



Repetitive Control Strategy: Design And Issues: A Survey

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Abstract: Various control strategies and methods are devised and developed from time to time in the field of process control through research or as the requirement arises. In the field of robust control, various iterative techniques are used like ILC (Iterative Learning Control), MPC (Model Predictive Control) etc. The iterative control methods provide for continuous up gradation of controller parameters from iteration to iteration. Several papers have been published in the field of iterative control methods. Repetitive control is one of the most extensively used techniques implemented in repetitive processes. Basic repetitive block consists of a periodic signal with positive feedback, summing the past and current errors; which is similar to an integrating control action. Most of the research has been done by implementing repetitive control strategy on a simple proportional controller. Here I am going to design and analyze a controller for a PID (Proportional Integral Derivative) mode which uses a repetitive control approach for set point or reference tracking. Dealing with the disturbance rejection, there are some issues due to positive feedback, but are mostly traded off with system performance. Major focus is on tracking which is a specialty of Repetitive control.

Keywords: iterative control, repetitive, PID, MRCS (Modified Repetitive Control Strategy), tracking

I. Introduction

In the field of control, various control methodologies and strategies have been devised, tested and implemented in process control and other such control applications. Favourite examples of controllers are proportional, Integral and derivative in parallel and/ or series combinations so far. Controller tuning is also an important aspect of process control. Speaking on control techniques, there are various techniques like hierarchical control, robust control (H-infinity loop shaping, sliding mode control), adaptive control, optimal control (LQG and MPC etc.), intelligent control (fuzzy logic, Genetic algorithms etc.), adaptive control (direct and indirect), etc

II. Control Strategies and Methodologies

A. Iterative learning control and repetitive control

First, the Iterative Learning Control (ILC) is a method of tracking control for systems that work in a repetitive mode. Repetition or iteration allows the system to improve tracking accuracy from repetition to repetition, in effect learning the required input needed to track the reference exactly. The 'learning processes' uses statistics from earlier repetitions to improve the control signal and eventually enabling an appropriate control action that can be found iteratively. The internal model principle yields conditions under which perfect tracking can be achieved but the design of the control algorithm still leaves many decisions to be made to suit the application.

B. The internal model principle ^[1]

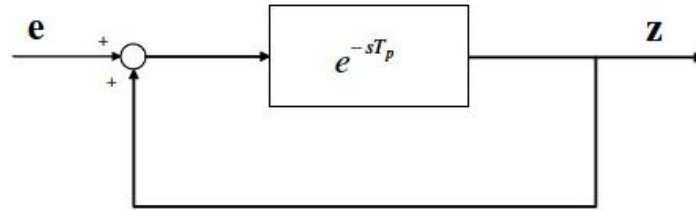
A familiar example of the Internal Model Principle applied in practice is through the use of an integrator (I) term in the common PID controller. Consider the case of a linear-motor driven positioning stage modeled as a free mass with a control force applied to it. Proportional and Derivative control alone are sufficient to stabilize the system, but any constant disturbance force (due to the process, gravity, cables, and etc.) requires some error between the reference and measured positions in order for the spring-like proportional control term to generate an output. A constant disturbance is modeled as a step input with a Laplace transform of $1/s$. Adding this term, an integrator, to the control algorithm allows the output to grow to a constant value as required to cancel the disturbance and achieve zero steady-state error.

III. Control of Periodic Processes

Before PID control continues to be one of the most popular control techniques due to its simplicity and usefulness. It is well known that for linear time-invariant systems the integral action in the PID controller is able

to give offset-free rejection of step disturbances. This zero steady state error is achieved by summing the output error over the sampling time index and giving a control signal proportional to this cumulative error. The integral action causes the controller output to change as long as an error persists in the controlled output. However, for periodic systems error at steady state is dynamic because it is periodic in nature. The PID controller is not able to remove the dynamic periodic error completely. The periodic error information is not accumulated over the run index and cyclic error persists at the steady state.^[2]

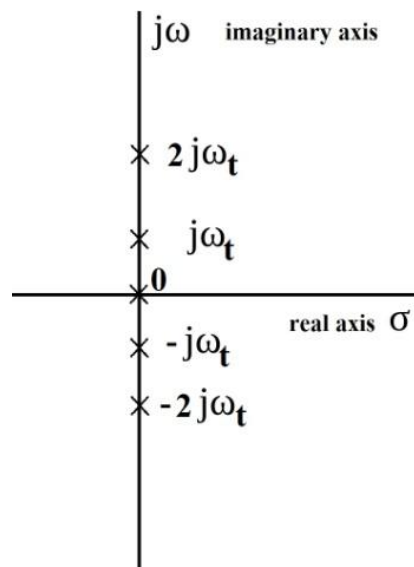
Figure 1 Block diagram of repetitive controller



A. Repetitive control(RC)

The goal of Repetitive Control is to ensure tracking of periodic reference trajectories or rejection of cyclic disturbances in continuous systems. Repetitive control was first introduced in the control of proton synchrotron magnetic supply, where in order to obtain the desired proton acceleration pattern it was necessary to control the current supply in a specific curve with a very high precision requirement. Since protons turn around in the synchrotron, the reference commands are periodic^[3]. A schematic block diagram of repetitive controller memory loop is shown in figure 1, where e is the error signal and z is the controller output. This is very similar to an oscillator circuit with a known frequency which is determined by the delay T_p .

Figure 2 Pole-zero locations of an oscillator on a complex s-plane



- **T-Periodic signals:** If the external signal is in the form of $y_d(t) = y_d(1-t)$, which can represent any periodic signal with a period T_p , then asymptotic tracking and/or asymptotic rejection can be achieved by incorporating the model $1/(1 - e^{-sT_p})$ into the closed-loop system (as shown in figure 1).
- **The pole locations:** They are shown in figure 2 for a signal generator with the period T_p . The transfer function of the generator has actually infinite number of poles on the imaginary axis.^[4]
- The Laplace transfer function of oscillator shown in figure 1 can be found as:

$$G(s) = \frac{z}{e} = \frac{e^{-sT_p}}{1 - e^{-sT_p}} \quad (1)$$

- The **infinite** number of **marginally stable poles** for this type of positive feedback loop can be mathematically stated as:

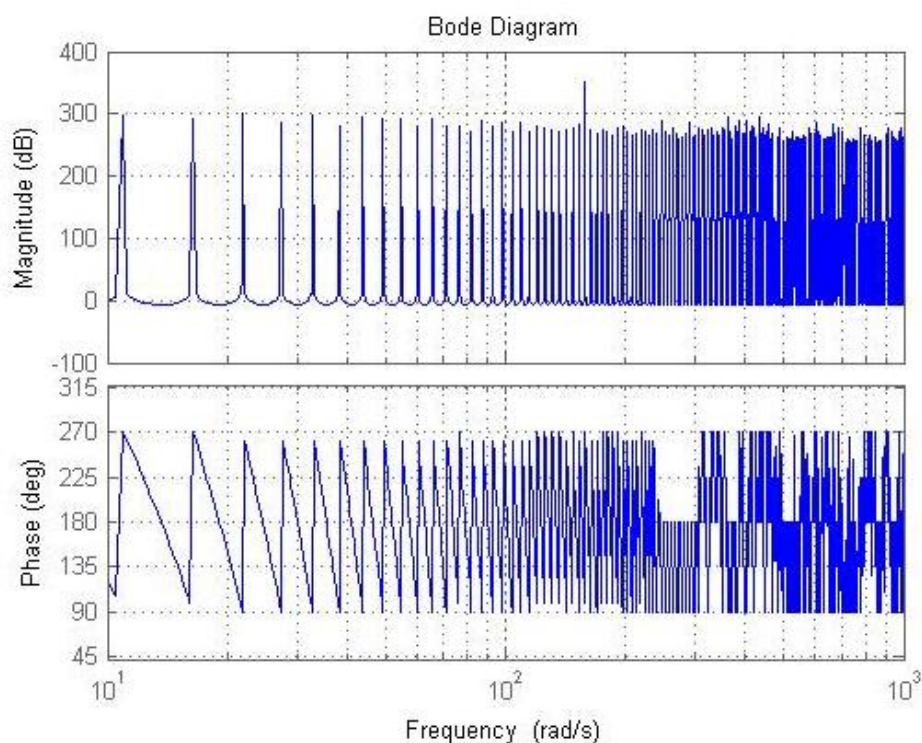
$$P_i = e^{-sT_p} = 1, \text{ for the values of } s \text{ to be } jk\omega_s, \text{ taken for every } k = \pm 1, \pm 2, \pm 3, \dots, \quad (2)$$

Where, $\omega_s = \frac{2\pi}{T_p}$

B. Sensitivity Analysis of the repetitive controller or the memory loop

The RC has proven very useful for asymptotic tracking and rejection of periodic signals. It has been successfully applied to areas such as mechanical manipulators, computer disk drives and spindle run out compensation. Repetitive control offers the potential to completely cancel the effect of a periodic disturbance by taking its periodic nature into account. However, to be able to reject disturbances completely RC needs to know the exact period of reference/disturbance. At first, one might think that a slight mismatch in period will not cause a major effect on the performance. But, as we shall see, even a small mismatch in the period diminishes the controller tracking/rejection performance significantly.^[3]

Figure 3 frequency response of the repetitive controller with single memory loop



The performance of the RC system depends on how precise the match is between the repetitive controller's signal generator period and the actual signal period. The most common approach for rejection of a periodic disturbance is based on the internal model principle, which states that a model of the disturbance should be included in the feedback. The internal model for any periodic signal with a known period can be generated by a free dynamic system that has a positive feedback around a pure time delay. The underlying property used is that a linear feedback system has perfect rejection at some frequency if the controller gain is infinite at that frequency. The repetitive controller generates a comb filter shaped closed loop sensitivity function, which seems evident from the frequency response shown in figure 3.

The notches in the comb filter provide substantial sensitivity reduction but they are extremely narrow. The error magnitude at the Fourier harmonic frequencies of the periodic signal is proportional to

$$1 - e^{-\frac{j2\pi n \tilde{T}_p}{T_p}} \quad (3)$$

If we approximate period mismatch ratio as

$$\varepsilon = \frac{\tilde{T}_p - T_p}{T_p} \quad (4)$$

Then, by approximation, the equation 3 becomes as follows ^[3]:

$$1 - j2\pi\varepsilon \quad (5)$$

C. Reasons of period mismatch

- **Unknown exact frequency:**

The noise source oftentimes has periodic components due to some rotating machinery generating the undesired signal and the frequency of these disturbances is usually not known a priori.

- **Frequency variation:**

In some situations the period of the disturbance signal is uncertain and slowly changing. For example, the periodic disturbance may be caused by an imbalance in the rotating machinery due to variations in the motor speed. Such situation requires an RC which can self-tune the respective controller signal generator period to match the external signal closely.

- **Non-divisible period:**

Typical repetitive controller assumes that the period of the disturbance can be represented as some integer multiple of the sample time of the digital control system. But, in general, period will not be an exact integer multiple except by chance. This causes serious issues because repetitive control during the present time step wants to examine the information from the previous period of disturbance at an appropriate corresponding time. As data is only available at discrete sample times, unless the period is an integer number of time, this issue can significantly degrade the performance.

D. Handling the mismatch

- **In case of unknown frequency/non-exact frequency:**

A sampled data recursive algorithm with a resolution finer than the sampling period is applied to fine tune the period. However, these algorithms have a major disadvantage: the accuracy of period identification greatly depends upon the sampling interval. Manayathara et al. ^[5] designed a discrete time repetitive controller for the continuous steel casting process that is used for the rejection of periodic load disturbances with unknown period.

A sampled data recursive algorithm with a resolution finer than the sampling period is applied to fine tune the period ^{[6], [7]}.

- **For Period of Non-integer Multiple of the Sampling Time/non-divisible period:**

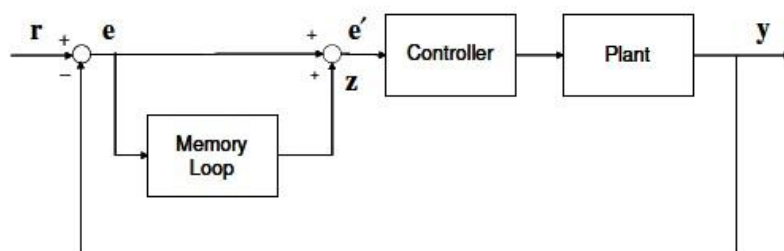
There are various approaches to handle this problem. Some of them include:

1. Changing the sampling rate ^{[6],[8]}
2. Extending the learning period ^[9]
3. Introducing virtual learning ^[10]

E. Non-Robust and Robust Repetitive Control

The non robust implementation of RC[†] uses only a single loop of the oscillator for generating the input signal to the intended controller in the desired control loop, as shown in figure 4, wherein, r is reference signal, e is the error signal, y is the output signal, z is the output of memory loop and é is the controller input. Controller here can be a simple proportional one, and the memory loop is shown in figure 1.

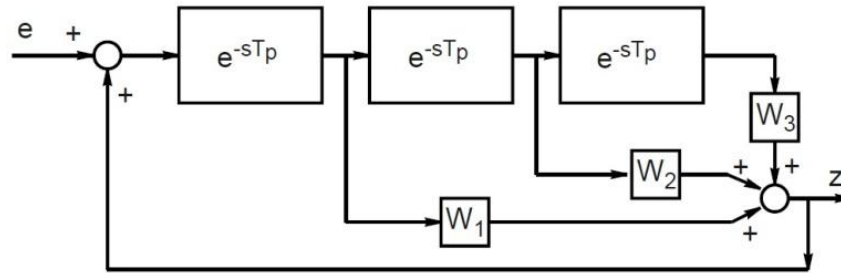
Figure 4 Block diagram of basic repetitive controller



For introducing robustness in the repetitive controller, some modifications have been done in the design of the memory loop of figure 4. Here, instead of the design of figure 1, Maarten Steinbuch ^[11] generalizes the repetitive controller by redesigning the oscillator using multiple weighted memory loops, which is much more reliable for

periodic disturbance rejection more accurately than the one with single memory loop as the gain of standard controller expands around harmonics. His design of a robust repetitive control block is shown in figure 5^[11], where, e is the error signal W_i are weights, and z is the controller input.

Figure 5 Generalized Repetitive Controller^[11]



The robust controller has the transfer function of equation 6^[11], equation 7^[11].

$$\frac{Z}{E} = G(s) = \frac{H(s)}{1 - H(s)} \quad (6)$$

Where,

$$H(s) = \sum_{i=1}^N W_i e^{-isT_p} \quad (7)$$

IV. Conclusion

Repetitive control strategy offers a superior solution to the tracking purpose for control in systems having periodic inputs just like integral control does for constant inputs. But for disturbance rejection, the basic structure has to be modified by placing an appropriate design within the basic one. Also there are certain stability issues due to the positive feedback (oscillator) in cascade with main controller. One important aspect for disturbance rejection is the model requirement of periodic disturbance. Still, the RC⁺ control remains a hot cake for controller designs for systems with periodic signals.

V. References

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