{3}$$

Putting the value of h in equation (2), then

$$y^3 + \left(3\frac{p^2}{9} - 2\frac{p^2}{3} + q\right)y + \left(-\frac{p^3}{27} + \frac{p^3}{9} - \frac{pq}{3} + r\right) = 0$$

$$y^3 + \left(q - \frac{p^2}{3}\right)y + \left(\frac{2p^3}{27} - \frac{pq}{3} + r\right) = 0 \quad \dots(3)$$

Take $y = u + v$ and using Carden Rule, then

$$y^3 = u^3 + v^3 + 3uv(u + v)$$

$$y^3 = u^3 + v^3 + 3uvy$$

$$y^3 - 3uvy - (u^3 + v^3) = 0 \quad \dots(4)$$

Equating equations (3) and (4), we get

$$-3uv = q - \frac{p^2}{3}$$

$$uv = \frac{p^2}{9} - \frac{q}{3}$$

$$uv = \frac{p^2 - 3q}{9} \quad \dots(5)$$

$$-(u^3 + v^3) = \frac{2p^3}{27} - \frac{pq}{3} + r$$

$$u^3 + v^3 = \frac{pq}{3} - \frac{2p^3}{27} - r$$

$$u^3 + v^3 = \frac{9pq - 2p^3 - 27r}{27} \quad \dots(6)$$

We write

$$u^3 - v^3 = \sqrt{(u^3 + v^3)^2 - 4u^3v^3}$$

$$= \sqrt{\frac{(9pq - 2p^3 - 27r)^2}{729} - 4\frac{(p^2 - 3q)^3}{729}}, \text{ Using equations (5) and (6)}$$

$$= \frac{\sqrt{(9pq - 2p^3 - 27r)^2 - 4(p^2 - 3q)^3}}{27}$$

$$u^3 - v^3 = \frac{\sqrt{(9pq - 2p^3 - 27r)^2 - 4(p^2 - 3q)^3}}{27} \quad \dots(7)$$

From equations (6) and (7)

$$u^3 + v^3 = \frac{9pq - 2p^3 - 27r}{27}$$

$$u^3 - v^3 = \frac{\sqrt{(9pq - 2p^3 - 27r)^2 - 4(p^2 - 3q)^3}}{27}$$

$$2u^3 = \frac{(9pq - 2p^3 - 27r) + \sqrt{(9pq - 2p^3 - 27r)^2 - 4(p^2 - 3q)^3}}{27}$$

$$u^3 = \frac{(9pq - 2p^3 - 27r) + \sqrt{(9pq - 2p^3 - 27r)^2 - 4(p^2 - 3q)^3}}{54}$$

$$2v^3 = \frac{(9pq - 2p^3 - 27r) - \sqrt{(9pq - 2p^3 - 27r)^2 - 4(p^2 - 3q)^3}}{27}$$

$$v^3 = \frac{(9pq - 2p^3 - 27r) - \sqrt{(9pq - 2p^3 - 27r)^2 - 4(p^2 - 3q)^3}}{54}$$

$$u^3 = \frac{\beta + \sqrt{\beta^2 - \mu}}{54}$$

$$v^3 = \frac{\beta - \sqrt{\beta^2 - \mu}}{54}$$

Where $\beta = 9pq - 2p^3 - 27r$

and $\mu = 4(p^2 - 3q)^3$

If $\beta^2 - \mu \geq 0$, then u and v both are real

If $\beta^2 - \mu < 0$, then u and v both are complex

Case 1. If $\beta^2 - \mu \geq 0$, then

$$x = y + h$$

$$x = u + v - \frac{p}{3}$$

Let roots of cubic equation are x_1, x_2, x_3 , then one of roots of cubic equation is

$$x_1 = u + v - \frac{p}{3}$$

Since x_1 is a root of cubic equation $x^3 + px^2 + qx + r = 0$

Hence $x - x_1$ is a factor of $x^3 + px^2 + qx + r$

Divide $x^3 + px^2 + qx + r = 0$ by $x - x_1$ on both sides, we get

Quadratic equation $x^2 + (p + x_1)x + (q + px_1 + x_1^2) = 0$

$$x = \frac{-(p + x_1) \pm \sqrt{(p + x_1)^2 - 4(q + px_1 + x_1^2)}}{2}, \text{ by Shridharacharya Method}$$

$$= \frac{-(p + x_1) \pm \sqrt{p^2 + x_1^2 + 2px_1 - 4q - 4px_1 - 4x_1^2}}{2}$$

$$= \frac{-(p + x_1) \pm \sqrt{p^2 - 2px_1 - 3x_1^2 - 4q}}{2}$$

$$= \frac{-(p + x_1) \pm \sqrt{p^2 + px_1 - 3px_1 - 3x_1^2 - 4q}}{2}$$

$$= \frac{-(p + x_1) \pm \sqrt{p(p + x_1) - 3x_1(p + x_1) - 4q}}{2}$$

$$= \frac{-(p+x_1) \pm \sqrt{(p+x_1)(p-3x_1)-4q}}{2}$$

Putting the value of x_1 , then

$$x = \frac{-\left(p+u+v-\frac{p}{3}\right) \pm \sqrt{\left(p+u+v-\frac{p}{3}\right)(p-3u-3v+p)-4q}}{2}$$

$$= \frac{-\frac{1}{3}(2p+3u+3v) \pm \sqrt{\frac{1}{3}(2p+3u+3v)(2p-3u-3v)-4q}}{2}$$

$$= \frac{-\frac{1}{3}\{2p+3(u+v)\} \pm \sqrt{\frac{1}{3}\{4p^2-9(u+v)^2\}-4q}}{2}$$

$$\text{Take } x_2 = \frac{-\frac{1}{3}\{2p+3(u+v)\} + \sqrt{\frac{1}{3}\{4p^2-9(u+v)^2\}-4q}}{2}$$

$$\text{and } x_3 = \frac{-\frac{1}{3}\{2p+3(u+v)\} - \sqrt{\frac{1}{3}\{4p^2-9(u+v)^2\}-4q}}{2}$$

Case 2. If $\beta^2 - \mu < 0$

$$u^3 = \frac{\beta + \sqrt{\beta^2 - \mu}}{54} = \frac{\beta}{54} + \frac{\sqrt{\beta^2 - \mu}}{54} = \frac{\beta}{54} + i \frac{\sqrt{\mu - \beta^2}}{54} = r(\cos \theta + i \sin \theta), \text{ say}$$

Where $i = \sqrt{-1}$

$$v^3 = \frac{\beta - \sqrt{\beta^2 - \mu}}{54} = \frac{\beta}{54} - \frac{\sqrt{\beta^2 - \mu}}{54} = \frac{\beta}{54} - i \frac{\sqrt{\mu - \beta^2}}{54} = r(\cos \theta - i \sin \theta), \text{ say}$$

Where $i = \sqrt{-1}$

$$u = r^{\frac{1}{3}}(\cos \theta + i \sin \theta)^{\frac{1}{3}}$$

$$= r^{\frac{1}{3}}\left\{\cos(2n\pi + \theta) + i \sin(2n\pi + \theta)\right\}^{\frac{1}{3}}, \text{ in general term}$$

$$= r^{\frac{1}{3}}\left\{\cos\left(\frac{2n\pi + \theta}{3}\right) + i \sin\left(\frac{2n\pi + \theta}{3}\right)\right\}, \text{ by De Moivre's theorem}$$

$$v = r^{\frac{1}{3}}(\cos \theta - i \sin \theta)^{\frac{1}{3}}$$

$$= r^{\frac{1}{3}}\left\{\cos(2n\pi + \theta) - i \sin(2n\pi + \theta)\right\}^{\frac{1}{3}}, \text{ in general term}$$

$$= r^{\frac{1}{3}}\left\{\cos\left(\frac{2n\pi + \theta}{3}\right) - i \sin\left(\frac{2n\pi + \theta}{3}\right)\right\}, \text{ by De Moivre's theorem}$$

Where $n = 0, 1, 2$

$$u + v = 2r^{\frac{1}{3}} \cos\left(\frac{2n\pi + \theta}{3}\right)$$

$$x = y + h$$

$$x = u + v - \frac{p}{3}$$

$$x = 2r^{\frac{1}{3}} \cos\left(\frac{2n\pi + \theta}{3}\right) - \frac{p}{3}$$

$$\text{Here } r = \sqrt{\frac{\beta^2}{54^2} + \frac{\mu - \beta^2}{54^2}}$$

$$= \frac{\sqrt{\mu}}{54}$$

$$= \frac{\mu^{\frac{1}{2}}}{54}$$

$$= \frac{2(p^2 - 3q)^{\frac{3}{2}}}{54}$$

$$= \frac{(p^2 - 3q)^{\frac{3}{2}}}{27}$$

$$r^{\frac{1}{3}} = \frac{\sqrt{p^2 - 3q}}{3}$$

$$\text{and } \theta = \tan^{-1} \frac{\sqrt{\mu - \beta^2}}{\beta}$$

$$\tan \theta = \frac{\sqrt{\mu - \beta^2}}{\beta}$$

$$\cos \theta = \frac{1}{\sqrt{1 + \tan^2 \theta}}$$

$$\cos \theta = \frac{1}{\sqrt{1 + \frac{\mu - \beta^2}{\beta^2}}}$$

$$= \frac{\beta}{\sqrt{\mu}}$$

$$x = 2r^{\frac{1}{3}} \cos\left(\frac{2n\pi + \theta}{3}\right) - \frac{p}{3}$$

$$x = \frac{2}{3} \sqrt{p^2 - 3q} \cos\left(\frac{2n\pi + \theta}{3}\right) - \frac{p}{3}$$

Putting $n = 0, 1, 2$, we get roots of the equation $x^3 + px^2 + qx + r = 0$

$$x_1 = \frac{2}{3} \sqrt{p^2 - 3q} \cos \frac{\theta}{3} - \frac{p}{3}$$

$$x_2 = \frac{2}{3} \sqrt{p^2 - 3q} \cos\left(\frac{2\pi + \theta}{3}\right) - \frac{p}{3}$$

$$x_3 = \frac{2}{3} \sqrt{p^2 - 3q} \cos\left(\frac{4\pi + \theta}{3}\right) - \frac{p}{3}$$

Working Rule: Consider a cubic equation in x of the form $x^3 + px^2 + qx + r = 0$, where p, q and r are all real constants.

Find β and μ

Where $\beta = 9pq - 2p^3 - 27r$

$$\text{and } \mu = 4(p^2 - 3q)^3$$

If $\beta^2 - \mu \geq 0$, then roots of cubic equation $x^3 + px^2 + qx + r = 0$

$$x_1 = u + v - \frac{p}{3}$$

$$x_2 = \frac{-\frac{1}{3}\{2p + 3(u+v)\} + \sqrt{\frac{1}{3}\{4p^2 - 9(u+v)^2\} - 4q}}{2}$$

$$x_3 = \frac{-\frac{1}{3}\{2p + 3(u+v)\} - \sqrt{\frac{1}{3}\{4p^2 - 9(u+v)^2\} - 4q}}{2}$$

Where u and v are given by

$$u^3 = \frac{\beta + \sqrt{\beta^2 - \mu}}{54} \text{ and } v^3 = \frac{\beta - \sqrt{\beta^2 - \mu}}{54}$$

If $\beta^2 - \mu < 0$, then roots of cubic equation $x^3 + px^2 + qx + r = 0$

$$x_1 = \frac{2}{3}\sqrt{p^2 - 3q} \cos \frac{\theta}{3} - \frac{p}{3}$$

$$x_2 = \frac{2}{3}\sqrt{p^2 - 3q} \cos \left(\frac{2\pi + \theta}{3} \right) - \frac{p}{3}$$

$$x_3 = \frac{2}{3}\sqrt{p^2 - 3q} \cos \left(\frac{4\pi + \theta}{3} \right) - \frac{p}{3}$$

Where θ is given by $\cos \theta = \frac{\beta}{\sqrt{\mu}}$.

Example 1. Solve $x^3 - 6x^2 + 11x - 6 = 0$.

Solution. $x^3 - 6x^2 + 11x - 6 = 0$

Equating with $x^3 + px^2 + qx + r = 0$, we get

$$p = -6, q = 11, r = -6$$

$$\beta = 9pq - 2p^3 - 27r$$

$$= 9 \times (-6) \times 11 - 2 \times (-6)^3 - 27 \times (-6)$$

$$= -594 + 432 + 162$$

$$= 0$$

$$\mu = 4(p^2 - 3q)^3$$

$$= 4(36 - 33)^3$$

$$= 4 \times 27$$

$$= 108$$

$$\beta^2 - \mu = 0 - 108 = -108$$

$$\beta^2 - \mu < 0$$

$$x_1 = \frac{2}{3}\sqrt{p^2 - 3q} \cos \frac{\theta}{3} - \frac{p}{3}$$

$$= \frac{2}{3}\sqrt{36 - 33} \cos \frac{\theta}{3} + 2$$

$$\begin{aligned}
 &= \frac{2}{3} \sqrt{3} \cos \frac{\theta}{3} + 2 \\
 &= \frac{2}{\sqrt{3}} \cos \frac{\theta}{3} + 2 \\
 x_2 &= \frac{2}{3} \sqrt{p^2 - 3q} \cos \left(\frac{2\pi + \theta}{3} \right) - \frac{p}{3} \\
 &= \frac{2}{3} \sqrt{36 - 33} \cos \left(\frac{2\pi + \theta}{3} \right) + 2 \\
 &= \frac{2}{3} \sqrt{3} \cos \left(\frac{2\pi + \theta}{3} \right) + 2 \\
 &= \frac{2}{\sqrt{3}} \cos \left(\frac{2\pi + \theta}{3} \right) + 2 \\
 x_3 &= \frac{2}{3} \sqrt{p^2 - 3q} \cos \left(\frac{4\pi + \theta}{3} \right) - \frac{p}{3} \\
 &= \frac{2}{3} \sqrt{36 - 33} \cos \left(\frac{4\pi + \theta}{3} \right) + 2 \\
 &= \frac{2}{3} \sqrt{3} \cos \left(\frac{4\pi + \theta}{3} \right) + 2 \\
 &= \frac{2}{\sqrt{3}} \cos \left(\frac{4\pi + \theta}{3} \right) + 2
 \end{aligned}$$

$$\cos \theta = \frac{\beta}{\sqrt{\mu}}$$

$$\cos \theta = 0$$

$$\theta = \frac{\pi}{2}$$

$$x_1 = \frac{2}{\sqrt{3}} \cos \frac{\pi}{6} + 2$$

$$= \frac{2}{\sqrt{3}} \frac{\sqrt{3}}{2} + 2$$

$$= 1 + 2$$

$$= 3$$

$$x_2 = \frac{2}{\sqrt{3}} \cos \frac{5\pi}{6} + 2$$

$$= -\frac{2}{\sqrt{3}} \frac{\sqrt{3}}{2} + 2$$

$$= -1 + 2$$

$$= 1$$

$$x_3 = \frac{2}{\sqrt{3}} \cos \frac{9\pi}{6} + 2$$

$$= 0 + 2$$

$$= 2$$

$$x_1 = 3, x_2 = 1, x_3 = 2.$$

Example 2. Solve $x^3 - 8x^2 + 29x - 52 = 0$.

Solution. $x^3 - 8x^2 + 29x - 52 = 0$

Equating with $x^3 + px^2 + qx + r = 0$, we get

$$p = -8, q = 29, r = -52$$

$$\beta = 9pq - 2p^3 - 27r$$

$$= 9 \times (-8) \times 29 - 2 \times (-8)^3 - 27 \times (-52)$$

$$= -2088 + 1024 + 1404$$

$$= 340$$

$$\mu = 4(p^2 - 3q)^3$$

$$= 4(64 - 87)^3$$

$$= 4 \times (-23)^3$$

$$= -48668$$

$$\beta^2 - \mu = (340)^2 - (-48668) = 164268$$

$$\beta^2 - \mu > 0$$

$$u^3 = \frac{\beta + \sqrt{\beta^2 - \mu}}{54}$$

$$= \frac{340 + \sqrt{164268}}{54}$$

$$= 13.8018498$$

$$u = 2.398717474$$

$$v^3 = \frac{\beta - \sqrt{\beta^2 - \mu}}{54}$$

$$= \frac{340 - \sqrt{164268}}{54}$$

$$= -1.209257203$$

$$v = -1.065384141$$

$$x_1 = u + v - \frac{p}{3}$$

$$= 2.398717474 - 1.065384141 + 2.666666667$$

$$= 4$$

$$x_1 = 4$$

$$x_2 = \frac{-\frac{1}{3}\{2p + 3(u+v)\} + \sqrt{\frac{1}{3}\{4p^2 - 9(u+v)^2\} - 4q}}{2}$$

$$\begin{aligned}
 &= \frac{-\frac{1}{3}\{-16+3(2.398717474-1.065384141)\} + \sqrt{\frac{1}{3}\{256-9(2.398717474-1.065384141)^2\}} - 116}{2} \\
 &= \frac{-\frac{1}{3}(-16+3.999999999) + \sqrt{\frac{1}{3}(256-15.99999999)} - 116}{2} \\
 &= \frac{-\frac{1}{3}(-12) + \sqrt{\frac{240}{3}} - 116}{2} \\
 &= \frac{4 + \sqrt{80-116}}{2} \\
 &= \frac{4 + \sqrt{-36}}{2} \\
 &= \frac{4+6i}{2} \\
 &= 2+3i
 \end{aligned}$$

$$x_3 = \frac{-\frac{1}{3}\{2p+3(u+v)\} - \sqrt{\frac{1}{3}\{4p^2-9(u+v)^2\}} - 4q}{2}$$

$$\begin{aligned}
 &= \frac{-\frac{1}{3}\{-16+3(2.398717474-1.065384141)\} - \sqrt{\frac{1}{3}\{256-9(2.398717474-1.065384141)^2\}} - 116}{2} \\
 &= \frac{-\frac{1}{3}(-16+3.999999999) - \sqrt{\frac{1}{3}(256-15.99999999)} - 116}{2} \\
 &= \frac{-\frac{1}{3}(-12) - \sqrt{\frac{240}{3}} - 116}{2} \\
 &= \frac{4 - \sqrt{80-116}}{2} \\
 &= \frac{4 - \sqrt{-36}}{2} \\
 &= \frac{4-6i}{2} \\
 &= 2-3i
 \end{aligned}$$

$$x_1 = 4, x_2 = 2+3i, x_3 = 2-3i.$$

Example 3. Solve $27x^3 + 54x^2 + 27x + 4 = 0$.

Solution. $27x^3 + 54x^2 + 27x + 4 = 0$

$$x^3 + 2x^2 + x + \frac{4}{27} = 0$$

Equating with $x^3 + px^2 + qx + r = 0$

$$p = 2, q = 1, r = \frac{4}{27}$$

$$\beta = 9pq - 2p^3 - 27r$$

$$= 9 \times 2 \times 1 - 2 \times 2^3 - 27 \times \frac{4}{27}$$

$$= 18 - 16 - 4$$

$$= -2$$

$$\mu = 4(p^2 - 3q)^3$$

$$= 4(4 - 3)^3$$

$$= 4$$

$$\beta^2 - \mu = 4 - 4 = 0$$

$$\beta^2 - \mu = 0$$

$$u^3 = \frac{\beta + \sqrt{\beta^2 - \mu}}{54}$$

$$= \frac{-2 + 0}{54}$$

$$u^3 = \frac{-1}{27}$$

$$u = \frac{-1}{3}$$

$$v^3 = \frac{\beta - \sqrt{\beta^2 - \mu}}{54}$$

$$= \frac{-2 - 0}{54}$$

$$v^3 = \frac{-1}{27}$$

$$v = \frac{-1}{3}$$

$$x_1 = u + v - \frac{p}{3}$$

$$= -\frac{1}{3} - \frac{1}{3} - \frac{2}{3}$$

$$x_1 = -\frac{4}{3}$$

$$x_2 = \frac{-\frac{1}{3}\{2p + 3(u + v)\} + \sqrt{\frac{1}{3}\{4p^2 - 9(u + v)^2\} - 4q}}{2}$$

$$= \frac{-\frac{1}{3}\left\{4 + 3\left(-\frac{1}{3} - \frac{1}{3}\right)\right\} + \sqrt{\frac{1}{3}\left\{16 - 9\left(-\frac{1}{3} - \frac{1}{3}\right)^2\right\} - 4}}{2}$$

$$\begin{aligned}
 &= \frac{-\frac{1}{3}(4-2) + \sqrt{\frac{1}{3}(16-4) - 4}}{2} \\
 &= \frac{-\frac{2}{3} + \sqrt{4-4}}{2} \\
 &= -\frac{1}{3} \\
 x_2 &= -\frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 x_3 &= \frac{-\frac{1}{3}\{2p+3(u+v)\} - \sqrt{\frac{1}{3}\{4p^2-9(u+v)^2\} - 4q}}{2} \\
 &= \frac{-\frac{1}{3}\left\{4+3\left(-\frac{1}{3}-\frac{1}{3}\right)\right\} - \sqrt{\frac{1}{3}\left\{16-9\left(-\frac{1}{3}-\frac{1}{3}\right)^2\right\} - 4}}{2} \\
 &= \frac{-\frac{1}{3}(4-2) - \sqrt{\frac{1}{3}(16-4) - 4}}{2} \\
 &= \frac{-\frac{2}{3} - \sqrt{4-4}}{2} \\
 &= -\frac{1}{3} \\
 x_3 &= -\frac{1}{3}
 \end{aligned}$$

$$x_1 = -\frac{4}{3}, x_2 = -\frac{1}{3}, x_3 = -\frac{1}{3}.$$

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