



Development of reliability based stochastic traffic assignment model due to uncertainties in demand and supply

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Abstract: In this paper a new stochastic user equilibrium traffic assignment model has been presented. This model is based on the variation of supply and demand due to the uncertainties caused by adverse weather conditions with different rainfall intensities. A travel time function is proposed to reflect the impact of rainfall variation. Traveler's perception error on travel time is also considered in the model. The influence of level of service under different weather conditions on O-D demand causing random condition in the network has been distinctly considered. A solution algorithm is proposed for solving the fixed point problem. Application of the proposed model and efficiency of solution algorithm is illustrated by numerical examples. The results indicate that the proposed algorithm can lead to a very stable solution for the medium size networks.

Key Words: Demand, Travel time, Adverse weather, Reliability

I. Introduction

In recent empirical studies on values of travel time and reliability, many have suggested that travelers are not only interested in travel time saving but also reduction in travel time variability. Variability plays an important role on traveler's route choice decision process. Congestion cannot be expressed simply in terms of average travel time. Hence it is necessary to identify the sources of uncertainty that will affect travel time. Mainly there are two factors for the travel time fluctuations:

- 1) Supply Side: Uncertainties are caused due to road closure, traffic accident; signal failure, maintenance work etc. Among the external factors rainfall affects the driving condition and consequently capacity of the road is also affected.
- 2) Demand Side: Seasonal and day-to-day demand fluctuation etc.

The coupling of the demand and supply uncertainties results in the recurrent variability and unreliability of travel time and traffic condition (Asakura and Kashiwadani, 1991). The best approach to study the random behavior of the uncertainties will be modeling of travel time as a function of stochastic behavior of the uncertainties by suitable techniques. Further this will also be helpful to observe the reliability of travel time due to the effect of uncertainties.

This paper develops a modeling framework taking into account reduction in capacity due to impact of rainfall. A generalized travel time function is introduced. The model also considers day-to-day demand variations. Numerical example is presented to illustrate the application of the proposed model and the remainder of the paper is organized as follows: The following section presents the brief literature review and section 3 gives the assumptions made in the study. Then, the section 4 explains the formulation of reliability based stochastic user equilibrium model, solution algorithm and numerical example on medium size network to check the efficiency of solution algorithm. Finally conclusions and references are given.

II. Background

Some of the previous works are briefly described below:

Vanderloop (2001) identifies the main causes of unreliability of travel time for Netherlands urban roads. According to his study, 74% of the unreliability in the travel time is mainly due to internal factors of the traffic. The remaining is due to weather (8%), road works (14%), accidents (3 to 12%) and combination factors contributes (2%). FHWA (2004) studies on traffic congestion and reliability. In this study they categorized the main seven sources which influence traffic events (includes traffic incidents, work zones and weather), traffic demand (includes fluctuations in normal traffic and special events) and physical highway features (includes traffic control devices and bottlenecks).

Bell (2000) estimated the route choice and the route choice probability by using a mixed-strategy Nash equilibrium approach. Chen et al (2002 b) proposed the capacity reliability as the probability that the network capacity can accommodate a certain traffic demand at a service level required, while taking into account traveler's route choice behavior. On the demand side, Clark and Watling (2005) proposed a stochastic network model with the Poisson distributed origin-destination (O-D) demand based on the original model proposed by Watling (2002). Sumalee et al (2006) applied this model to the reliable network design problem. Nakayama and Takayama (2003) applied a similar stochastic model but with the Binomial distribution of the travel demand. Shao et al (2006) extended the Lo et al's study (2006) and presented a demand driven travel time reliability based user equilibrium model to consider the effects of daily demand fluctuations. Recent empirical results found that travel time reliability is an important criterion for route choice decision (Abdel and Kitamura, 1995; Kazimi et al, 2000; Lam 2000; Lam and Small, 2002; de Palma and Picard 2005). Therefore, investigation of the traveler's route choice behaviors under network uncertainty may be helpful for the transportation planners understand traveler's responses to the congested networks at the strategic planning level.

III. Assumptions

The following basic assumptions are made in the formulation of the model.

- 1) It is assumed that the path flow is the product of path choice proportion and O-D travel demand where the path choice proportion is deterministic variable.
- 2) Route choice probability is given by logit formulae to reflect traveler's perception errors.
- 3) Traffic assignment model proposed in this paper falls within the category of static model i.e. traveler's will not change their paths en-route.
- 4) The travel demands between each O-D pair are assumed to follow Normal distributions similar to the assumptions in the studies (Asakura and Kashiwadani, 1991, Chen et al 2003).

IV. Stochastic traffic assignment model

In this section brief description of the formulation of reliability based stochastic traffic assignment model has been mentioned. Travel time reliability is defined as the probability that a traveler can arrive at the destination within a given travel time threshold. Travel time reliability is an increasing concern of travelers, because it allows travelers to make better use of their own time. Confronted with travel time variations, travelers do not certainly know the outcome when making their route choice decisions. Therefore if travelers plan their trips based on the average travel time, they may be late or early arrive at destination. Thus travelers must consider the travel time variations if they want to arrive on time. Under this circumstance travelers are interested in not only travel time saving but enhancement of travel time reliability in their route choices.

Such a risk-averse behavior in the context of path choice model has been confirmed by several empirical studies (e.g. Bruinsma et al. 1999; Bates et al., 2001; Lam and Small 2001). In particular Lam and Small (2001) found that travelers are likely to set up a travel time safety margin to avoid late arrival. The safety margin is the extra time allowed for a trip to cope with the travel time uncertainty (Hall, 1983). Various models (Uchida and Iida 1983, Lo et al 2006, Shao et al 2006) were proposed to capture the risk based path choice behavior using the concept of effective travel time or travel time budget. The effective travel time is defined as the summation of mean travel time and safety margin. Following are the equations and proposed solution algorithm for the formulation of reliability based stochastic equilibrium model.

A. Travel Time Function

To capture the rain effects and supply uncertainty, a new link travel time function is proposed denoted as Universal Bureau of Public Roads (UBPR) travel time function

$$t_a(x_a, I) = t_{ao}f_{tao}(I) + \frac{\beta t_{ao}}{f_{ca}(I)} \left(\frac{x_a}{c_a}\right)^n \quad (1)$$

Where c_a = Link capacity

I = Hourly average of rainfall intensity ($I \geq 0$)

t_{ao} = Free flow travel time on link a.

β and n = Parameters in the conventional BPR function

X_a = Flow on link a $\in A$

$f_{tao}(I)$ = Scaled functions on free flow time (≥ 1)

$f_{ca}(I)$ = Scaled function on link capacity (≤ 1)

$f_{tao}(I) = 1$ and $f_{ca}(I) = 1$ meaning that there is no rain, the new travel time function is conventional BPR function

Note: It is already proved that higher the rainfall the lower the free flow speed and the link capacity.

If we observe equation (1), it can be found that the higher the rainfall intensity, f_{tao} (I) will be higher which is in numerator and f_{ca} (I) will be lower which is in denominator, and hence finally the travel time will be higher. This property is in accordance with the empirical studies by Tam et al (2007) and Rakha et al (2008).

By empirical validation, functional forms of free flow travel time and capacity have been developed for the study areas which are as follows.

$$f_{tao}(I) = e^{(0.04)I}, \text{ C.O.R} = 0.98 \quad (2)$$

$$f_{ca}(I) = 0.863 e^{(-0.01)I}, \text{ C.O.R} = 0.95 \quad (3)$$

B. Effective travel time

Due to travel time variations, travelers do not exactly know how long their journeys are, or whether they can arrive on time. Therefore, a traveler usually sets up a travel time safety margin to improve the likelihood of arrival on time. As mentioned previously, the safety margin is the extra time that a traveler allows for his (or her) trip (Hall, 1983). Under this condition, the effective travel time for a trip is defined as the summation of mean travel time and safety margin.

$$c_k^{lm} = t_k^{lm} + s_k^{lm} \quad \forall k \in K_{lm} \text{ and } lm \in C \quad (4)$$

Where l = origin node, m = Destination node, K = Path set, C = Set of O-D pair

Clearly, a higher travel time reliability he (or she) is concerned with, a larger safety margins the traveler budgets. If travelers want to choose a route with travel time reliability not less than a confidence level ρ (say 95 %) and at the same time they intend to minimize their effective travel time on the route by adjusting their travel time safety margin, then for each route, it follows that

$$\min_{s_k^{lm}} c_k^{lm} = t_k^{lm} + s_k^{lm} \quad \forall k \in K_{lm} \text{ and } lm \in C \quad (5)$$

Such that $P[T_k^{lm} \leq c_k^{lm}] \geq \rho$

Where ρ is the confidence level. It can be explained that a traveler should allow a safety margin to his (or her) trip in order to arrive on time for $\rho = 95\%$ of all trips. The above chance constrained model is a simple optimization problem with only one decision variable and one constraint.

The above minimization problem indicates that the effective travel time can be regarded as the minimum criteria, which a traveler allows for a trip, to satisfy the travel time reliability chance constrained defined by equation (5). The minimization model can be rewritten as bellow

$$\min_{s_k^{lm}} \int_{-\infty}^{t_k^{lm} + s_k^{lm}} \frac{1}{\sqrt{2\pi} \sigma_{t_k^{lm}}} \exp\left(-\frac{(x - t_k^{lm})^2}{2(\sigma_{t_k^{lm}})^2}\right) dx \geq \rho \quad (6)$$

Performing some simple manipulations on the minimization problem (6), the solution has been deduced. The equation of safety margin is as follows;

$$s_k^{lm} = \sigma_{t_k^{lm}} \Phi^{-1}(\rho) \quad (7)$$

If $\rho = 95\%$, the above equation can be written as

$$s_k^{lm} = 1.65 \sigma_{t_k^{lm}} \text{ where } \sigma = \text{Standard Deviation}$$

Hence first we have to find out standard deviation of travel time for any path to calculate safety margin.

C. Elastic travel demand analysis

When there is chances of high rainfall (either it is found from weather forecast or by experience) travelers feel that there will be traffic jam on the route. In this condition they may cancel/postpone the trip or change their routes. Thus due to possibility of rainfall of varying intensities service condition will be affected and consequently O-D demand will change. The travel demand between O-D pair l - m can be expressed as function of minimum perceived effective travel time and is given by the following expressions.

$$Q_{lm} = q_{lm} + \epsilon_{lm} \\ q_{lm} = d_{lm} c_k^{lm} \quad (8)$$

Where q_{lm} is the mean travel demand between the O-D pair lm , ϵ_{lm} = perception Error

d_{lm} is the elastic demand function for the O-D pair lm which is increasing function w.r.t c_k^{lm}

c_k^{lm} = Expected minimum perceived effective travel time which is given by the following formula.

$$c_k^{lm} = -\frac{1}{\theta} \ln \sum_{k \in K_{lm}} \exp(-\theta c_k^{lm}) \quad \forall lm \in C \quad (9)$$

$\theta > 0$ is the dispersion parameter used to measure the degree of traveler's perception errors on the effective travel time. The equation (9) implies that the higher the rainfall intensity information, the larger the value of c_k^{lm} perceived by the travelers.

D. Path choice Proportion

Random term ϵ_{lm} representing traveler's perception errors are assumed to be identically and independently Gumbel distributed random variables with mean zero and identical S.D, the path choice proportion or route choice probability is given by formula:

$$p_k^{lm} = \frac{\exp(-\theta c_k^{lm})}{\sum_{j \in K_{lm}} \exp(-\theta c_j^{lm})} \quad \forall k \in K_{lm} \quad (10)$$

Where θ = Dispersion Parameter used to measure the degrees of perception error.

c_k^{lm} = effective travel time

A higher θ means smaller perception errors, and hence better information quality. In the limiting case, when θ approaches infinity, the corresponding route flow pattern approaches reliability- based deterministic user equilibrium.

E. RSUE condition and model formulation

It is reasonable to assume that every traveler will try to minimize his or her perceived effective travel time when travelling from an origin to destination. Consequently the network equilibrium could be reached, in which

*For each O-D pair and each user class, no traveler can reduce his or her perceived effective travel time by unilaterally changing routes.

*In addition the O-D demand of each O-D pair satisfy the demand function $q_{lm} = d_{lm} c_k^{lm}$

The logit based RSUE condition can be expressed as

$$y_k^{lm} = p_k^{lm} q_{lm} \quad \forall k \in K_{lm}, \quad \forall lm \in C \quad (11)$$

Where y_k^{lm} be the mean route flow on path k between O-D pair lm

$$q_{lm} = d_{lm} c_k^{lm} \quad (12)$$

c_k^{lm} = Expected minimum perceived effective travel time

F. Rainfall Category adopted

Rainfall Categories	Expected hourly average of rainfall intensity (mm)
No Rain	0
Light Rain	2.5
Medium Rain	7.5
Heavy Rain	18
Very heavy Rain	32

G. Solution Algorithm Steps

Detailed steps for the proposed solution algorithm are presented as follows:

1) Input: a) stopping tolerance (convergence value = 0.01)

b) O-D route incidence matrix

Origin node Destination node Number of O-D pairs Number of paths

c) Initial Mean O-D Demand

d) Dispersion parameter θ

e) $\beta, n \Rightarrow$ Parameter of travel time function

f) Free flow travel time of each link

g) Capacity of each link.

2) Take initial value of mean O-D demand for each O-D pair (q_{lm}), mean path flow (y_k^{lm})

3) By using mean path flow, calculate link flow

4) By using link flow, capacity, free flow travel time, calculate mean travel time, safety margin and effective travel time for each path.

5) Calculate expected perceived effective travel time for each O-D pair and O-D demand.

6) Calculate path choice probability for each path (p_k^{lm})

7) Calculate mean path flow by using $y_k^{lm} = p_k^{lm} q_{lm}$

8) Calculate the value of convergence

$$M = \frac{\|y^j - p^j q^j\|}{\|y^j\|} \quad (13)$$

9) If $M \leq .01$

Stop

Otherwise go to next iteration

10) Step size $\alpha^j = \frac{1}{j}$ (M.S.A) (14)

11) For new iteration compute mean path flow and mean O-D demand by using following equations

$$y_i^j = y_i^{j-1} + \alpha^j (y_i^{new} - y_i^{j-1}) \quad (15)$$

$$q_{lm}^j = q_{lm}^{j-1} + \frac{1}{j} (q_{lm}^{new} - q_{lm}^{j-1}) \quad (16)$$

Where j is number of iteration

12) By using values of step (11) go to step (3) and continue iteration till $\rho = 0.01$

In large networks for making route choices, there can be a number of convenient alternatives. Previously some persons have used column generation procedure to limit the alternative routes. In this work to avoid this complication it is assumed that the route set is given and fixed (i.e. route set is not changed from iteration to iteration).

H. Numerical example

The purpose of numerical example is to illustrate the efficiency of the proposed solution algorithm. The example is based on a medium size network located at Mira Road – E in Thane district having 6 nodes and 11 links. All the links are having identical speed-density relationship. Details of network are given below.

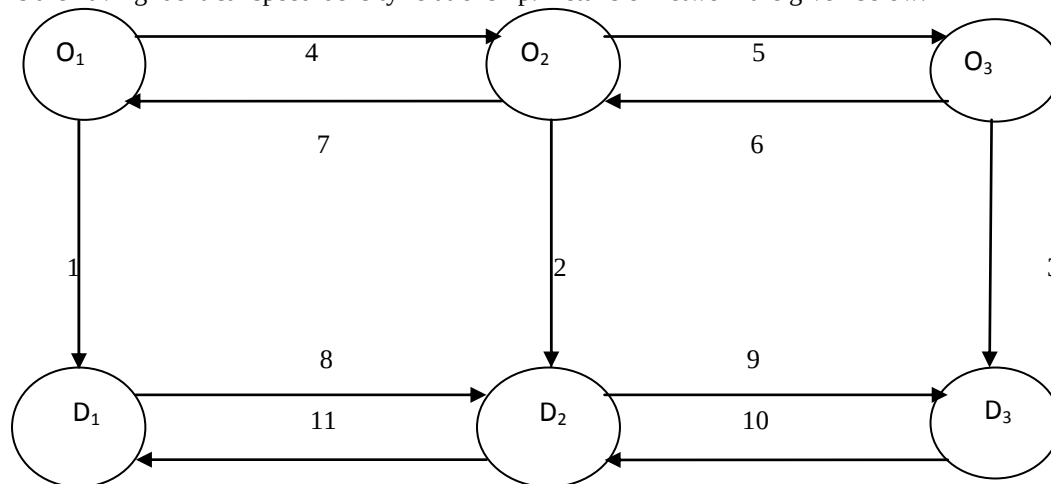


Figure 1: SUE Network

Table I: List of path and link

Origin	Destination	Path(Number)	Link
O ₁	D ₁	O ₁ -D ₁ (1), O ₁ -O ₂ -D ₂ -D ₁ (2), O ₁ -O ₂ -O ₃ -D ₃ -D ₂ -D ₁ (3)	O ₁ -D ₁ , O ₁ -O ₂ , O ₂ -D ₂ , D ₂ -D ₁ , O ₁ -O ₂ , O ₂ -O ₃ , O ₃ -D ₃ , D ₃ -D ₂ , D ₂ -D ₁
O ₁	D ₂	O ₁ -D ₁ -D ₂ (4), O ₁ -O ₂ -D ₂ (5), O ₁ -O ₂ -O ₃ -D ₃ -D ₂ (6)	O ₁ -D ₁ , D ₁ -D ₂ , O ₁ -O ₂ , O ₂ -D ₂ , O ₁ -O ₂ , O ₂ -O ₃ , O ₃ -D ₃ , D ₃ -D ₂
O ₁	D ₃	O ₁ -O ₂ -O ₃ -D ₃ (7), O ₁ -O ₂ -D ₂ -D ₃ (8), O ₁ -D ₁ -D ₂ -D ₃ (9)	O ₁ -O ₂ , O ₂ -O ₃ , O ₃ -D ₃ , O ₁ -O ₂ , O ₂ -D ₂ , D ₂ -D ₃ , O ₁ -D ₁ , D ₁ -D ₂ , D ₂ -D ₃
O ₂	D ₁	O ₂ -O ₁ -D ₁ (10), O ₂ -D ₂ -D ₁ -D ₂ (11), O ₂ -O ₃ -D ₃ -D ₂ -D ₁ (12)	O ₂ -O ₁ , O ₁ -D ₁ , O ₂ -D ₂ , D ₂ -D ₁ , O ₂ -O ₃ , O ₃ -D ₃ , D ₃ -D ₂ , D ₂ -D ₁
O ₂	D ₂	O ₂ -D ₂ (13), O ₂ -O ₁ -D ₁ -D ₂ (14), O ₂ -O ₃ -D ₃ -D ₂ (15)	O ₂ -D ₂ , O ₂ -O ₁ , O ₁ -D ₁ , D ₁ -D ₂ , O ₂ -O ₃ , O ₃ -D ₃ , D ₃ -D ₂
O ₂	D ₃	O ₂ -O ₃ -D ₃ (16), O ₂ -D ₂ -D ₃ (17), O ₂ -O ₁ -D ₁ -D ₂ -D ₃ (18)	O ₂ -O ₃ , O ₃ -D ₃ , O ₂ -D ₂ , D ₂ -D ₃ , O ₂ -O ₁ , O ₁ -D ₁ , D ₁ -D ₂ , D ₂ -D ₃
O ₃	D ₁	O ₃ -O ₂ -O ₁ -D ₁ (19), O ₃ -O ₂ -D ₂ -D ₁ (20), O ₃ -D ₃ -D ₂ -D ₁ (21)	O ₃ -O ₂ , O ₂ -O ₁ , O ₁ -D ₁ , O ₃ -O ₂ , O ₂ -D ₂ , D ₂ -D ₁ , O ₃ -D ₃ , D ₃ -D ₂ , D ₂ -D ₁
O ₃	D ₂	O ₃ -O ₂ -O ₁ -D ₁ -D ₂ (22), O ₃ -O ₂ -D ₂ (23), O ₃ -D ₃ -D ₂ (24)	O ₃ -O ₂ , O ₂ -O ₁ , O ₁ -D ₁ , D ₁ -D ₂ , O ₃ -O ₂ , O ₂ -D ₂ , O ₃ -D ₃ , D ₃ -D ₂
O ₃	D ₃	O ₃ -D ₃ (25), O ₃ -O ₂ -O ₁ -D ₁ -D ₂ -D ₃ (26), O ₃ -O ₂ -D ₂ -D ₃ (27)	O ₃ -D ₃ , O ₃ -O ₂ , O ₂ -O ₁ , O ₁ -D ₁ , D ₁ -D ₂ , D ₂ -D ₃ , O ₃ -O ₂ , D ₂ -D ₃

Table II: O-D Matrix of network

Origin	Destination	Number of O-D pairs	Number of path	Number of link
O ₁ -O ₂ -O ₃	D ₁ -D ₂ -D ₃	09	27	11

Table III: Origin-Destination Demand table

Demand (PCU/hr)	O ₁	O ₂	O ₃
D ₁	5000	2500	1500
D ₂	3000	10000	2000
D ₃	1600	1500	4000

Table IV: Free flow travel time (t_{ao}) and capacity (c_a)

Link Number	1	2	3	4	5	6	7	8	9	10	11
t_{ao} (hour)	0.5	0.34	0.50	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
c_a (PCU/hr)	6000	7000	9000	8000	8000	8000	8000	8000	8000	8000	8000

UBPR Parameter

$$n = 1, \beta = 0.5$$

Type of rain condition = Heavy rain (10-25 mm/hr)

Calibrated Scaled functions from field data

$$f_{\text{tao}}(I) = e^{(0.04)I}$$

$$f_{\text{ca}}(I) = 0.863 e^{(-0.01)I}$$

Elastic demand function (Adopted)

$$q_{lm} = q_{lm}^{\max} \exp(-0.7 c_k^{lm}) \quad \forall lm \in C$$

c_k^{lm} = Expected minimum perceived effective travel time

Dispersion parameter (θ) = 4

ρ is the stopping tolerance. = .01

Table V: Mean path flow (PCU/hr)

Path Number	1	2	3	4	5	6	7	8	9	10
Mean path flow	1610	190	02	236	782	10	94	255	77	196
Path Number	11	12	13	14	15	16	17	18	19	20
Mean path flow	653	08	4313	46	57	04	377	139	72	239
Path Number	21	22	23	24	25	26	27			
Mean path flow	88	05	503	185	1361	01	131			

Table VI: Mean O-D Demand

Demand (PCU/hr)	O ₁	O ₂	O ₃
D ₁	1803	858	400
D ₂	1029	4417	694
D ₃	427	521	1494

Figure 1 shows the network. It also shows the direction of travel on the path. Origin and destination is connected by link which results path and is shown in table 1. Table 2 indicates details of O-D matrix. Origin-destination demand adopted is given in table 3 and free flow travel time and capacity of each link is shown in table 4. First to see the performance and convergence of the proposed solution algorithm, it is applied on the above network by using search direction based on method of successive average and then based on by method of successive Valued Average. By running the programme using Matlab software, the resulting mean path flow and mean O-D demand is shown in table 5 and 6 and number of iterations are given below. The convergence of the proposed algorithm is depicted in figure 2 in terms of M.

MSA method

Number of iterations = 9372, Time of convergence = 40 seconds

MSVA method

Number of iterations = 33, Time = 3 Seconds

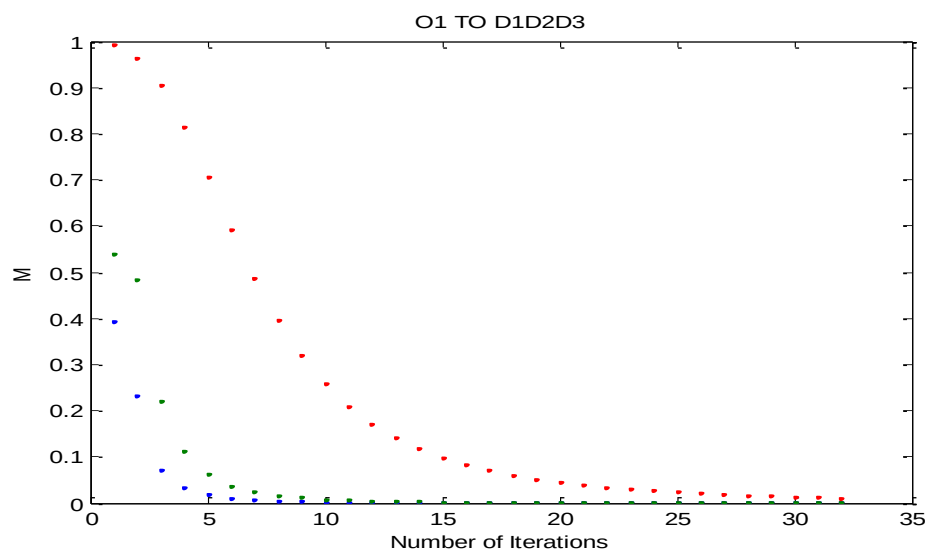


Figure 2: Plot of convergence Vs Number of iterations

V. Analysis of result

The proposed solution algorithm is applied on a medium size network taking different rainfall conditions and perception errors. First using the method of successive averages for search direction for fixed point problem (Sheffi, 1985), the main programme terminates after 9723 iterations. Then method of successive average was

modified and method of successive valued average was developed. MSVA allocates more weights to the latter iterations instead of equal averages in MSA. Due to applications of MSVA method, the main programme terminates after 33 iterations and time of iterations is also reduced. The results show that the RSUE condition is achieved as M is almost equal to zero after the iterations. This illustrates the proposed heuristic algorithm can converge to a stable solution for the medium size network.

VI. Conclusions

In this paper a new reliability based stochastic user equilibrium (RSUE) model has been presented. The direct effect of rainfall intensities on the link travel time has been considered in this model. Adverse weather's effect on the O-D demand was modeled by using a modified elastic demand function. Traveler's perception error has been taken into account. RSUE traffic assignment problem has been formulated as equivalent path based fixed point problem. Solution algorithm based on MSA and modified MSA i.e MSVA was proposed for solving the fixed point problem. One example for medium size network is presented to illustrate the application of the proposed model and efficiency of the solution algorithm. The result shows that proposed algorithm is satisfying the convergence criteria and hence can be applied on such type of networks for finding out demand and path flow at equilibrium condition.

The study developed Method of Successive Valued Average (MSVA) method to modify Method of Successive Average (MSA) for fixing step size. The result shows that convergence criteria is very fast and gives equilibrium solution after lesser number of iterations.

The traditional User Equilibrium (UE) model ignores the travel time variation in the traveler's route choice decision process whereas Reliability based Stochastic User Equilibrium (RSUE) model considers the reliability aspect of acceptable travel time to account for risk. Reliable travel time act as a performance measure for transportation system users, planners and decision makers. RSUE model can be applied for network design problem so as to assess the performance of network improvements on travel time reliability.

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