



EXACT SOLUTION OF MIXED CONVECTIVE HEAT AND MASS TRANSFER FLOW ALONG AN INFINITE POROUS PLATE EMBEDDED IN A POROUS MEDIUM

K.K.Tiwari¹ Aarti Tiwari²

¹Deptt. of Basic Sciences, S.R.M.S. College of Engg. & Tech., Bareilly (U.P.), India

²Deptt. of Basic Sciences, S.R.M.S. Women's College of Engg. & Tech., Bareilly (U.P.), India

Abstract: We analyze the unsteady laminar three-dimensional flow with mass and heat transfer of a viscous incompressible fluid having foreign mass along an infinite vertical porous plate in a porous medium. The vertical plate is assumed to be porous and is subjected to a periodic suction and oscillating free stream. The governing equations are solved using separation of variables and the successive perturbation techniques. The effect of the various physical parameters on the velocity, temperature and mass concentration distributions are discussed.

Keywords: MHD convection, laminar flow, heat transfer, mass transfer, perturbation, skin function

I. Introduction

The problem of laminar flow control has interesting applications in the field of aeronautical engineering. Various theoretical and experimental studies of various arrangements of suction holes were considered. The effects of transverse sinusoidal suction on the steady flow past a porous flat plate were considered by Gersten and Gross [1]. Soundalgekar and Wavre [2] investigated the two-dimensional transient free convection flow in the presence of foreign mass, past an infinite vertical porous plate with oscillating wall temperature and constant suction. Soundalgekar [3] investigated the oscillating flow past an infinite vertical porous plate with mass transfer. Bansal et al. [4] have studied heat, mass and momentum transfer in unsteady hydromagnetic free convection flow past an accelerated vertical plate. Singh et al. [5] have studied the effect of mass transfer on three-dimensional unsteady forced and free convective flow past an infinite vertical plate with periodic suction. Sharma and Mishra [6] investigated the effect of mass transfer on unsteady MHD flow and heat transfer past an infinite porous vertical moving plate. Abdel-Naby et al. [7] studied the radiation effects on MHD unsteady free convection flow over vertical plate with variable surface temperature and obtained a numerical solution for the governing equations. Sarangi and Jose [8] studied the unsteady MHD free convection flow and mass transfer through porous medium with variable suction but with constant heat flux. Zakaria [9] considered the thermal boundary layer equation for a MHD flow of a perfectly conducting fluid. In the present paper, the effect of porosity of the medium on three-dimensional unsteady laminar flow heat and mass transfer, of a viscous incompressible fluid with foreign mass along an infinite vertical porous plate with periodic suction and oscillating free stream is investigated. The flow in the porous medium deals with the analysis in which the differential equation governing the fluid motion is based on the Darcy's law which accounts for the drag exerted by the porous medium [10].

II-Problem Formulation

The unsteady flow of an incompressible viscous fluid with foreign mass along an infinite porous plate lying vertically on the xz -plane in such a way that the x -axis is oriented in the direction of the flow in the upwards direction, and the y -axis is perpendicular to the plate with oscillatory free stream velocity. The temperature of the plate is assumed to vary with time periodically. The concentration level of the foreign mass is considered to be very small. The flow in the porous medium deals with the analysis in which the differential equation governing the fluid motion is based on the Darcy's law which accounts for the drag exerted by the porous medium [10]. The constant mean suction velocity is treated as the same for the fluid and the species inside it. Since the plate is infinite in the x -direction therefore all the physical quantities are not varying with x , however the flow remains three-dimensional due to the variable suction velocity. Considering the usual Boussinesq approximation, the flow is governed by the following equations [7-9]

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

$$\rho \left(\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = g(\rho_\infty - \rho) + \mu \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) - \frac{\mu}{K} (u - U(t)) \quad (2)$$

$$\rho \left(\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) - \frac{\mu}{K} v \quad (3)$$

$$\rho \left(\frac{\partial w}{\partial t} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) - \frac{\mu}{K} w \quad (4)$$

$$\begin{aligned} \rho c_p \left(\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) &= k \left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \\ &+ \mu \left(2 \left(\left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right) + \left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right)^2 \right) \end{aligned} \quad (5)$$

$$\frac{\partial \psi}{\partial t} + v \frac{\partial \psi}{\partial y} + w \frac{\partial \psi}{\partial z} = D \left(\frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right) \quad (6)$$

where t is the time, u, v, w are the velocity components in the x, y, z directions, respectively, $U(t)$ is the free stream velocity, p is the fluid pressure, T is the temperature of the fluid, ρ is the density of the fluid, ρ_∞ is the density of the fluid in free stream, μ is the coefficient of viscosity, c_p is the specific heat at constant pressure, k is the thermal conductivity, ψ is the concentration of foreign mass species, D is the diffusion coefficient, and K is the Darcy permeability. The last term in the right side of each of eqs. (2), (3) and (4) represents the porosity force.

The boundary conditions are given by

$$y = 0: u = 0, v = -v_0 \left(1 + \varepsilon \cos \frac{\pi v_0 z}{v} \right), w = 0, T = T_0 + \varepsilon(T_0 - T_\infty)e^{i\omega t}, \psi = \psi_w \quad (7a)$$

$$y \rightarrow \infty: u \rightarrow U(t) = U_0(1 + \varepsilon e^{i\omega t}), w \rightarrow 0, p \rightarrow p_\infty, T \rightarrow T_\infty, \psi \rightarrow \psi_\infty \quad (7b)$$

where v is the kinematic viscosity, T_0 is the characteristic temperature, T_∞ is the free stream temperature, v_0 is the constant mean suction velocity, U_0 is the mean part of the free stream velocity, ω is the frequency, ψ_w is the concentration of foreign mass species at the plate and ε is very small parameter ($0 < \varepsilon < 1$).

III-Exact Solution

For $\beta\Delta T < 1$ and $\beta\Delta C < 1$, we can determine $(\rho_\infty - \rho)$ in terms of $(T - T_\infty)$ and $(\psi - \psi_\infty)$ as follows

$$g(\rho_\infty - \rho) = g\beta\rho(T - T_\infty) + g\beta^*\rho(\psi - \psi_\infty) \quad (8)$$

where β is the volumetric coefficient of thermal expansion and β^* is the volumetric coefficient of expansion with concentration. Introducing the following non-dimensional quantities [7-9]

$$\bar{y} = \frac{v_0 y}{v}, \bar{z} = \frac{v_0 z}{v}, \bar{t} = \frac{v_0^2 t}{4v}, \bar{u} = \frac{u}{U_0}, \bar{v} = \frac{v}{U_0}, \bar{w} = \frac{w}{U_0}, \bar{\omega} = \frac{4v\omega}{v_0^2} \quad (9a)$$

$$\bar{T} = \frac{T - T_\infty}{T_0 - T_\infty}, \bar{\psi} = \frac{\psi - \psi_\infty}{\psi_w - \psi_\infty}, \bar{p} = \frac{p}{\rho v_0^2}, \bar{P}_r = \frac{\mu c_p}{k}, \bar{S}_c = \frac{v}{D} \quad (9b)$$

$$\bar{G}_r = \frac{v g \beta (T_0 - T_\infty)}{U_0 v_0^2}, \bar{\lambda} = \frac{v_0}{U_0}, \bar{G}_c = \frac{v g \beta^* (\psi_w - \psi_\infty)}{v_0^2 U_0}, \bar{E}_c = \frac{U_0^2}{c_p (T_0 - T_\infty)}, \bar{\gamma} = \frac{v^2}{v_0^2 K} \quad (9c)$$

and substituting eqs. (8) and (9) into eqs. (1) to (7), we get (The bars will be dropped for convenience)

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (10)$$

$$\frac{1}{4} \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \bar{G}_r T + \bar{G}_c \psi + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} - \gamma(u - 1 - \varepsilon e^{i\omega t}) \quad (11)$$

$$\frac{1}{4} \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} - \gamma v \quad (12)$$

$$\frac{1}{4} \frac{\partial w}{\partial t} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} - \gamma w \quad (13)$$

$$\begin{aligned} \bar{P}_r \left(\frac{1}{4} \frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) &= \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + 2\bar{P}_r \bar{\lambda}^2 \bar{E}_c \left(\left(\frac{\partial^2 v}{\partial y^2} \right)^2 + \left(\frac{\partial^2 w}{\partial z^2} \right)^2 \right) + \bar{P}_r \bar{E}_c \left(\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial u}{\partial z} \right)^2 \right) \\ &+ \bar{P}_r \bar{\lambda}^2 \bar{E}_c \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right)^2 \end{aligned} \quad (14)$$

$$S_c \left(\frac{1}{4} \frac{\partial \psi}{\partial t} + v \frac{\partial \psi}{\partial y} + w \frac{\partial \psi}{\partial z} \right) = \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \quad (15)$$

$$y = 0: u = 0, v = -(1 + \varepsilon \cos \pi z), w = 0, T = 1 + \varepsilon e^{i\omega t}, \psi = 1 \quad (16a)$$

$$y \rightarrow \infty: u \rightarrow 1 + \varepsilon e^{i\omega t}, w \rightarrow 0, p \rightarrow p_\infty, T \rightarrow 0, \psi \rightarrow 0 \quad (16b)$$

where G_r is the Grashoff number, G_c is the modified Grashoff number, E_c is the Eckert number, P_r is the Prandtl number, S_c is the Schmidt number, λ is the suction parameter and γ is the porosity parameter.

Following the procedures discussed in the literature in treating similar problems [7-9]. We assume a perturbed solution in the neighbourhood of the plate in the form

$$H(y, z, t) = H_0(y) + \varepsilon H_1(y, z, t) + O(\varepsilon^2) \quad (17)$$

where H stands for any of u, v, w, p, T or ψ .

Skin-friction coefficient at the plate is given by

$$\tau_{fx} = \frac{\nu}{\mu U_0 v_0} (\tau_{xy})_{y=0} = (P(y) + iQ(y))_{y=0} \quad (18)$$

where

$$\begin{aligned} P(y) = & u'_{00}(y) + E_c u'_{01}(y) \\ & + \varepsilon \left((H'_1(y) + E_c H'_3(y)) \cos \omega t \right. \\ & \left. - \sin \omega t (H'_2(y) + E_c H'_4(y)) \{u'_{120}(y) + E_c u'_{121}(y)\} \cos \pi z \right) \end{aligned} \quad (19)$$

$$Q(y) = \varepsilon \left(\sin \omega t (H'_1(y) + E_c H'_3(y)) + \cos \omega t (H'_2(y) + E_c H'_4(y)) \right) \quad (20)$$

Then, the skin-friction coefficient at the plate is given by

$$\tau_f = |\tau_{fx}| = 1/\sqrt{P^2(0) + Q^2(0)} \quad (21)$$

The rate of heat transfer in terms of the Nusselt number at the plate is given by

$$\begin{aligned} N_u = & \frac{\nu q}{k v_0 (T_0 - T_\infty)} \\ = & T'_{00}(0) + E_c T'_{01}(0) + \varepsilon (H'_7(0) \cos \omega t - H'_8(0) \sin \omega t) + \varepsilon (T'_{120}(0) + E_c T'_{121}(0)) \cos \pi z \\ & + i\varepsilon (H'_7(0) \sin \omega t - H'_8(0) \cos \omega t) \end{aligned} \quad (22)$$

The rate of mass transfer in terms of the Stanton number St_m at the plate is given by

$$St_m = \frac{D}{v_0(\psi_w - \psi_\infty)} \left(\frac{\partial \psi}{\partial y} \right)_{y=0} = -1 - \frac{\varepsilon}{S_c} (a_3 I_{19} + (a_2 + S_c) I_{20} + (\pi + S_c) I_{21}) \cos \pi z \quad (23)$$

References

- [1] Gersten, K. and Gross, J. F., "Flow and Heat Transfer along a Plane Wall with Periodic Suction," *ZAMP*, Vol. 25, pp. 399-408 (1974).
- [2] Soundalgekar, V. M. and Wavre, P. D., "Unsteady Free Convection Flow Past an Infinite Vertical Plate with Variable Suction and Mass Transfer," *Int. J. Heat Mass Transfer*, Vol. 20, pp. 1375-1380 (1977).
- [3] Soundalgekar, V.M. and Wavre, P. D., "Unsteady Free Convection Flow Past an Infinite Vertical Plate with Constant Suction and Mass Transfer," *Int. J. Heat Mass Transfer*, Vol. 20, pp. 1363-1374 (1977).
- [4] Bansal, J. L., Dave, A. and Jat, R. N., "Heat, Mass and Momentum Transfer in Unsteady Hydromagnetic Free Convection Flow Past an Accelerated Vertical Plate," *Proc. Natn. Acad. Sci. India*, Vol. 60(A), pp. 211-226 (1990).
- [5] Singh, K. D., Verma, G. N. and Kumar, S., "Effect of Mass Transfer on Three-Dimensional Unsteady Forced and Free Convective Flow Past an Infinite Vertical Plate with Periodic Suction," *Proc. Natn. Acad. Sci. India*, Vol. 65(A), pp. 293-308 (1995).
- [6] Sharma, P. R. and Mishra, U., "Effects of Mass Transfer on Unsteady MHD Flow and Heat Transfer Past an Infinite Porous Vertical Moving Plate," *Ind. J. Theo. Phys.*, Vol. 50, pp. 109-115 (2002).
- [7] Abdel-Naby, M. A., "Finite Difference Solution of Radiation Effects on MHD Unsteady Free Convection Flow over Vertical Plate with Variable Surface Temperature," *JAM*, Vol. 2, pp. 65-86 (2003).
- [8] Sarangi, K. C. and Jose, C. B., "Unsteady MHD Free Convection Flow and Mass Transfer through Porous Medium with Variable Suction and Constant Heat Flux," *J. Indian Acad. Math.*, Vol. 26, pp. 115-126 (2004).
- [9] Zakaria, M., "Thermal Boundary Layer Equation for MHD Flow of a Perfectly Conducting Fluid," *J. Appl. Math. Comput.*, Vol. 148, pp. 67-79 (2004).
- [10] Khaled, A. R. A. and Vafai, K., "The Role of Porous Media in Modeling Flow and Heat Transfer in Biological Tissues," *Int. J. Heat Mass Transf.*, Vol. 46, pp. 4989-5003 (2003).