



Intuitionistic Z-Numbers

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Abstract: In 2011 Zadeh introduced the concept of Z-number. Z-numbers look deceptively simple but they embody a subtle and a very useful technique in fuzzy logic. In this paper we introduce a type of Z-number-the intuitionistic Z-number and describe the operations on intuitionistic Z-numbers.

Keywords: Z-number, Z-valuation, Fuzzy event probability, Restriction, Intuitionistic number Fuzzy set, Intuitionistic fuzzy set, .

I. Introduction

In 2011 Zadeh introduced the concept of Z-number. Z-numbers look deceptively simple but they embody a subtle and a very useful technique in fuzzy logic. In particular they play a significant role in the field of "computing with words". Ronald R Yager [5] has made a study of Z-numbers and Z-valuations. Shankar K.Pal et al. have provided insight about how Z-numbers can be applied in the field of "computing with words"[3]. In his 2011 paper Zadeh had mentioned that Z-numbers could be generalized in many ways. In this paper we introduce a generalization of Z-number-the intuitionistic Z-number.

II. Preliminary Definitions

Definition 1: Fuzzy Set

A fuzzy set A defined on a universe X may be given as:

$A = \{(x, \mu_A(x)) / x \in X\}$ where $\mu_A: X \rightarrow [0,1]$ is the membership function of A . The membership value $\mu_A(x)$ describes the degree of belonging of $x \in X$ in A [8].

Definition 2: Intuitionistic Fuzzy Set (IFS)

An intuitionistic fuzzy set A , on a universe X is defined as [2]

$A = \{(x, \mu_A(x), \gamma_A(x)) / x \in X\}$, where $\mu_A(x) + \gamma_A(x) \leq 1$

For each $x \in X$, $\mu_A(x)$ and $\gamma_A(x)$ are called the degree of membership and the degree of non-membership, respectively of x to A .

Definition 3: Restrictions

A restriction may be viewed as generalized constraint. Suppose A is a fuzzy set. The statement $R(X) : X \text{ is } A$

is referred to as a possibilistic restriction ([11],[9]). Here A plays the role of possibility distribution of X .

The statement " $R(X) : X \text{ is } A$ " is to be understood as "The Possibility ($X=u$) is $\mu_A(u)$ " where μ_A is the membership function of A and u is a generic value of X .

When X is a random variable, the probability distribution of X plays the role of probabilistic restriction on X ([9],[4]).

A probabilistic restriction is expressed as:

$R(X) : X \text{ is } f$, where f is the p.d.f. of X .

That statement " $R(X) : X \text{ is } f$ " is to be understood as "Prob($u \leq x \leq u+du$) = $f(u)du$ "

Definition 4 : Fuzzy event probability [10]

Let X be a random variable taking real values and A a fuzzy set defined on the real line. The fuzzy event probability of " $X \text{ is } A$ " is

$$FEP(x \in A) = \int \mu_A(u) p(u) du$$

Definition 5: Z-number

Zadeh defines Z-number as follows [6]

A Z-number is an ordered pair of fuzzy numbers $Z = (A, B)$, associated with a real-valued uncertain variable X , with the first component A , a restriction on the values which X can take and the second component B , a measure of reliability of the first component.

Definition 6: Z-valuation

A Z-valuation is an ordered triple (X, A, B) where A and B are fuzzy numbers [6]

A Z-valuation is equivalent to an assignment statement "X is (A,B)", where X is an uncertain variable' A is a restriction on the values which X can take and B is referred to as certainty.

Z-valuation (X,A,B) may be viewed as a restriction on X defined by

FEP(X is A) is B

More explicitly

Possibility(FEP(xεA) = u) = $\mu_B(u)$

Definition7: Z-restriction

A Z-restriction is expressed as

$$R(X) : X \text{ is } Z$$

where Z is a combination of possibilistic and probabilistic restriction ([11],[9]) defined as

Z : EEP(X is A) is B in which A and B are fuzzy numbers.

Equivalently, a Z-valuation (X,A,B) is a Z-restriction on X.

$$(X,A,B) \rightarrow X \text{ is } (A,B)$$

Extension Principle

A more general version was described in[12]. In this version, we have

$$Y = f(X)$$

$$R(X): g(X) \text{ is } A \text{ (constraint on } u \text{ is } \mu_A(g(u)))$$

$$R(Y): \mu_Y(v) = \sup_u \mu_A(g(u))$$

subject to

$$v = f(u)$$

For a function with two arguments, the extension principle reads:

$$Z = f(X, Y)$$

$$R(X): g(X) \text{ is } A \text{ (constraint on } v \text{ is } \mu_A(g(u)))$$

$$R(Y): h(Y) \text{ is } B \text{ (constraint on } v \text{ is } \mu_B(h(u)))$$

$$R(Z): \mu_Z(w) = \sup_{u,v} (\mu_X(g(u)) \wedge \mu_Y(h(v))), \wedge = \min$$

subject to

$$w = f(u, v)$$

III. Intuitionistic Z-number

Definition

An ordered pair (A,B) is said to be a intuitionistic Z-number (IFS Z-number) if A is a fuzzy set defined on the real line and B is a intuitionistic fuzzy set defined on the the interval [0,1].

Z-valuation of IFS Z-number

Let X be a real random variable and (A,B) be an IFS Z-number. Let A(x) be the membership function of A. Let B(x) be the membership function of B, and b(x) be the non-membership function of B .Then (the ordered triple (X,A,B) is a Z-valuation which is equivalent to the following statement:

FEP(X is A) is B

More explicitly we may write:

Possibility(FEP(X is A) =u) = B(u)

Impossibility(FEP(X is A) = u) =b(u)

IV. Operations on IFS Z-numbers

Let X,Y be random variables.Given the IFS valued Z-valuations (X,A,B) and (Y,E,F) and an operation * the problem is to determine the Z-valuation of X * Y is as follows:

Let p(x) and q(x) be the underlying probability density functions of X and Y respectively. The known information may be summarized as

$$\mu_B(\int A(x)p(x)dx = u) = B(u)$$

$$\gamma_B(\int A(x)p(x)dx = u) = b(u)$$

$$\mu_F(\int E(x)q(x)dx = u) = F(u)$$

$$\gamma_F(\int E(x)q(x)dx = u) = f(u)$$

Notice that p,q are not known. Also $0 \leq B(u) + b(u) \leq 1$ and $0 \leq F(u) + f(u) \leq 1$.

Let us denote the IFS valued Z-valuation of W by (W,G,H), and the underlying probability density function of W by p_w . The membership function of G is determined by application of Extension Principle:

$$G(w) = \sup_{u,v} (A(u) \wedge E(v))$$

where the supremum is taken over all u,v such that $w = u * v$.

The next step is to determine the possibility distribution of p_w .Note that

$$p_w(t) = \sum_{t=u*v} p(u)q(v)$$

We shall denote the above equation by $p_w = p * q$. By applying the extension principle we have

$$\mu_{p_w}(p_w) = \sup_{\{p,q/p_w=p*q\}} (B(\int A(u)p(u)du) \wedge F(\int E(v)q(v)dv))$$

Also we have

$$\gamma_{p_w}(p_w) = \inf_{\{p,q/p_w=p*q\}} (b(\int A(u)p(u)du) \vee f(\int E(v)q(v)dv))$$

It is clear that $0 \leq \mu_{p_w}(p_w), \gamma_{p_w}(p_w)$.

Let $R(p) = \int A(u)p(u)du$ and $S(q) = \int E(v)q(v)dv$.

By the definition of an IFS we have

$$B(R(p)) + b(R(p)) \leq 1$$

$$F(S(q)) + f(S(q)) \leq 1$$

Without loss of generality we may assume $b(R(p)) \vee f(S(q)) = b(R(p))$

Also we have $B(R(p)) \wedge F(S(q)) \leq B(R(p))$

So $b(R(p)) \vee f(S(q)) + B(R(p)) \wedge F(S(q)) \leq b(R(p)) + B(R(p)) \leq 1$

$\inf_{\{p,q/p_w=p*q\}} (b(R(p)) \vee f(S(q))) \leq b(R(p_0)) \vee f(S(q_0)) \leq 1 - \{B(R(p_0)) \wedge F(S(q_0))\}$ for all p_0, q_0 .

Therefore $\{B(R(p_0)) \wedge F(S(q_0))\} \leq 1 - \inf_{\{p,q/p_w=p*q\}} (b(R(p)) \vee f(S(q)))$

So

$$\sup_{\{p_0,q_0/p_w=p_0*q_0\}} \{B(R(p_0)) \wedge F(S(q_0))\} \leq 1 - \inf_{\{p,q/p_w=p*q\}} \{b(R(p)) \vee f(S(q))\}$$

$$\mu_{p_w}(p_w) \leq 1 - \gamma_{p_w}(p_w)$$

$$\text{That is } \mu_{p_w}(p_w) + \gamma_{p_w}(p_w) \leq 1$$

This inequality proves that μ_{p_w} and γ_{p_w} satisfy the requirement to be the membership and non-membership functions of an IFS.

H-the membership function and h-the non-membership function of the intuitionistic fuzzy set H are determined to be

$$H(t) = \sup_{p_w/t = \int p_w(s)G(s)ds} (\mu_{p_w}(p_w))$$

where the supremum is taken over the set $\{p_w/t = \int p_w(s)G(s)ds\}$

$$h(t) = \inf_{p_w/t = \int p_w(s)G(s)ds} (\gamma_{p_w}(p_w))$$

where the supremum is taken over the set $\{p_w/t = \int p_w(s)G(s)ds\}$

Again it is clear that $h(t) \geq 0, H(t) \geq 0$.

Using

$$\mu_{p_w}(p_w) + \gamma_{p_w}(p_w) \leq 1$$

we get $h(t) = \inf_{p_w/t = \int p_w(s)G(s)ds} (\gamma_{p_w}(p_w)) \leq \gamma_{p_w}(p_{w_0})$ for all p_{w_0}

$$\leq 1 - \mu_{p_w}(p_{w_0})$$

That is $\mu_{p_w}(p_{w_0}) \leq 1 - \inf_{p_w/t = \int p_w(s)G(s)ds} (\gamma_{p_w}(p_w))$ for all p_{w_0}

So $H(t) = \sup_{p_w/t = \int p_w(s)G(s)ds} (\mu_{p_w}(p_w)) \leq 1 - \inf_{p_w/t = \int p_w(s)G(s)ds} (\gamma_{p_w}(p_w)) = 1 - h(t)$

That is $H(t) + h(t) \leq 1$

Hence the pair of functions H(t), h(t) define an IFS. This concludes the theoretical procedure for computing the IFS valued Z-valuation of $W = X * Y$.

V. Conclusion

In this paper we have introduced a novel type of Zadeh's Z-number. The procedure for computation with IFS Z-numbers has also been given.

VI. References

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