



Effectiveness Analysis of a Two Unit Priority Standby System with Replacement Under Warranty Scheme

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Abstract: In this paper a two non identical unit system model is investigated and analyzed under warranty with single repair facility and the concept of priority, as one unit may get priority for both operation and repair. The priority unit that is operative initially and other unit is kept as cold standby. The priority unit has three modes; minor, major and catastrophic failure and the other unit has one mode only i.e. normal failure. Single repair facility is available with the system. Priority is being given to first unit for both operation and repair. The various measures of system effectiveness such as transition probability in steady state, mean time to system failure, availability etc.

Keywords: Stochastic Modeling; Reliability; Mean time to system failure; Mean sojourn time; Steady State Availability; Busy period.

I. Introduction

The needs of modern society are growing with the time and advancement in the science and technology. Sophistication, accuracy and effectiveness are the most emphasized issues of the present time. Thus the improvement in effectiveness of such complex systems has acquired special importance in recent years. The effectiveness of a system means the suitability of the system for the fulfillment of the intended tasks and the efficiency of utilising the means put into it. The suitability of performance definite tasks is primarily determined by the reliability and quality of the system. System effectiveness is the probability that the system can successfully meet an operational demand within a given time under specified conditions. The probability of survival, mean time to system failure, availability, mean down time, frequency of failures are some of the measures of system effectiveness.

Two units standby redundant system are discussed widely in the literature. Gupta [5], Gupta and Bansal [2], Gupta and Choudhary [3], Gupta *et al.* [4] Joorel [10], Joorel *et al* [9] and Sharma *et al* [12]. However, few researchers has drawn the attention of many by incorporating the concept of system effectiveness. The concepts of warranty caught the special attention of industrialists and marketing professional due to increasing competition. Different warranty schemes were discussed for obtaining a 'fair chunk' of the market by winning the confidence of the consumer. Jack and Murthy [8] discussed a sub-optimal servicing strategy under one-dimensional warranty scheme for the sold items under warranty period, Chen and Popova [1] and Jack and Dagpunar [6] studied different maintenance policies with a two-dimensional warranty, Jack *et al* [7] reviewed literature on different maintenance strategies and two-dimensional warranty schemes. Recently, Sharma and Agrawal [11] has studied the effectiveness analysis of a system by incorporating the concept of warranty scheme at different failure modes, which did not get much attention earlier. This warranty scheme provided replacement cover for the catastrophic failure and repair cover for any failure. It also provides repair cover for the non-priority unit, called standby unit, for failure.

In this paper a two non identical unit system model is investigated and analyzed with single repair facility and the concept of priority, as one unit may get priority for both operation and repair. Here we have considered the system which consists of two non identical units namely 'P' and 'C'. 'P' has three modes: minor, major and catastrophic failure and the unit 'C' has one mode only i.e. normal failure. Priority in operation and repair is given to 'P'. Priority is being given to first unit for both operation and repair. The various measures of system effectiveness such as transition probability in steady state, mean time to system failure, availability etc.

II. System Description

(a) **Assumptions:** The priority redundant system consists of two dissimilar units. The priority unit is put online initially and gets priority in operation. The non-priority unit is kept as cold standby. Priority is being given to first unit for both operation as well as repair. The failure of the priority unit may be minor, marginal or

catastrophic. The standby unit fails in a single mode. The warranty scheme adopted here provides replacement cover for catastrophic failure of the priority unit. It also provides a repair cover for other failures of the priority unit as well as for the non-priority unit (standby unit). The switchover time is instantaneous. There is single repair/replacement facility. A repaired unit is as good as new. The failure time distributions for the units are negative exponential while repair and replacement time distribution are arbitrary.

(b) States of the system

The system may be in one of the following states:

$$S_0 = (P_o, C_s) \quad S_1 = (P_{m_r}, C_o) \quad S_2 = (P_{n_r}, C_o) \quad S_3 = (P_{c_r}, C_o) \quad S_4 = (P_{m_R}, C_w) \quad S_5 = (P_{n_R}, C_w) \\ S_6 = (P_{c_R}, C_w) \quad S_7 = (P_o, C_r) \quad S_8 = (P_{m_r}, C_w) \quad S_9 = (P_{n_r}, C_w) \quad S_{10} = (P_{c_r}, C_w)$$

The $S_0, S_1, S_2, S_3, S_7, S_8, S_9$ and S_{10} are regenerative states while S_4, S_5 and S_6 are non-regenerative states

(c) Notations:

P_o : Priority unit on online	C_s : Standby unit
P_{m_r} : Priority unit under repair having minor failure.	C_o : Standby unit under operation.
C_r : Standby unit under repair	C_w : Standby unit under waiting for repair
P_{n_r} : Priority unit under repair having major failure	
P_{c_r} : Priority unit under replacement having catastrophic failure	
P_{m_R} : Priority unit repair continued having minor failure	
P_{n_R} : Priority unit repair continued having major failure	
P_{c_R} : Priority unit replacement continued having catastrophic failure.	

The following notations are used in the paper:

$\alpha_1, \alpha_2, \alpha_3$: Constant failure rates of priority unit in minor, marginal and catastrophic failure modes

β : Constant failure rate of standby unit

$f(\cdot), F(\cdot)$: pdf and cdf of replacement time

$g_1(\cdot), G_1(\cdot)$: pdf and cdf of repair time in minor failure mode

$g_2(\cdot), G_2(\cdot)$: pdf and cdf of repair time in marginal failure mode

$k(\cdot), K(\cdot)$: Pdf and cdf of repair time of non priority

μ_i : Mean sojourn time in state S_i .

$M_i(t)$: Probability that the system sojourns in state S_i upto time t .

$*$: Symbol for Laplace transforms i.e. $f^*(s) = \int_0^\infty e^{-st} f(t) dt$

$\sim$: Symbol for Laplace -Stieltjes transforms i.e. $\tilde{F}(s) = \int_0^\infty e^{-st} dF(t)$

$\pi_i(t)$: cdf of time to system failure where starting from up state S_0

$A_i(t)$: P [system is up at epoch t]

$B_i(t)$: P [repairman is busy in repair at an epoch t]

$\odot$: symbol for Laplace convolution

$\sum m_{ij} = \mu_i$, where m_{ij} be the mean elapsed time by the system in the state S_i before transiting to other regenerative state S_j .

$$\sum \alpha = \sum_{i=1}^3 \alpha_i$$

III. Transition probabilities and mean sojourn times

Let $T_0, T_1, T_2, \dots$ denotes the epochs at which the system enters any state $S_i \in E$ and X_n be the state visited at epoch $T_n + 1$ i.e just after n th transition at T_n . Then $\{X_n, T_n\}$ is a Markov- renewal process with state space E and $Q_{ij} = P \left\{ X_{n+1} = j, T_{n+1} - T_n \leq \frac{t}{x_n} = i \right\}$ is the semi- Markov Kernel over E . The transition probability matrix of the embedded Markov chain is

$$P \equiv (p_{ij}) \equiv (Q_{ij}(\infty)) \equiv Q(\infty).$$

By simple probabilistic arguments, the non-zero elements $P \equiv (p_{ij})$ are:

$$P_{01} = \frac{\alpha_1}{\sum \alpha}; \quad P_{02} = \frac{\alpha_2}{\sum \alpha}; \quad P_{03} = \frac{\alpha_3}{\sum \alpha}; \quad P_{10} = G_1(\beta); \quad P_{17}^{(4)} = 1 - G_1(\beta); \quad P_{20} = G_2(\beta) \\ P_{27}^{(5)} = 1 - G_2(\beta); \quad P_{30} = F(\beta); \quad P_{37}^{(6)} = 1 - F(\beta); \quad P_{70} = R(\sum \alpha); \quad P_{78} = \frac{\alpha_1}{\sum \alpha} [1 - R(\sum \alpha)] \\ P_{79} = \frac{\alpha_2}{\sum \alpha} [1 - R(\sum \alpha)]; \quad P_{7,10} = \frac{\alpha_3}{\sum \alpha} [1 - R(\sum \alpha)]; \quad P_{87} = P_{97} = P_{10,7} = 1 \quad (1)$$

Evidently

$$P_{01} + P_{02} + P_{03} = 1; \quad P_{10} + P_{17}^{(4)} = 1; \quad P_{20} + P_{27}^{(5)} = 1; \quad P_{30} + P_{37}^{(6)} = 1; \quad P_{70} + P_{78} + P_{79} + P_{7,10} = 1 \quad (2)$$

For any state S_i , the mean sojourn time μ_i is the taken by the system in state S_j before transiting to any other state. Let the survival time in state S_i be T_i , then the mean sojourn time μ_i in state $S_i \in E$ is given by

$$\mu_i = E(T_i) = \int_0^{\infty} P(T_i > t) dt$$

Thus

$$\mu_0 = \frac{1}{\Sigma \alpha}; \mu_1 = \frac{1}{\beta} [1 - \tilde{G}_1(\beta)]; \mu_2 = \frac{1}{\beta} [1 - \tilde{G}_2(\beta)]; \mu_3 = \frac{1}{\beta} [1 - \tilde{F}(\beta)]; \mu_7 = \frac{1}{\Sigma \alpha} [1 - \tilde{K}(\beta)] \quad (3)$$

IV. MEAN TIME TO SYSTEM FAILURE

On the basis of arguments used for regenerative processes, we obtain the following recursive relations for

$$\begin{aligned} \pi_1(t) &= Q_{01}(t) \otimes \pi_1(t) + Q_{02}(t) \otimes \pi_2(t) + Q_{03}(t) \otimes \pi_3(t) \\ \pi_0(t) &= Q_{10}(t) \otimes \pi_0(t) + Q_{14}(t) \\ \pi_2(t) &= Q_{20}(t) \otimes \pi_0(t) + Q_{25}(t) \\ \pi_3(t) &= Q_{30}(t) \otimes \pi_0(t) + Q_{36}(t) \end{aligned} \quad (4-7)$$

On taking Laplace- Stieltjes transform in (4-7) and solving them $\tilde{\pi}_0(s)$, we get the following expression for $\tilde{\pi}_0(s)$

$$\tilde{\pi}_0(s) = \frac{\tilde{Q}_{01}\tilde{Q}_{14} + \tilde{Q}_{02}\tilde{Q}_{25} + \tilde{Q}_{03}\tilde{Q}_{36}}{1 - \tilde{Q}_{01}\tilde{Q}_{10} - \tilde{Q}_{02}\tilde{Q}_{20} - \tilde{Q}_{03}\tilde{Q}_{30}} \quad (8)$$

(for sake simplicity we write $\tilde{Q}_{ij}(s) = \tilde{Q}_{ij}$)

On using $\tilde{Q}_{ij}(s) \rightarrow p_{ij}$ as $s \rightarrow 0$ and $m_{ij} = -Q'_{ij}(0) = -\frac{d}{ds} \int e^{-st} dQ_{ij}(t) |_{s=0}$ in (8), the following expression for MTSF is obtained:

$$MTSF = \frac{\mu_0 + \mu_1 P_{01} + \mu_2 P_{02} + \mu_3 P_{03}}{1 - P_{01} P_{10} - P_{02} P_{20} - P_{03} P_{30}} \quad (9)$$

On putting the values of μ_i and p_{ij} from (1) and (3) in (9), the expression for mean time to system failure becomes

$$MTSF = \frac{\beta + \Sigma \alpha - \tilde{G}_1(\beta) - \tilde{G}_2(\beta) - \tilde{F}(\beta)}{\beta [\Sigma \alpha - \alpha_1 \tilde{G}_1(\beta) - \alpha_2 \tilde{G}_2(\beta) - \alpha_3 \tilde{F}(\beta)]} \quad (10)$$

V. Pointwise and Steady State Availability Analysis

By using elementary probabilistic arguments, the recursive relations for $A_i(t)$ are given as,

$$\begin{aligned} A_0(t) &= M_0(t) + q_{01}(t) \otimes A_1(t) + q_{02}(t) \otimes A_2(t) + q_{03}(t) \otimes A_3(t) \\ A_1(t) &= M_1(t) + q_{10}(t) \otimes A_0(t) + q_{17}^{(4)}(t) \otimes A_7(t) \\ A_2(t) &= M_2(t) + q_{20}(t) \otimes A_0(t) + q_{27}^{(5)}(t) \otimes A_7(t) \\ A_3(t) &= M_3(t) + q_{30}(t) \otimes A_0(t) + q_{37}^{(6)}(t) \otimes A_7(t) \\ A_7(t) &= M_7(t) + q_{70}(t) \otimes A_0(t) + q_{78}(t) \otimes A_8(t) + q_{79}(t) \otimes A_9(t) + q_{7,10}(t) \otimes A_{10}(t) \\ A_8(t) &= q_{87}(t) \otimes A_7(t); A_9(t) = q_{97}(t) \otimes A_7(t); A_{10}(t) = q_{10,7}(t) \otimes A_7(t) \end{aligned} \quad (11-16)$$

where $M_i(t)$ be the probability that the system starting from state $S_i \in E$ at $t = 0$ is up continuously up to epoch t without making any transition to any other regeneration state or returning to itself. Thus

$$M_0(t) = e^{-\Sigma \alpha t}; M_1(t) = e^{-\beta t} \tilde{G}_1(t); M_2(t) = e^{-\beta t} \tilde{G}_2(t); M_3(t) = e^{-\beta t} \tilde{F}(t); M_7(t) = e^{-\Sigma \alpha t} \tilde{K}(t);$$

Taking Laplace Transformation in (11-16) and solving them for A_0^* we get

$$A_0^* = N_2(s) / D_2(s) \quad (17)$$

where

$$\begin{aligned} N_2(s) &= M_0^* + q_{01}^* M_1^* + q_{02}^* M_2^* + q_{03}^* M_3^* [1 - q_{78}^* q_{87}^* - q_{79}^* q_{97}^* - q_{7,10}^* q_{10,7}^*] \\ &\quad + [q_{01}^* q_{17}^{(4)*} + q_{02}^* q_{27}^{(5)*} + q_{03}^* q_{37}^{(6)*}] M_7^* \end{aligned}$$

And

$$\begin{aligned} D_2(s) &= [1 - q_{01}^* q_{10}^* - q_{02}^* q_{20}^* - q_{03}^* q_{30}^*] [1 - q_{78}^* q_{87}^* - q_{79}^* q_{97}^* - q_{7,10}^* q_{10,7}^*] \\ &\quad - q_{70}^* [q_{01}^* q_{17}^{(4)*} + q_{02}^* q_{27}^{(5)*} + q_{03}^* q_{37}^{(6)*}] \end{aligned}$$

The steady-state availability of the system may written in the following form

$$A_0 = A_0(\infty) = \lim_{s \rightarrow 0} s A_0^*(s) = N_2(0) / D_2'(0) \quad (18)$$

On using $\tilde{Q}_{ij}(s) \rightarrow p_{ij}$ as $s \rightarrow 0$, $m_{ij} = -Q'_{ij}(0) = -\frac{d}{ds} \int e^{-st} dQ_{ij}(t) |_{s=0}$, $\Sigma m_{ij} = \mu_1$ and the values of μ_i and p_{ij} from (1) and (3) in (17), the values of $N_2(0)$ and $D_2'(0)$ can be obtained and which are given by

$$\begin{aligned} N_2(0) &= \frac{\tilde{K}(\Sigma \alpha)}{\beta} \left[1 + \frac{(1-\beta)}{\Sigma \alpha} [\alpha_1 \tilde{G}_1(\beta) + \alpha_2 \tilde{G}_2(\beta) + \alpha_1 \tilde{F}] \right] + \\ &\quad \frac{1}{\Sigma \alpha} \left[1 - \frac{1}{\Sigma \alpha} [\alpha_1 \tilde{G}_1(\beta) + \alpha_2 \tilde{G}_2(\beta) + \alpha_1 \tilde{F}] \right] \end{aligned} \quad (19)$$

and

$$D_2'(0) = R(\Sigma \alpha) \left[\frac{1}{\beta} - \left(\frac{1}{\beta} - \frac{1}{\Sigma \alpha} (\alpha_1 \int_0^\infty \tilde{G}_1(t) + \alpha_2 \int_0^\infty \tilde{G}_2(t) + \alpha_3 \int_0^\infty \tilde{F}(t)) (\alpha_1 \tilde{G}_1(\beta) + \alpha_2 \tilde{G}_2(\beta) + \alpha_1 \tilde{F}) \right) \right] + \left[1 - \frac{1}{\Sigma \alpha} (\alpha_1 \tilde{G}_1(\beta) + \alpha_2 \tilde{G}_2(\beta) + \alpha_1 \tilde{F}) \right] R \quad (20)$$

$$\text{where } R = \left[\frac{1}{\Sigma \alpha} (1 + \alpha_1 \int_0^\infty \tilde{G}_1(t) + \alpha_2 \int_0^\infty \tilde{G}_2(t) + \alpha_3 \int_0^\infty \tilde{F}(t)) \right]$$

VI. Busy Period Analysis

The recursive relations for $B_i(t)$ are given below:

$$\begin{aligned} B_0(t) &= q_{01}(t) \otimes B_1(t) + q_{02}(t) \otimes B_2(t) + q_{03}(t) \otimes B_3(t) \\ B_1(t) &= W_1(t) + q_{10}(t) \otimes B_0(t) + q_{17}^{(4)}(t) \otimes B_7(t) \\ B_2(t) &= W_2(t) + q_{20}(t) \otimes B_0(t) + q_{27}^{(5)}(t) \otimes B_7(t) \\ B_3(t) &= W_3(t) + q_{30}(t) \otimes B_0(t) + q_{37}^{(6)}(t) \otimes B_7(t) \\ B_7(t) &= W_7(t) + q_{70}(t) \otimes B_0(t) + q_{78}(t) \otimes B_8(t) + q_{79}(t) \otimes B_9(t) + q_{7,10}(t) \otimes B_{10}(t) \\ B_8(t) &= W_8(t) + q_{87}(t) \otimes B_7(t); \quad B_9(t) = W_9(t) + q_{97}(t) \otimes B_7(t); \\ B_{10}(t) &= W_{10}(t) + q_{10,7}(t) \otimes B_7(t) \end{aligned} \quad (21-27)$$

where

$$W_1(t) = e^{-\beta t} \tilde{G}_1(t); W_2(t) = e^{-\beta t} \tilde{G}_2(t); W_3(t) = e^{-\beta t} \tilde{F}(t); W_7(t) = e^{-\beta t} \tilde{K}(t) \\ W_8(t) = \tilde{G}_1(t); W_9(t) = \tilde{G}_2(t); W_{10}(t) = \tilde{F}(t)$$

On taking Laplace transformation in (21-27) and solving for B_0^* we

$$B_0^* = N_3(s)/D_2(s) \quad (28)$$

where

$$N_3(s) = [q_{01}^* W_1^* + q_{02}^* W_2^* + q_{03}^* W_3^*] [1 - q_{78}^* q_{87}^* - q_{79}^* q_{97}^* - q_{10,7}^* q_{10,7}^*] + [q_{01}^* q_{17}^{(4)*} + q_{02}^* q_{27}^{(5)*} + q_{03}^* q_{37}^{(6)*}] W_7^* \\ D_2(s) \text{ is given by (17)}$$

In the long run, the fraction of time for which the repair facility is busy is given by

$$B_0 = \lim_{t \rightarrow \infty} B_0 = \lim_{s \rightarrow 0} s B_0^*(s) = N_3(0)/D_2'(0) \quad (29)$$

where

$$\begin{aligned} N_3(0) &= [p_{01}^* W_1^* + p_{02}^* W_2^* + p_{03}^* W_3^*] [1 - p_{78}^* q_{87}^* - p_{79}^* q_{97}^* - p_{10,7}^* p_{10,7}^*] + [p_{01}^* p_{17}^{(4)*} + p_{02}^* p_{27}^{(5)*} + p_{03}^* p_{37}^{(6)*}] W_7^* \\ &= \frac{1}{\Sigma \alpha} \left[\frac{R(\Sigma \alpha)}{\beta} \left[1 - \frac{1}{\Sigma \alpha} (\alpha_1 \tilde{G}_1(\beta) + \alpha_2 \tilde{G}_2(\beta) + \alpha_1 \tilde{F})(1 - \beta) \right] + \left[1 - \frac{1}{\Sigma \alpha} (\alpha_1 \tilde{G}_1(\beta) + \alpha_2 \tilde{G}_2(\beta) + \alpha_1 \tilde{F}) \right] \right] \end{aligned} \quad (30)$$

and $D_2'(0)$ is given by (20).

VII. Expected Number of Visits by the Repair Facility

The recursive relations of $V_i(t)$ are given as

$$\begin{aligned} V_0(t) &= Q_{01}(t) \otimes [1 + V_1(t)] + Q_{02}(t) \otimes [1 + V_2(t)] + Q_{03}(t) \otimes [1 + V_3(t)] \\ V_1(t) &= Q_{10}(t) \otimes V_0(t) + Q_{17}^{(4)}(t) \otimes V_7(t) \\ V_2(t) &= Q_{20}(t) \otimes V_0(t) + Q_{27}^{(5)}(t) \otimes V_7(t) \\ V_3(t) &= Q_{30}(t) \otimes V_0(t) + Q_{37}^{(6)}(t) \otimes V_7(t) \\ V_7(t) &= Q_{70}(t) \otimes V_0(t) + Q_{78}(t) \otimes V_8(t) + Q_{79}(t) \otimes V_9(t) + Q_{7,10}(t) \otimes V_{10}(t) \\ V_8(t) &= Q_{87}(t) \otimes V_7(t); \quad V_9(t) = Q_{97}(t) \otimes V_7(t); \quad V_{10}(t) = Q_{10,7}(t) \otimes V_7(t) \end{aligned} \quad (31-38)$$

Taking Laplace-Stieltjes transformation of above equations and solving for $V_0^*(s)$, the following expression is obtained

$$V_0^*(s) = N_4(s)/D_3(s)$$

where

$$N_4(s) = [\tilde{Q}_{01} + \tilde{Q}_{02} + \tilde{Q}_{03}] [1 - \tilde{Q}_{78} \tilde{Q}_{87} - \tilde{Q}_{79} \tilde{Q}_{97} - \tilde{Q}_{7,10} \tilde{Q}_{10,7}] + [\tilde{Q}_{01} \tilde{Q}_{17}^{(4)} + \tilde{Q}_{02} \tilde{Q}_{27}^{(5)} + \tilde{Q}_{03} \tilde{Q}_{37}^{(6)}] \quad (39)$$

And $D_3(s)$ is same as of (17) on replacing q_{ij}^* by $\tilde{Q}_{ij}$.

In steady state, number of visits per unit time is given by

$$V_0 = \lim_{t \rightarrow \infty} V_0 = \lim_{s \rightarrow 0} s V_0^*(s) = N_4(s)/D_3'(0) \quad (40)$$

Where $N_4(s) = \tilde{R}(\Sigma \alpha)$ and $D_3'(0)$ is given by (20).

VIII. Cost Benefit Analysis

The cost benefit analysis of the system can be studied by considering different characteristics of the model such as mean up time of the system, expected busy period of the repair facility and expected number of visits by the repairman, total operation cost, total support cost, total cost due to failure only.

Thus the expected profit incurred in $(0,t]$ may be defined as

$$G(t) = C_R \mu_{up}(t) - C_O(t) - C_S(t) - C_F(t) - C_B \mu_b(t) - C_V V_0(t) \quad (41)$$

where C_R is revenue generated per unit up time, C_B is the cost per unit time for which the system is under repair, C_V is the cost per visit by the repairman, $C_O(t)$ and $C_S(t)$ are the total operation and total support costs respectively in time $(0,t]$ and $C_F(t)$ is the total cost due to failure, $\mu_{up}(t)$ is the expected up time of the system and $\mu_b(t)$ is the expected busy period of the repairman only in time $(0,t]$. In the long run this profit becomes

$$G = \lim_{t \rightarrow \infty} G(t)/t = \lim_{s \rightarrow 0} G^*(s) = C_R A_0 - C_O - C_S - C_F - C_B B_0 - C_V V_0 \quad (42)$$

where, C_O, C_S and C_F are steady state operating, support and failure costs respectively. A_0, B_0 and V_0 are given by (18), (29) and (40). It is summarized that the reliability of the system can be increased by using superior quality parts initially. No doubt, onetime acquisition cost will increase, but it decreases long term operating costs e.g. less consumption of energy and fuel, minimization of down time etc. as the frequency of failure decreases, the failure costs and support costs decrease due to saving of repair and supply resources. Furthermore, the reliability of the system can also be improved by replacing the parts, if needed, without any compromise in quality, providing proper training to the operators and repair persons, following a regular maintenance schedule and maintaining proper inventory of the parts.

IX. TRANSITION STATES DIAGRAM

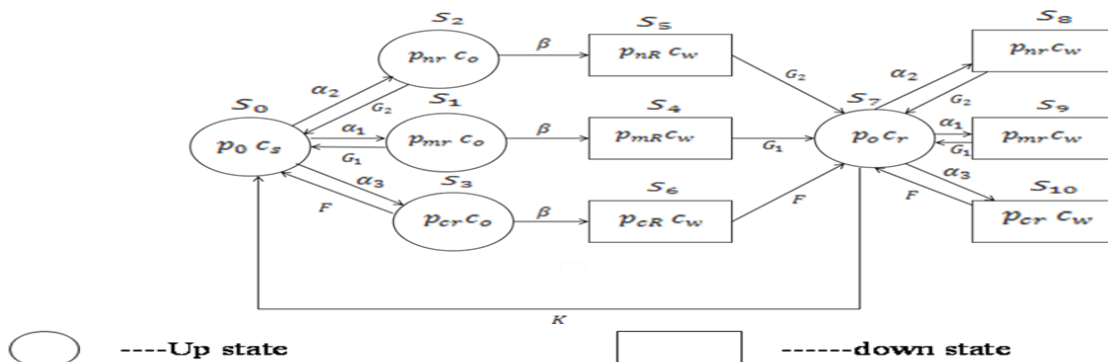


Fig. 1

X. Graphical Study of System Behaviour

The behavior of MTSF and availability of the system is studied graphically in this section and to plot their graphs, the replacement and repair time distributions are also assumed to be distributed exponentially with constant rates β_1, β_2 and β_3 respectively. The graphs of MTSF are depicted by Fig. 2-4 and that of availability are depicted by Fig. 5-10 with respect to the different parameters. From Fig. 2-5, it is observed that the MTSF decreases uniformly as the failure rates of the system increases irrespective of the other fixed parameters. It may also be concluded that expected life of the system can be increased with decreasing failure rate of priority unit. Further, from Fig. 5-7 it is observed that the availability of the system is gradually decreases with increasing failure rates irrespective of type of failure. However, from Fig. 8-10, we note that availability increases with increasing repair rates. Thus, we can conclude that the expected life of the system can be increased by increasing repair rate of priority unit.

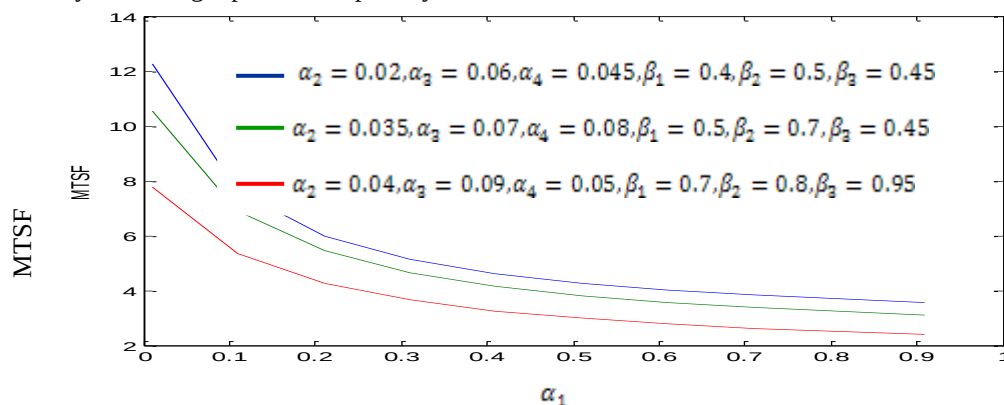


Fig.2: BEHAVIOUR OF MTSF W.R.T. α_1 FOR DIFFERENT VALUES OF $\alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3$

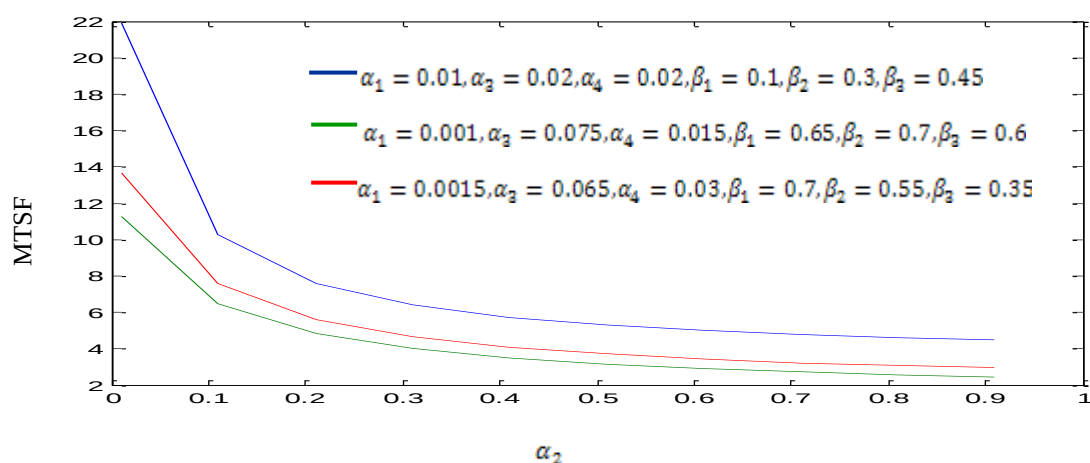


Fig.3: BEHAVIOUR OF MTSF W.R.T. α_2 FOR DIFFERENT VALUES OF $\alpha_1, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3$

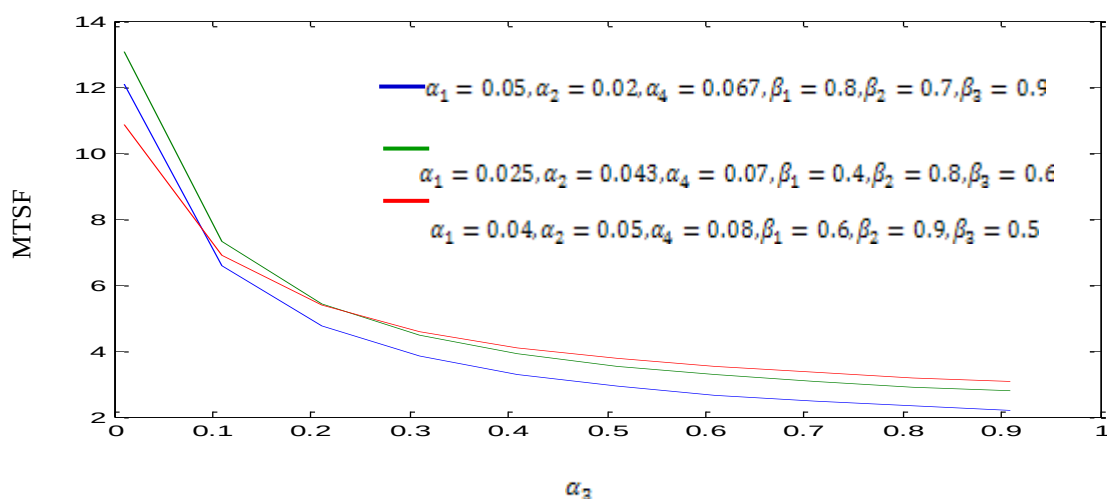


Fig.4: BEHAVIOUR OF MTSF W.R.T. α_3 FOR DIFFERENT VALUES OF $\alpha_1, \alpha_2, \alpha_4, \beta_1, \beta_2, \beta_3$

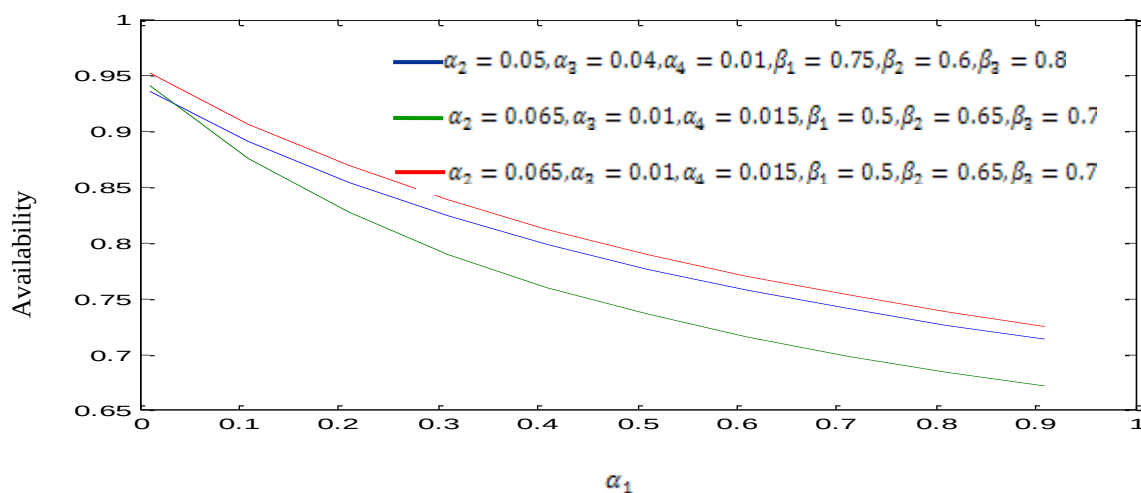


Fig. 5: BEHAVIOUR OF STEADY STATE AVAILABILITY W.R.T. α_1 FOR DIFFERENT VALUES OF $\alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3$

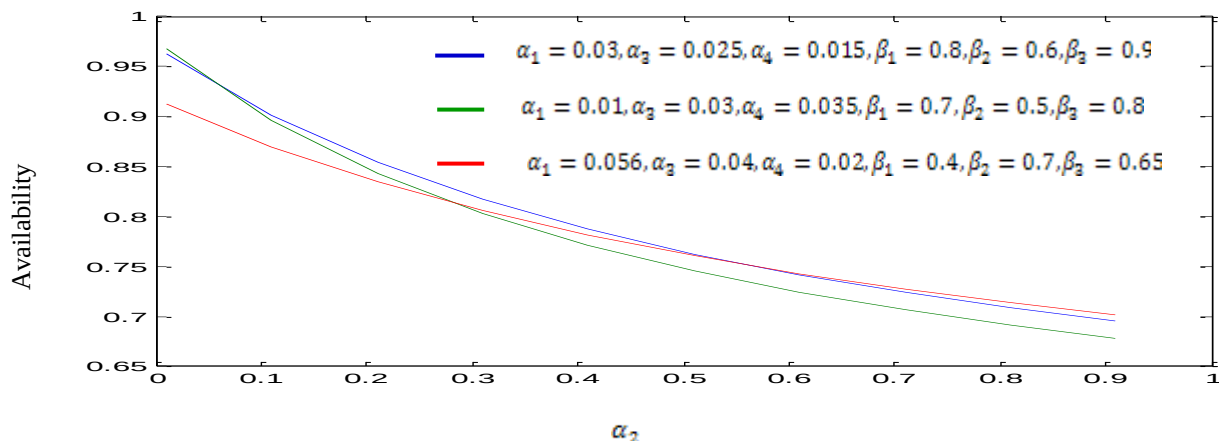


Fig.6: BEHAVIOUR OF STEADY STATE AVAILABILITY W.R.T. α_2 FOR DIFFERENT VALUES OF $\alpha_1, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3$

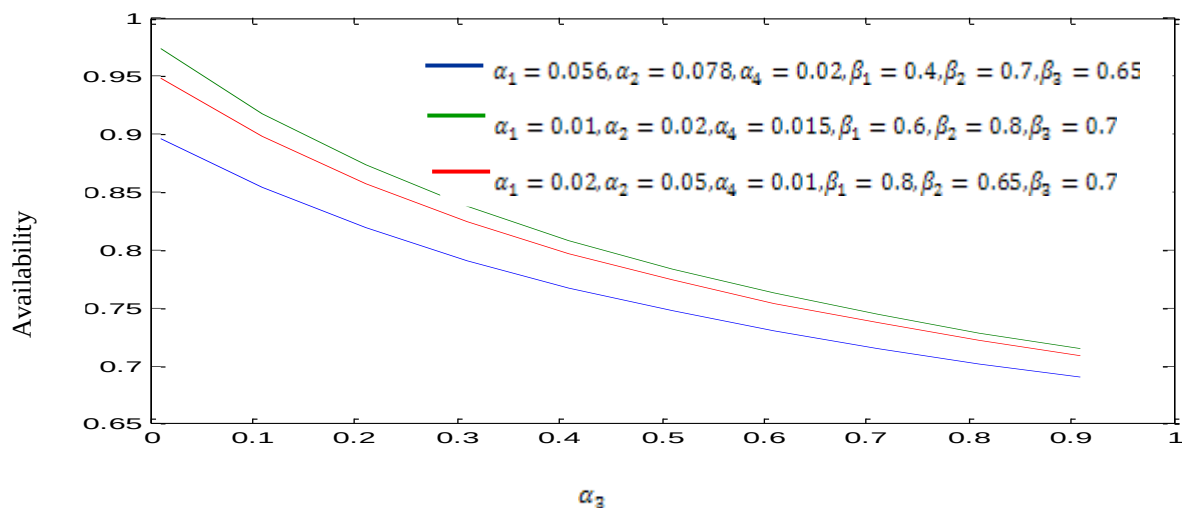


Fig.7: BEHAVIOUR OF STEADY STATE AVAILABILITY W.R.T. α_3 FOR DIFFERENT VALUES OF $\alpha_1, \alpha_2, \alpha_4, \beta_1, \beta_2, \beta_3$

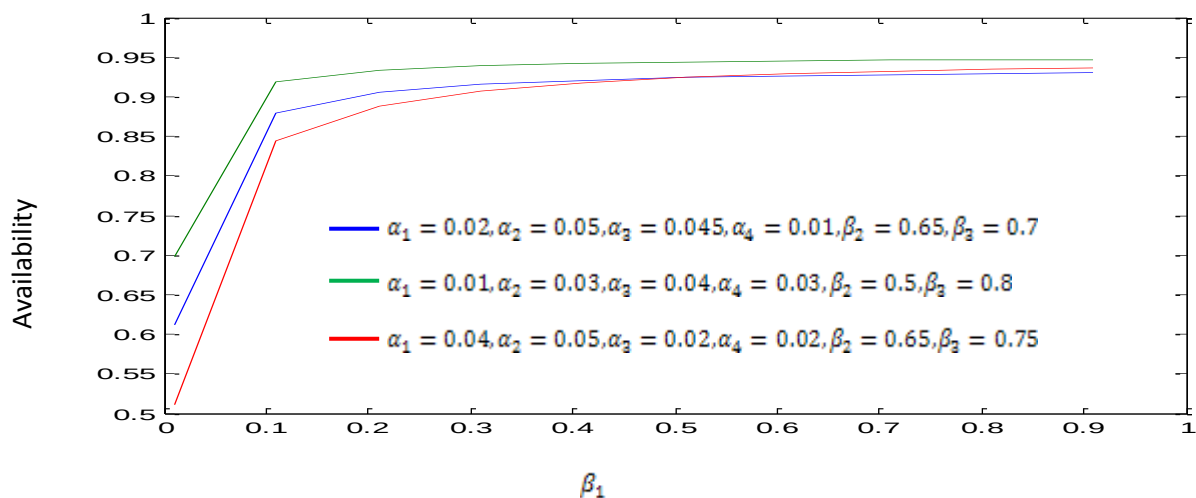


Fig.8: BEHAVIOUR OF STEADY STATE AVAILABILITY W.R.T. β_1 FOR DIFFERENT VALUES OF $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_2, \beta_3$

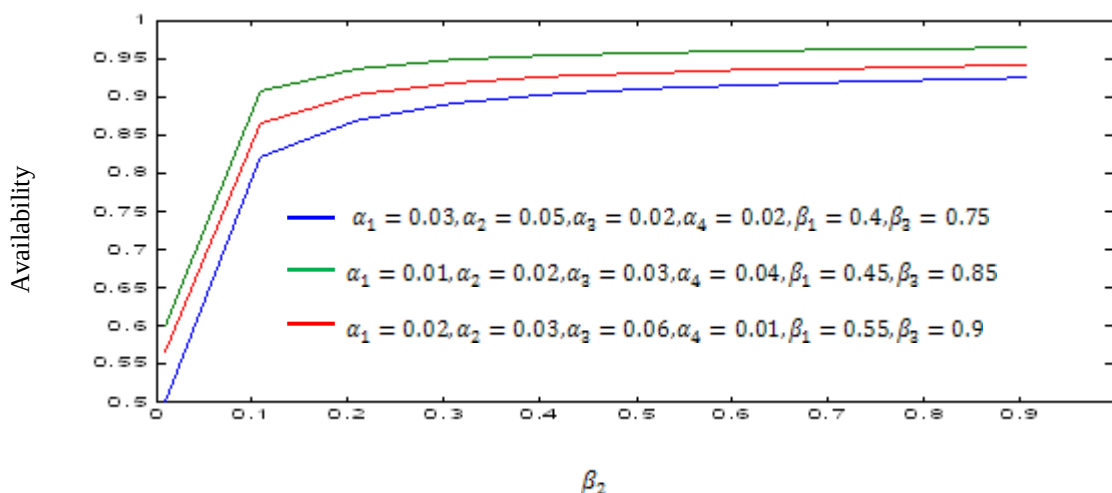


Fig.9 BEHAVIOUR OF STEADY STATE AVAILABILITY W.R.T. β_2 FOR DIFFERENT VALUES OF $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_3$

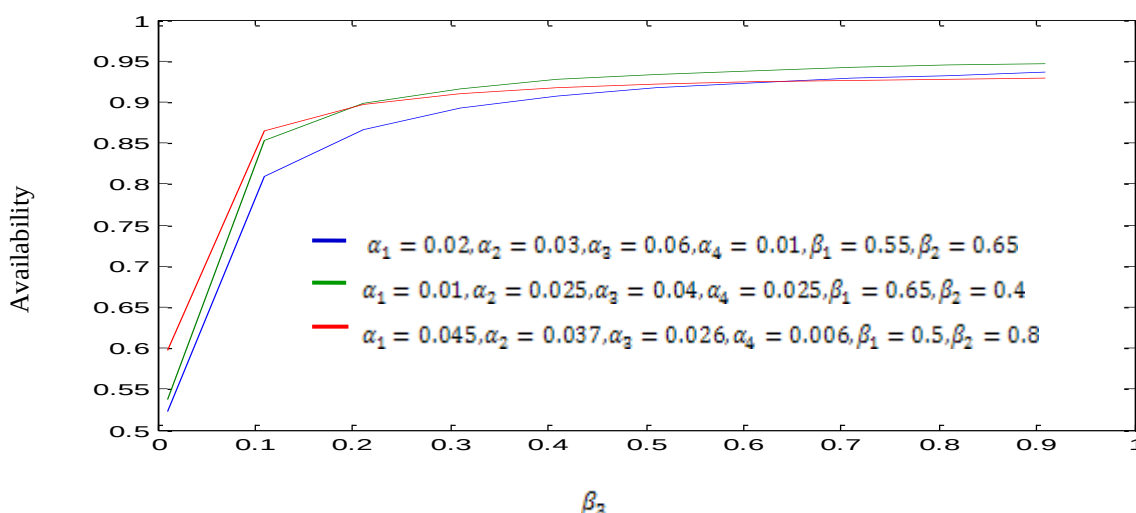


Fig.10 BEHAVIOUR OF STEADY STATE AVAILABILITY W.R.T. β_3 FOR DIFFERENT VALUES OF $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2$

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